Optimization 2

Exercise Sheet 10
Submission: Wednesday, 17.01.2018, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two.

Homepage of the Lecture: http://www.zib.de/ws17_Optimierung_II
Questions?: beckenbach@zib.de

Exercise 10.1 2+4+2 Points
Let $G = (V,E)$ be a connected graph. The graphic matroid $M_G$ associated to $G$ has $E$ as its ground set and $F \subseteq E$ is independent if and only if $G[F]$ contains no cycle. Denote the set of independent sets of $M_G$ by $I_G$.

(a) Show that $M_G$ is indeed a matroid.

(b) How do the bases and circuits of $M_G$ look like?

(c) Give a formula for the rank function of $M_G$.

Exercise 10.2 6 Points
Give a closed formula for the rank functions of the following matroids.

(a) The free matroid $(E, 2^E)$.

(b) The $k$-uniform matroid $U_{k,n}$ on a ground set of size $n$, i.e.
$$U_{k,n} := (\{1,\ldots,n\}, \{I \subseteq \{1,\ldots,n\} : |I| \leq k\}).$$

(c) The trivial matroid $(E, \{\emptyset\})$.

PLEASE TURN OVER
Exercise 10.3  

Let $G = (V, E)$ be a connected graph. Look at the following independence systems and prove or disprove that they are matroids.

(a) $\mathcal{M}_1 := (E, \mathcal{I}_1)$ where $\mathcal{I}_1$ is the set of all matchings in $G$, i.e.,

$$\mathcal{I}_1 = \{ M \subseteq E : \deg_M(v) \leq 1 \ \forall v \in V \}.$$ 

(b) $\mathcal{M}_2 := (V, \mathcal{I}_2)$ where $\mathcal{I}_2$ is the set of all vertex subsets covered by some matching of $G$, i.e.,

$$\mathcal{I}_2 = \{ U \subseteq V : \exists \text{ matching } M \text{ with } U \subseteq V(M) \}.$$