Dr. Niels Lindner
ZIB 3007 (Rundbau)
lindner@zib.de

# Problem Set 5

due: May 28, 2018

## Exercise 1 **5 points**

Fix $T \in \mathbb{N}$. For $x \in \mathbb{R}$, define $[x]_T$ as the unique $y \in [0, T)$ with $x \equiv y \mod T$. Show that:

(a) $[x]_T = x - T\lfloor x/T \rfloor$ for all $x \in \mathbb{R}$,

(b) $[[x]_T]_T = [x]_T$ for all $x \in \mathbb{R}$,

(c) $[x + nT]_T = [x]_T$ for all $n \in \mathbb{Z}$, $x \in \mathbb{R}$,

(d) $[x + y]_T \leq [x]_T + [y]_T$ for all $x, y \in \mathbb{R}$,

(e) $0 \leq \sum_{i=1}^n [x_i]_T - \left[\sum_{i=1}^n x_i\right]_T \leq (n-1)T$ for all $n \in \mathbb{N}$, $x_1, \ldots, x_n \in \mathbb{R}$.

## Exercise 2 **5 points**

Let $\mathcal{E} = (V, E)$ be an event-activity network. Suppose that

(1) $V = V_{\text{arr}} \,\dot\cup\, V_{\text{dep}}$ and $\#V_{\text{arr}} = \#V_{\text{dep}}$,

(2) $\#\delta^-(v) = 1$ for all $v \in V_{\text{arr}}$ and $\#\delta^+(v) = 1$ for all $v \in V_{\text{dep}}$.

Let $E_t := E \cap (V_{\text{arr}} \times V_{\text{dep}})$. Moreover, define a *circulation* in $\mathcal{E}$ as a disjoint union of directed circuits in $\mathcal{E}$. Prove that there is a bijective map

$$\{\text{perfect matchings of } (V, E_t)\} \to \{\text{circulations in } (V, E) \text{ visiting each vertex } v \in V\}.$$

## Exercise 3 **3 points**

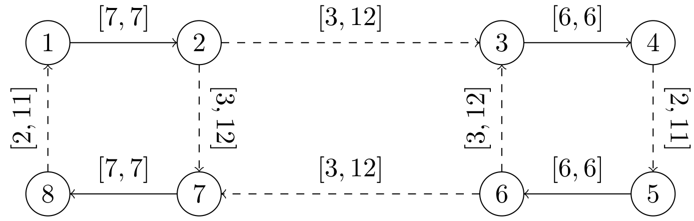
Find a $2 \times 2$-matrix $A$ over $\mathbb{Z}/10\mathbb{Z}$ such that $\#\{x \in (\mathbb{Z}/10\mathbb{Z})^2 \mid Ax = 0\} = 2$.

**Please turn over!**

Dr. Niels Lindner
ZIB 3007 (Rundbau)
lindner@zib.de

## Exercise 4 **7 points**

Let $\mathcal{E} = (V, E)$ denote the following event-activity network with period time $T = 10$:




Driving activities have a solid line, transfer activities are dashed. The lower bounds $\ell$ and upper bounds $u$ on each activity are indicated as an interval $[\ell_e, u_e]$.

(a) Compute all solutions $\pi \in (\mathbb{Z}/10\mathbb{Z})^8$ satisfying

$$\pi_j - \pi_i \equiv_{10} \ell_{ij} \quad \text{for all driving activities } ij \in E.$$

(b) Compute an optimal solution to the PESP on $\mathcal{E}$, where each transfer activity has weight 1 and each driving activity has weight 0.