Dr. Niels Lindner (lindner@zib.de)
Pedro Maristany de las Casas (maristany@zib.de)

Problem Set 13

due: January 27, 2020

Exercise 1 — 5 points

Find an example of a periodic vehicle scheduling instance $(\mathcal{N}, T, \pi, x)$ and an optimal periodic vehicle schedule S_p such that for some optimal aperiodic vehicle schedule $S_{a,n}$ for $(\mathcal{T}_n, \preceq)$ as defined in the lecture holds $\nu(S_{a,n}) < \nu(S_p)$.

Exercise 2 — 10 points

The *Returning ATSP* is the following: Given a complete digraph K_n^* on n vertices with cost function $c \in \mathbb{R}^E_{\geq 0}$ and a distinguished vertex $v \in V(K_n^*)$, find a closed walk C in K_n^* satisfying all of the following properties:

1. C is of minimum cost w.r.t. c,
2. C visits each vertex in $V(K_n^*) \setminus \{v\}$ exactly once,
3. C visits v at least once.

(a) Prove that *Returning ATSP* is NP-complete.

(b) Construct a polynomial-time reduction from the *single-depot aperiodic vehicle scheduling problem* to *Returning ATSP*.

(c) Let $\pi : V(K_n^*) \to \mathbb{R}_{\geq 0}$ be a map with $\pi(v) = 0$. Suppose that

$$c_{ij} = \begin{cases} \pi(j) - \pi(i) & \text{if } \pi(j) > \pi(i) \text{ and } j \neq v, \\ \infty & \text{if } \pi(j) \leq \pi(i) \text{ and } j \neq v, \\ 0 & \text{if } j = v, \end{cases} \qquad \text{holds for all } ij \in E(K_n^*).$$

Subject to these restrictions, give a polynomial-time algorithm for *Returning ATSP*.

Exercise 3 — 5 points

Let $(\mathcal{T}, \preceq)$ be a single-depot aperiodic vehicle scheduling instance with optimal fleet size ν. Let $Q \subseteq \mathcal{T}$ be a subset of trips with the property that $t \in Q$ implies $t' \in Q$ for all trips $t' \in \mathcal{T}$ for which there is a chain $t' = t_1 \preceq \cdots \preceq t_r = t$. Define

$$X := \{p\} \cup \{d_t \mid t \in Q\} \cup \{a_t \mid t \in Q\} \subseteq V(\mathcal{N}(\mathcal{T}, \preceq)).$$

Prove that if f is an optimal p-q-flow in $\mathcal{N}(\mathcal{T}, \preceq)$ of value ν, then $\sum_{e \in \delta^+(X)} f_e = \nu$.