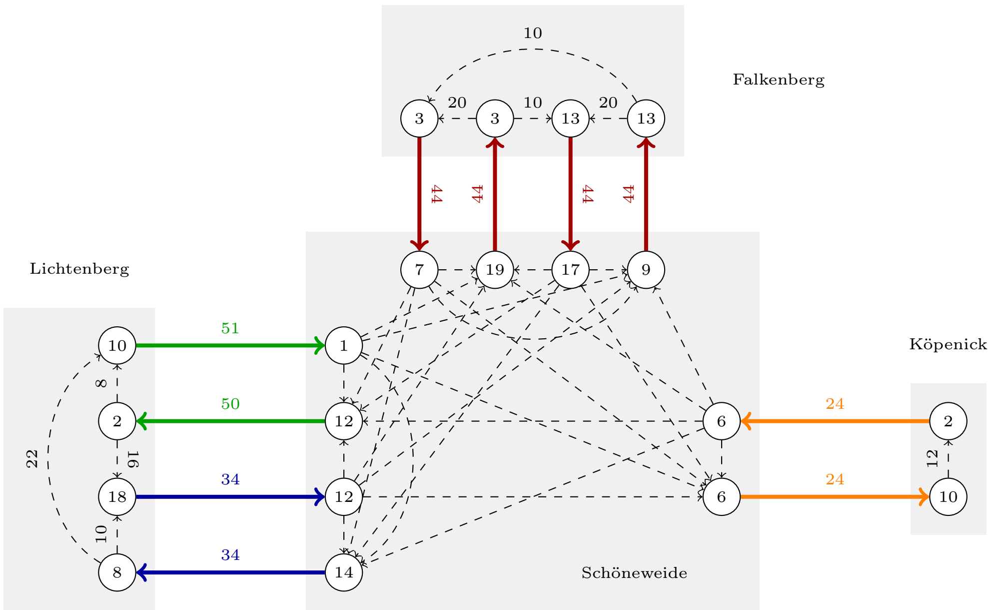
Dr. Niels Lindner
ZIB 3007 (Rundbau)
lindner@zib.de

Problem Set 9

due: June 25, 2018

Exercise 1 **10 points**

Consider the following event-activity network $\mathcal{E} = (V, E)$ with period time $T = 20$, periodic timetable $\pi : V \to [0, 20)$ and tensions $x : E \to \mathbb{R}_{\geq 0}$:



The set E decomposes into driving activities E_d (bold and colored), and turnaround activities E_t (dashed). The turnaround activities at a stop (gray box) with n arrival events and n departure events form a complete bipartite graph $K_{n,n}$ with n^2 edges. For each turnaround activity $ij \in E$ holds $x_{ij} = [\pi_j - \pi_i - 4]_{20} + 4 \in [4, 24)$. Recall that each activity $ij \in E$ carries a *periodic offset* $p_{ij} := (x_{ij} - \pi_j + \pi_i)/T$.

Solve the periodic vehicle scheduling problem on $\mathcal{E}$:

(a) Compute a minimum cost circulation (w.r.t. x or p) covering each driving activity exactly once.

(b) Compute a minimum-weight perfect matching (w.r.t. x or p) of the subgraph (V, E_t).

(c) How many vehicles does an optimal periodic vehicle schedule require?

(d) Find the number of distinct periodic vehicle schedules in $\mathcal{E}$.

Dr. Niels Lindner
ZIB 3007 (Rundbau)
lindner@zib.de

Exercise 2 **10 points**

Consider the 7×8-matrix

$$A := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Why is this matrix totally unimodular?

(b) Denote by I the 8×8-identity matrix. Show that

$$\begin{pmatrix} A & 0 \\ 0 & A \\ I & I \end{pmatrix}$$

is not totally unimodular by explicitly giving a submatrix B with $\det(B) \notin \{-1, 0, 1\}$.

(c) Give an example of a minimum cost 2-commodity flow problem P on a digraph G with integer cost, balance and capacity functions with the property that P has a non-integer optimal solution.