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Project B1

Strategic planning in public transport

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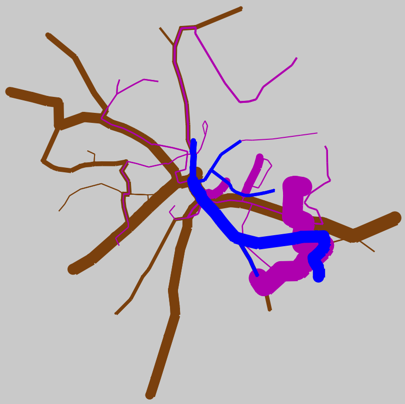
Strategic planning

Planning process in public transport:

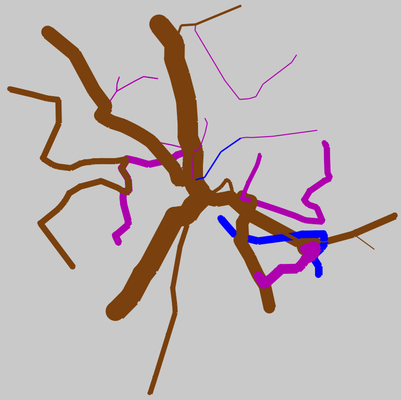
1. Strategic planning
   - ▷ Network design
   - ▷ Line planning
   - ▷ Timetable construction
   - ▷ Fare planning
2. Operational planning
3. Online planning

Strategic planning deals with the fundamental design aspects of public transport systems. The goal of this project is to develop a mathematical optimization toolbox to support the key decisions of planners in this area.

Optimized lines in Potsdam



current lines



optimized lines

Line planning

Given a network, demands d_{st} (OD-matrix), operating costs, and travel times, find line paths and frequencies such that the demand is satisfied and the following two objectives are balanced: the travel times of passengers are minimized and fixed and operating costs are minimized.

Multi-commodity flow model:

min $\sum_{\ell}(C_\ell\, x_\ell + c_\ell\, f_\ell) \;+\; \lambda \sum_p \tau_p\, y_p$ cost and travel time

i) $\sum_{p\in\mathcal{P}_{st}} y_p = d_{st} \quad \forall\, s,t$ transport passengers

ii) $\sum_{p\ni a} y_p \le \sum_{\ell\ni a} \kappa_\ell\, f_\ell \quad \forall\, a$ arc capacities

iii) $f_\ell \le F\, x_\ell \quad \forall\, \ell$ frequency bounds

Variables:
- $y_p \in \mathbb{R}_+$ passenger flow on path p
- $f_\ell \in \mathbb{R}_+$ frequency of line ℓ
- $x_\ell \in \{0,1\}$ decision variable for line ℓ

Selected theoretical results

Theorem: Already the LP relaxation of the line planning problem is NP-hard.

Theorem: If the lengths of the lines are restricted to be $O(\log n)$, where n is the number of nodes, the LP relaxation can be solved in polynomial time.

Discrete-choice model for fare planning

- ▷ Alternatives A: single ticket (1), Bahn-Card (2), car (3)
- ▷ Variables: x_d = distance dependent fare, x_b = basic fare
- ▷ Prices: $x_d \cdot \mathrm{km}_{st}$ = single ticket; $x_b + \frac{1}{2}\cdot x_d \cdot \mathrm{km}_{st}$ = Bahn-Card
- ▷ Utility: V^a_{st} = Utility-function for alternative a
- ▷ X_{st} = random number of trips from s to t

⇝ nonlinear optimization problem: maximize expected revenue

$$\max \sum_{k=1}^{N} \sum_{(s,t)} d_{st} \cdot \frac{\mathbb{P}[X_{st}=k]}{\sum_{b\in A} e^{V^b_{st}(\boldsymbol{x},k)}} \cdot \Big[(x_d \cdot \mathrm{km}_{st} \cdot k)\cdot e^{V^1_{st}(\boldsymbol{x},k)} + (x_b + \tfrac{1}{2}\, x_d \cdot \mathrm{km}_{st} \cdot k)\cdot e^{V^2_{st}(\boldsymbol{x},k)}\Big]$$
$$s.t.\quad \boldsymbol{x} \ge \mathbf{0}.$$

Fare planning

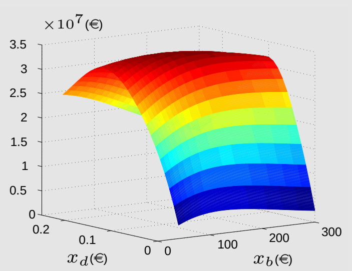
Background: OD-matrix depends on fares.

Given: demand functions between s and t, depending on the fares

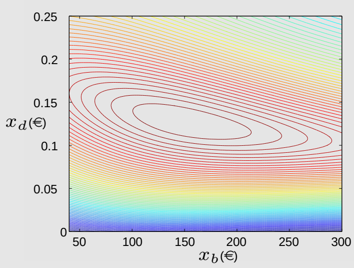
Problem: design fare system (fares for single ticket, Bahn-Card, ...)

Goal: maximize revenue

Optimized fares for the Dutch IC network



revenue function $r(x_b, x_d)$



contour plot of revenue function

Industry cooperation

ViP Verkehrsbetrieb Potsdam GmbH
Stadt Potsdam
IVU Traffic Technologies AG

MATHEON cooperation

Project B3 – exploit similarities of telecommunications and public transport
Project B5 – integration of line planning and timetabling
Project F4 – visualization of line plans