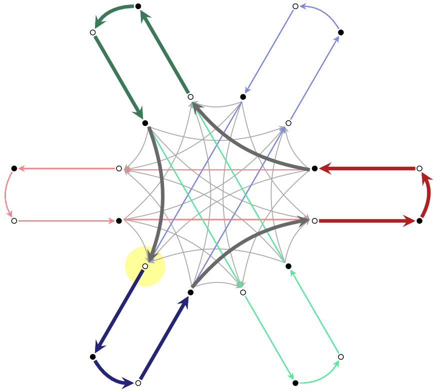

Forward Cycle Bases and Periodic Timetabling

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ATMOS 2021

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Periodic Event Scheduling Problem (PESP)

Given

$G = (V, A)$	event-activity network,
$T \in \mathbb{N}$	period time,
$\ell \in \mathbb{Z}^A$	lower bounds,
$u \in \mathbb{Z}^A$	upper bounds,
$w \in \mathbb{R}^A_{\geq 0}$	weights,

find

$\pi \in [0, T)^V$	periodic timetable,
$x \in \mathbb{R}^A$	periodic tension

such that

(1) $\pi_j - \pi_i \equiv x_{ij} \bmod T$ for all $ij \in A$,

(2) $\ell \leq x \leq u$,

(3) $w^\top x$ is minimum,

or decide that no such (π, x) exists.

(Serafini and Ukovich, 1989)

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Cycle-based MIP formulation:

(Nachtigall, 1994, Liebchen and Peeters, 2009)

$$\begin{aligned} \text{Minimize} \quad & w^\top x \\ \text{s.t.} \quad & \Gamma x = Tz, \\ & \ell \leq x \leq u, \\ & z \in \mathbb{Z}^B \end{aligned}$$

$B \subseteq \mathbb{Z}^A$	integral cycle basis of G
$\Gamma \in \mathbb{Z}^{B \times A}$	cycle matrix of B
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Cycle inequalities:

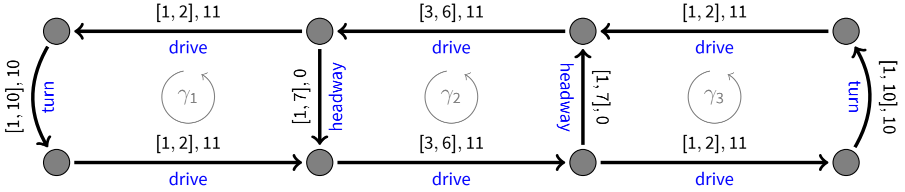
(Odijk, 1994)

$$\left\lceil \frac{\gamma_+^\top \ell - \gamma_-^\top u}{T} \right\rceil \leq \frac{\gamma^\top x}{T} \leq \left\lfloor \frac{\gamma_+^\top u - \gamma_-^\top \ell}{T} \right\rfloor$$

for each oriented cycle $\gamma \in \{-1, 0, 1\}^A$.

A Small PESP Instance: Cycle Inequalities

PESP instance with period time $T = 10$:



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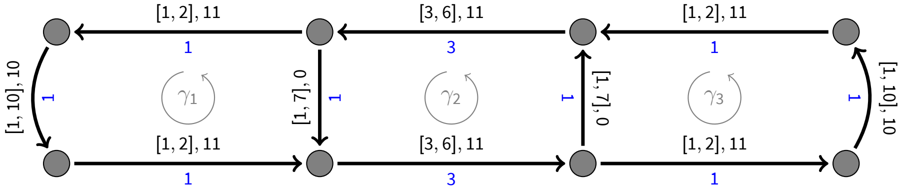

Relaxation	Optimal weighted slack
LP relaxation	0

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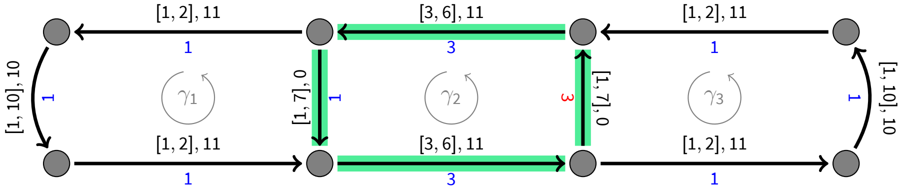

Relaxation		Optimal weighted slack
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$+\gamma_2^\top x \geq 10 \left\lceil \frac{3+1+3+1}{10} \right\rceil = 10$	$\gamma_2^\top x = 3 + 1 + 3 + 3 = 10$	0

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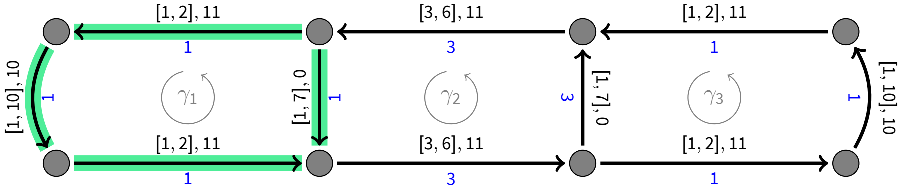

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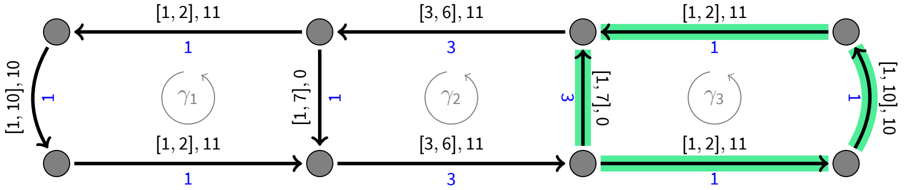

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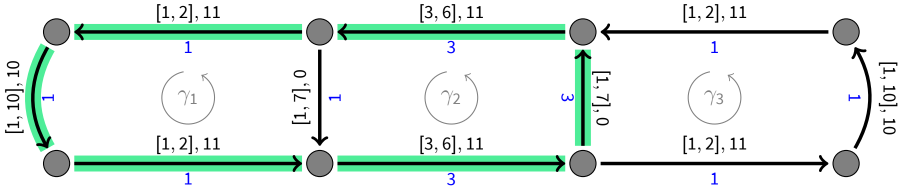

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$+\ (\gamma_1 + \gamma_2)^\top x \geq 10 \left\lceil \frac{10}{10} \right\rceil = 10$	$(\gamma_1 + \gamma_2)^\top x = 12$	0

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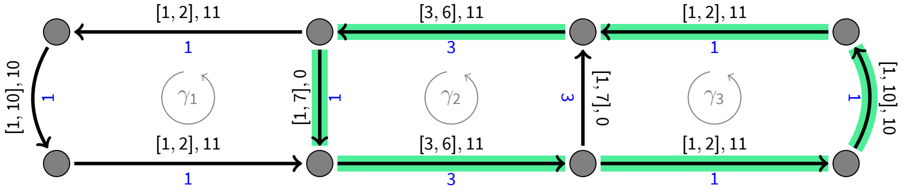

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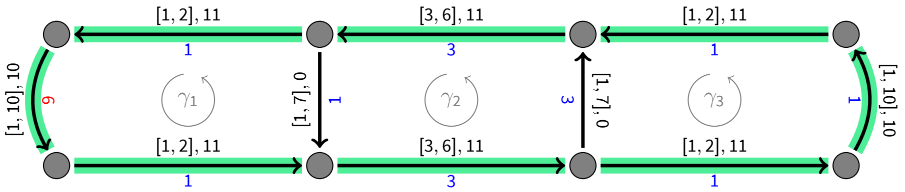

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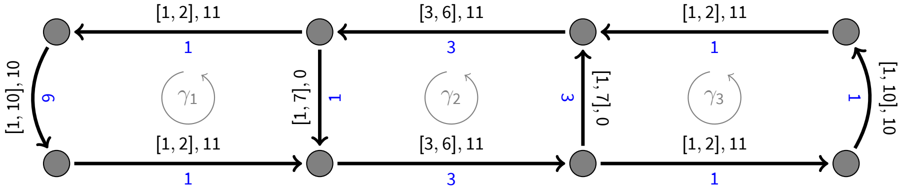

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PESP MIP		80

A Small PESP Instance: Conclusions

Observation

- Cycle inequalities derived from the planar cycle basis $\{\gamma_1, \gamma_2, \gamma_3\}$ are useless. This is also the integral cycle basis with minimum span $u - \ell$.
- The only contributing cycle inequalities come from the *forward* cycles γ_2 and $\gamma_1 + \gamma_2 + \gamma_3$.
- If the cycle basis contains the ”vehicle rotation” $\gamma_1 + \gamma_2 + \gamma_3$, then the LP relaxation closes the MIP optimality gap at the root node.
- $\gamma_1 + \gamma_2 + \gamma_3$ is the only cycle where are arcs have positive weight.

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Look for cycle bases consisting of forward or heavy-weight cycles.

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Some Benefits of Forward Cycles

- cycle inequalities $=$ change-cycle inequalities.
- increasing the modulo parameters correlates with increasing objective value

Cycle Space and Cycle Bases

Let $G = (V, A)$ be a digraph.

Cycle space:

$$\mathcal{C} := \left\{ \gamma \in \mathbb{Z}^A \,\middle|\, \forall v \in V : \sum_{a \in \delta^+(v)} \gamma_a = \sum_{a \in \delta^-(v)} \gamma_a \right\} \quad \text{(abelian group)}$$

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(1)	B basis of $\mathbb{R}$-vector space $\mathcal{C} \otimes \mathbb{R}$	*directed* cycle basis
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Hierarchy: (5) $\Rightarrow$ (4) $\Rightarrow$ (3) $\Rightarrow$ (2) $\Rightarrow$ (1) (Kavitha et al., 2009)

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Forward cycle basis: cycle basis B consisting only of forward cycles

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Theorem (Seymour and Thomassen, 1987)

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Example: Non-Existence of Forward Strictly Fundamental Bases

Every digraph has a spanning forest, and hence a strictly fundamental cycle basis.
But: Not every strongly connected G has a *forward* strictly fundamental cycle basis.

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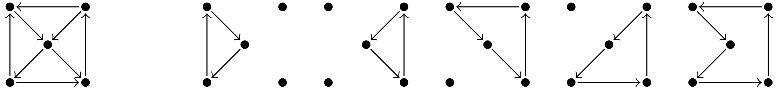

G directed Hamiltonian $\Rightarrow$ G strongly connected
no spanning tree with exclusively forward fundamental cycles
forward weakly fundamental cycle basis by first 4 cycles

A Standard Construction

Question

How can we ensure existence of forward *integral* cycle bases for PESP instances?

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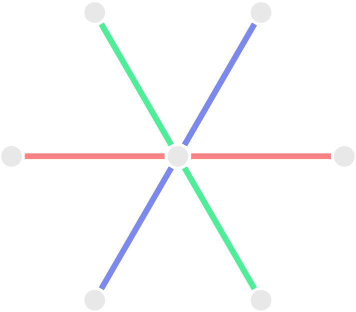
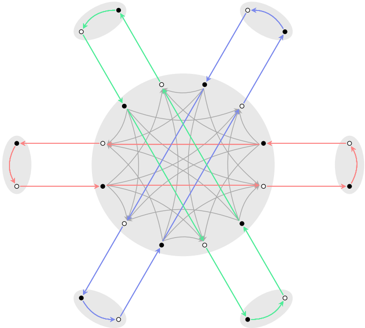
Line-Based Event-Activity Networks



line network
3 bidirectional lines

→



event-activity network
drive, dwell, turnaround, transfer activities

ILTY Cycles

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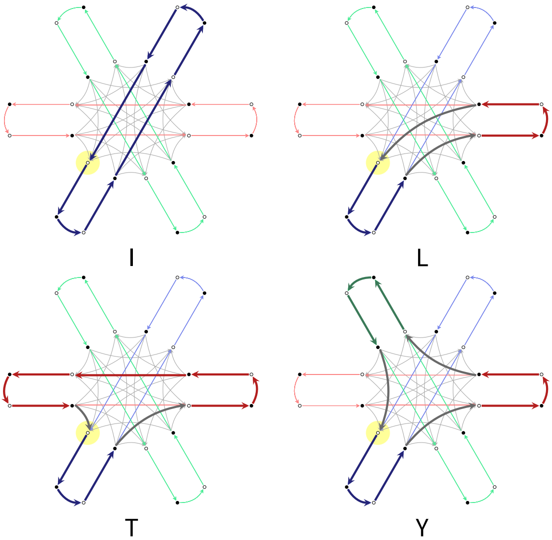
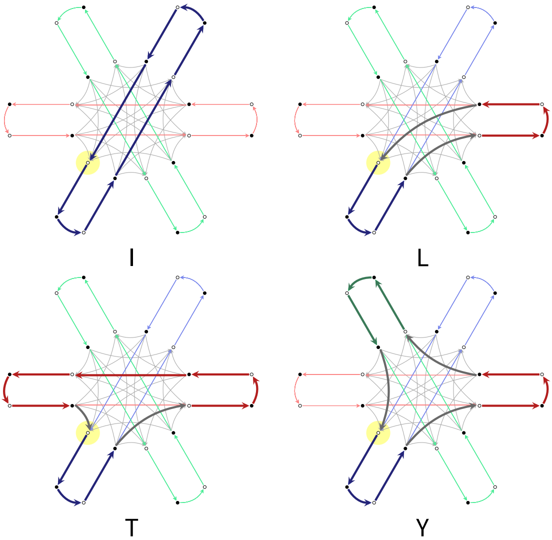


ILTY cycles at a station:

	# dwell	# transfer
I	2	0
L	0	2
T	1	2
Y	3	0

ILTY Cycles



ILTY cycles at a station:

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Y	3	0

Theorem (LLM, 2021)

The set B_s of ILTY cycles at a station s through a fixed event at s is a weakly fundamental basis for the space spanned by all ILTY cycles at s.

There is B' s.t. $B' \cup \bigcup_{s \in S} B_s$ is a forward integral cycle basis, *and B' projects to a strictly fundamental cycle basis of the line network.*

Minimum Forward Cycle Bases

Weights for Cycle Bases

Let $c \in \mathbb{R}^A_{\geq 0}$ be a weight vector.

Weight of a cycle basis: $c(B) = \sum_{\gamma \in B} \sum_{a \in \gamma} \gamma_a$

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Motivation: weight of B w.r.t. $u - \ell \approx \log(\#$ possible modulo parameters $z \in \mathbb{Z}^B)$

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	type	complexity (oriented)
(1)	directed	P (Horton's algorithm, 1987)
(2)	undirected	P (Horton's algorithm, 1987)
(3)	integral	?
(4)	weakly fund.	APX-hard (Rizzi, 2007)
(5)	strictly fund.	APX-hard (Galbiati et al., 2007)

Minimum Forward Cycle Bases

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	type	complexity (oriented)	complexity (forward)
(1)	directed	P (Horton's algorithm, 1987)	P (Gleiss et al., 2003)
(2)	undirected	P (Horton's algorithm, 1987)	P (Gleiss et al., 2003)
(3)	integral	?	?
(4)	weakly fund.	APX-hard (Rizzi, 2007)	?
(5)	strictly fund.	APX-hard (Galbiati et al., 2007)	?

Forward Cycles in Practice

Recapitulation

- We want to use forward integral cycle bases for solving the PESP MIP.
- Forward cycle bases exist in strongly connected digraphs.
- A forward integral cycle basis can be constructed in line-based networks by means of ILTY cycles.
- Minimum weight forward (un)directed cycle bases can be computed by a modification of Horton's algorithm.

Forward Cycles in Practice

Recapitulation

- We want to use forward integral cycle bases for solving the PESP MIP.
- Forward cycle bases exist in strongly connected digraphs.
- A forward integral cycle basis can be constructed in line-based networks by means of ILTY cycles.
- Minimum weight forward (un)directed cycle bases can be computed by a modification of Horton's algorithm.

PESPlib

- benchmarking library of PESP instances by Goerigk
- networks are not strongly connected
- but they are very close to line-based networks!

Reverse Engineering PESPlib Instances

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- The network is now bipartite. $\rightarrow$ arrivals and departures
- Remove all arcs with $u_a - \ell_a = T - 1 = 59$. $\rightarrow$ transfers (all start at arrivals)
- All remaining arcs have $u_a - \ell_a \leq 17$. $\rightarrow$ drive or dwell activities
- The network decomposes into 110 directed paths. $\rightarrow$ lines
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Remark

The structure of all 16 PESPlib railway instances follows this pattern.

Computational Set-Up

Solver: Concurrent PESP solver (Borndörfer et al., 2020) with Gurobi 9.1, up to 8 threads, 1h wall time

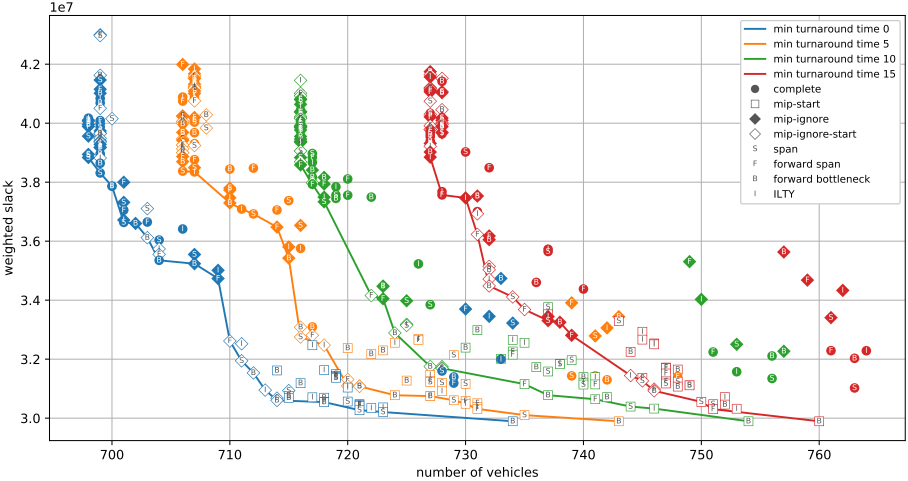
Scenarios: R1L1v with...

- 4 minimum turnaround times: $\ell_a = 0, 5, 10, 15$
- 7 turnaround weights: $w_a = 0, 2\,500, 5\,000, 10\,000, 20\,000, 40\,000, 80\,000$
- 4 cycle bases: span, forward span, forward bottleneck, ILTY
- 6 solution strategies:

Strategy	MIP	Initial solution	Ignore light arcs	Other
complete	✓		✓	✓
mip	✓			
mip-start	✓	✓		
mip-ignore	✓		✓	
mip-ignore-start	✓	✓	✓	
dual	✓	✓		

- 2 evaluation criteria: weighted passenger slack (i.e., without turnaround activities), number of vehicles (vehicles stay on line)

Pareto Front



Results: Primal Side

Unsurprising Results

- The higher the turnaround weights, the lower the number of vehicles.
- With the passenger-optimized initial timetable, the number of vehicles tends to be higher.
- Within 1h, reaching the passenger slack of the PESPlib incumbent is impossible, but the best number of vehicles goes down to the theoretical minimum $+1$.

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Impact of Cycle Bases

- The “mip” strategy without initial solution and without further heuristics performs bad in all cases.
- The picture is quite diffuse. For the 4 other strategies and for all 4 cycle bases, we find at least one non-dominated solution each.
- Comparing the 4 cycle bases, the difference is at most 2.6% in passenger slack and 0.6% in number of vehicles on average.

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Conclusion: The choice of cycle basis does not matter.

Results: Dual Side

- After 1h, the best dual bound for the traditional oriented minimum span basis is on average 17.6% *worse* than with ILTY.
- With minimum turnaround time 0 and turnaround weight 0, dual bounds are valid for the original R1L1:

instance	cycle basis	dual bound
`R1L1v`	span	20 638 013
`R1L1v`	forward span	20 609 801
`R1L1v`	forward bottleneck	20 591 564
`R1L1v`	ILTY	20 901 883
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(24h wall time, with CPLEX 12.10 and flip inequality separation)

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New Challenges: PESPlib has grown by 2 instances with turnarounds (R1L1v and R4L4v).

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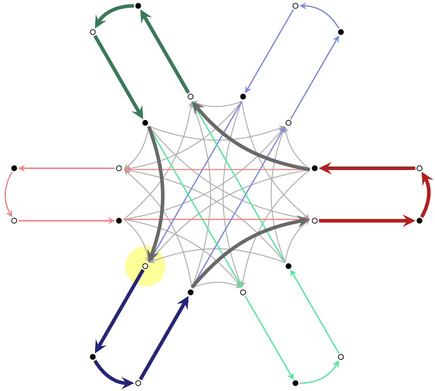

Forward Cycle Bases and Periodic Timetabling

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