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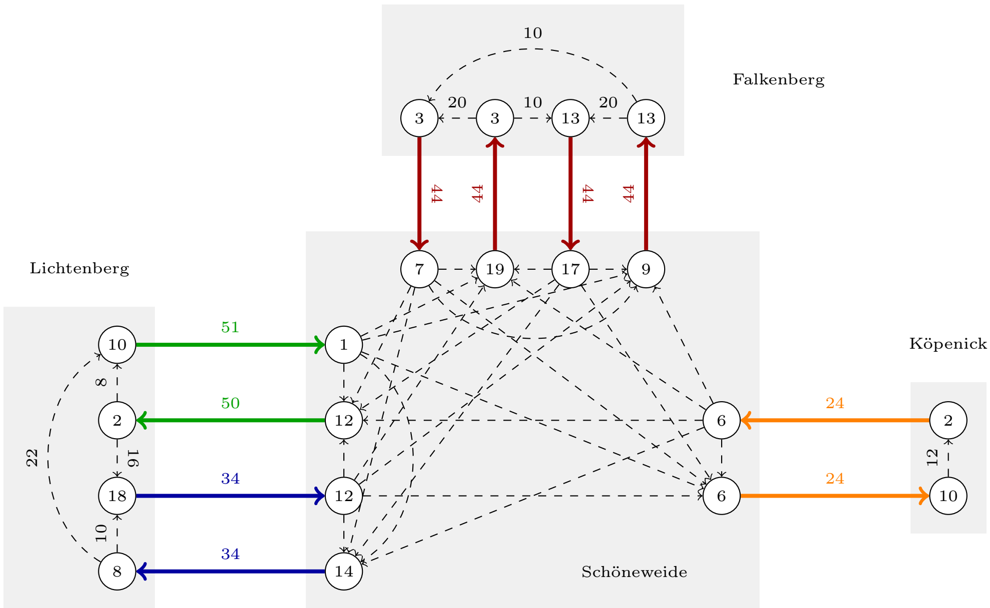
Problem Set 12

due: January 20, 2020

Exercise 1 **10 points**

Consider the following event-activity network $\mathcal{N} = (V, E)$ with period time $T = 20$, periodic timetable $\pi \in [0, 20)^V$ and activity durations $x \in \mathbb{R}^E_{\geq 0}$:



The set E decomposes into driving activities E_d (bold and colored), and turnaround activities E_t (dashed). The turnaround activities at a stop (gray box) with n arrival events and n departure events form a complete bipartite graph $K_{n,n}$ with n^2 edges. For each turnaround activity $ij \in E$ holds $x_{ij} = [\pi_j - \pi_i - 4]_{20} + 4 \in [4, 24)$. Recall that each activity $ij \in E$ carries a *periodic offset* $p_{ij} := (x_{ij} - \pi_j + \pi_i)/T$.

Solve the periodic vehicle scheduling problem on $\mathcal{N}$:

(a) Compute a minimum-weight perfect matching (w.r.t. x or p) of the subgraph (V, E_t).

(b) Deduce from (a) a minimum cost circulation (w.r.t. x or p) covering each driving activity exactly once.

(c) How many vehicles does an optimal periodic vehicle schedule require?

(d) Find the number of distinct periodic vehicle schedules in $\mathcal{N}$.

Exercise 2 **5 points**

Let $\mathcal{N} = (V, E)$ be an event-activity network. Suppose that

(1) $V = V_{\text{arr}} \,\dot\cup\, V_{\text{dep}}$ and $\#V_{\text{arr}} = \#V_{\text{dep}}$,

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(2) $E = E_d \,\dot{\cup}\, E_t$, $E_d \subseteq V_{\text{dep}} \times V_{\text{arr}}$ and $E_t \subseteq V_{\text{arr}} \times V_{\text{dep}}$,

(3) $\#\delta^-(v) = 1$ for all $v \in V_{\text{arr}}$ and $\#\delta^+(v) = 1$ for all $v \in V_{\text{dep}}$.

Define a *circulation* in $\mathcal{N}$ as a (vertex-)disjoint union of directed circuits in $\mathcal{N}$. Prove that

$$\{\text{circulations in } (V,E) \text{ containing each } e \in E_d\} \to \{\text{perfect matchings in } (V, E_t)\}$$
$$(\gamma_e)_{e \in E} \mapsto (\gamma_e)_{e \in E_t}$$

is a bijective map.

Exercise 3

5 points

Let $\mathcal{N} = (V, E)$ be an event-activity network. For a period time $T \in \mathbb{N}$, let $\pi \in [0, T)^V$ be a periodic timetable with activity durations $x \in \mathbb{R}^E$ such that $\pi_w - \pi_v \equiv x_{vw} \bmod T$ holds for all $vw \in E$. Suppose that $\gamma \in \mathbb{R}^E$ satisfies $\sum_{e \in \delta^-(v)} \gamma_e = \sum_{e \in \delta^+(v)} \gamma_e$ at every event $v \in V$. Show that $\gamma^t x \equiv 0 \bmod T$.