Freie Universität Berlin
FB Mathematik und Informatik
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Prof. Dr. Ralf Borndörfer
Isabel Beckenbach

Optimization 2

Exercise Sheet 11

Submission: Wednesday, 24.01.2018, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 11.1 — 10 Points

Let $G = (V_1 \cup V_2, E)$ be a bipartite graph with sides V_1, V_2, i.e., $|e \cap V_1| = |e \cap V_2| = 1$ for all $e \in E$. The set of matchings in G induces an independence system $\mathcal{M} = (E, \mathcal{I})$ where $\mathcal{I}$ is the set of all matchings in G. Show that $\mathcal{M}$ is the intersection of two matroids.

Give an example of a bipartite graph G together with a weight function such that the (max. weight) greedy algorithm does not return a maximum weight matching. Prove that the weight of a matching returned by the greedy algorithm is as least half of the weight of an optimum matching.

Exercise 11.2 — 2+4+4 Points

Let $\mathcal{M} = (E, \mathcal{I})$ be a matroid and $S \subseteq E$ a subset of the ground set E. The *deletion* of S is the pair $\mathcal{M} \setminus S := (E \setminus S, \{I \subseteq E \setminus S : I \in \mathcal{I}\})$. The contraction of S is defined as $\mathcal{M}/S = (\mathcal{M}^* \setminus S)^*$.

(a) Show that $\mathcal{M} \setminus S$ and $\mathcal{M}/S$ are matroids.

(b) Let $r_{\mathcal{M}}, r_{\mathcal{M}\setminus S}, r_{\mathcal{M}/S}$ be the rank functions of $\mathcal{M}$, $\mathcal{M}\setminus S$, and $\mathcal{M}/S$, respectively. Show that $r_{\mathcal{M}\setminus S}(F) = r_{\mathcal{M}}(F)$ and $r_{\mathcal{M}/S}(F) = r_{\mathcal{M}}(F \cup S) - r_{\mathcal{M}}(S)$ for all $F \subseteq E \setminus S$.

(b) Let G be a graph and $\mathcal{M}(G)$ its associated graphic matroid. Show that for every $e \in E$ the matroid $\mathcal{M}(G) \setminus \{e\}$ is isomorphic to $\mathcal{M}(G - e)$ where $G - e$ is the graph obtained form G by deleting e. Furthermore, show that $\mathcal{M}(G)/\{e\}$ is isomorphic to $\mathcal{M}(G/e)$ for all $e \in E$ where G/e is obtained from G by contracting e.