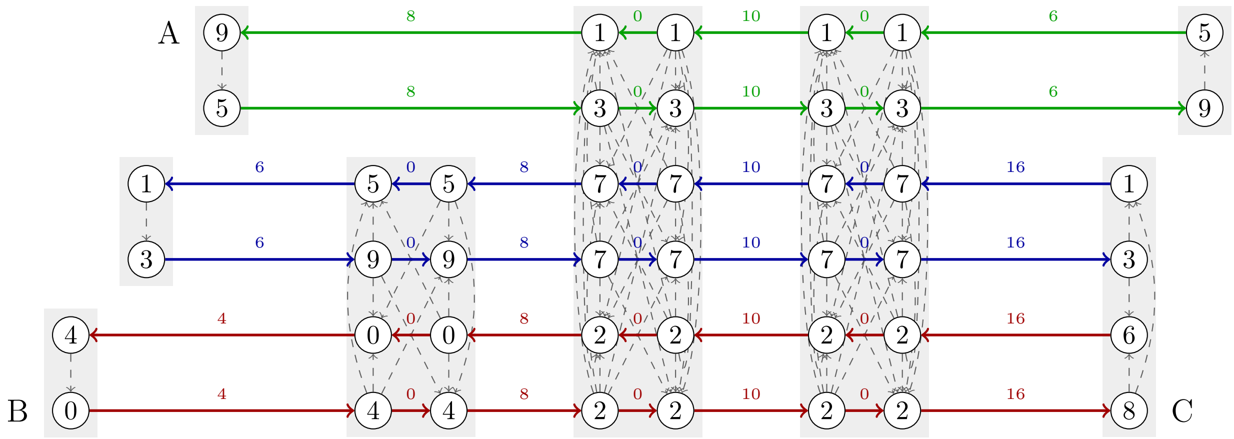
Dr. Niels Lindner
ZIB 3007 (Rundbau)
lindner@zib.de

Test Exam

discussed on July 16, 2018

Exercise 1 **15 points**

Consider the following event-activity network $\mathcal{E}$ with period time $T = 10$, periodic timetable $\pi : V(\mathcal{E}) \to \{0, 1, \ldots, 9\}$, and periodic tensions $x : E(\mathcal{E}) \to \mathbb{N}_0$:



The solid activities are driving and waiting activities of three lines (green, blue, and red). All dashed activities are transfer activities. The periodic tension x_{ij} of any activity $ij \in \mathcal{E}(E)$ is given by

$$x_{ij} \begin{cases} > 0 & \text{if } ij \text{ is a driving activity,} \\ = 0 & \text{if } ij \text{ is a waiting activity,} \\ = [\pi_j - \pi_i - 2]_{10} + 2 & \text{if } ij \text{ is a transfer activity.} \end{cases}$$

(a) Is the timetable π symmetric?

(b) Suppose that the timetable is operated from 7.00 to 24.00, repeating every 10 minutes. Starting 8.08 at A, what is the earliest arrival time at B?

Now consider transfer activities as turnaround activities for vehicles.

(c) If every vehicle is only allowed to run on a single line, how many vehicles are required to operate the timetable π?

(d) Does permitting turnarounds from red to blue or vice versa at stop C decrease the number of required vehicles?

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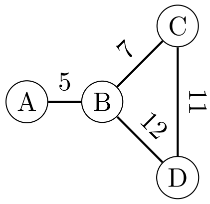
Exercise 2 **5 points**

Consider the following undirected graph G with cost function $c : E(G) \to \mathbb{R}_{\geq 0}$:



Compute a shortest TSP tour visiting each vertex at least once.

Exercise 3 **5 points**

Fix $T \in \mathbb{N}$. For $a, b \in \mathbb{R}$ with $a \leq b$ define

$$[a, b]_T := \bigcup_{k \in \mathbb{Z}} [a + kT, b + kT].$$

(a) Let $\ell_1, \ell_2, u_1, u_2 \in \mathbb{R}$ such that $0 \leq \ell_1 \leq u_2 \leq \ell_2 \leq u_1 < T$. Show that

$$[\ell_1, u_1]_T \cap [\ell_2, u_2 + T]_T = [\ell_1, u_2]_T \cup [\ell_2, u_1]_T.$$

(b) Let i, j be events of an event-activity network $\mathcal{E}$. How can the periodic event scheduling constraint $\pi_j - \pi_i \in [1, 3]_{10} \cup [7, 9]_{10}$ be modeled in $\mathcal{E}$?

Exercise 4 **5 points**

Find two connected undirected graphs G and M with the following properties:

- G has an octilinear embedding into the plane,
- M cannot be octilinearly embedded into the plane,
- M is a minor of G.

Exercise 5 **5 points**

The *set cover problem* is the following: Given a finite non-empty set X, a collection $\mathcal{C}$ of subsets of X such that $\bigcup_{C \in \mathcal{C}} C = X$, and a natural number k, is there a subcollection $\mathcal{S} \subseteq \mathcal{C}$ of size $|\mathcal{S}| \leq k$ such that $X = \bigcup_{S \in \mathcal{S}} S$?

Construct a polynomial-time reduction from the set cover problem to the following cost-oriented line planning problem: Given a graph G, a line pool $\mathcal{L}_0$, and lower and upper frequency bounds $f^{\min} \leq f^{\max} : E(G) \to \mathbb{N}_0$, find a line plan $(\mathcal{L}, f)$ with the minimum number of lines subject to

$$f_e^{\min} \leq \sum_{\ell \in \mathcal{L} \colon e \in E(\ell)} f_\ell \leq f_e^{\max} \quad \text{for all } e \in E(G).$$