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Problem Set 4

due: November 11, 2019

Exercise 1 **8 points**

In the last exercise session we proved that the decision version of the *Resource Constrained Shortest Path Problem* is $\mathcal{NP}$-complete. However, instances of the problem where all arc weights are equal can be solved in polynomial time.

Unit-Weight Resource Constrained Shortest Path Problem
INSTANCE: Graph $G = (V, E)$, start and end nodes $s, t \in V$, edge lengths $l_e \in \mathbb{N}$ for $e \in E$, edge weight $w \in \mathbb{N}$ for all edges, and weight limit $W \in \mathbb{N}$.
TASK: Find a shortest (s, t)-path in G with respect to the edge lengths l_e such that the sum of the edge weights along the path does not exceed W.

Find a polynomial-time algorithm for the *Unit-Weight Resource Constrained Shortest Path Problem* and analyze its running time.

Exercise 2 **6 points**

Show that the decision version of the *Longest Circuit Problem* is $\mathcal{NP}$-complete.

Longest Circuit Problem
INSTANCE: Graph $G = (V, E)$, edge lengths $l_e \in \mathbb{N}$ for $e \in E$, threshold $L \in \mathbb{N}$.
QUESTION: Is there a circuit C in G such that the sum of the edge lengths in C is at least L?

Hint: For the reduction use the *Hamiltonian Circuit Problem* presented in the lecture.

Exercise 3 **6 points**

Let $G = (V, E)$ be a connected undirected graph. A *bridge* of G is an edge in E whose removal disconnects G. Let $T \subseteq V$ be the set of odd-degree vertices of G.
Prove that an edge $e \in E$ is a bridge if and only if e is contained in every T-join.