Freie Universität Berlin
FB Mathematik und Informatik
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# Optimization III

## Exercise Sheet 2

Submission: Thursday, 03.05.2018, 14:00

**Exercises:**
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `maristany@zib.de`

### Exercise 2.1 — 10 Points

Consider a network $\mathcal{N} := (D := (V, A), u, s, t, )$. A preflow $f$ is said to be a *maximum preflow*, if it maximizes $ex_f(t)$.

a) Show that for every maximum preflow $f$ there exists a maximum flow $f'$ such that $f'(a) \leq f(a)$ for all $a \in A$. (Hint: construct a small example in which $f'$ and $f$ do not coincide.)

b) Show how a maximum preflow can be converted into a maximum flow in $\mathcal{O}(nm)$ time. (Hint: use a slightly modified version of Edmonds-Karp Algorithm.)

### Exercise 2.2 — 10 Points

Prove that the Push-Relabel algorithm performs $\mathcal{O}(n^2m)$ non-saturating pushes. Hints:

Consider a potential function

$$\Phi := \sum_{\text{active vertices v}} \Psi(v).$$

At the beginning there holds $\Phi = 0$ and the value of $\Phi$ remains positive at all times. The algorithm performs three different types of operations: node relabelings, saturating pushes, and non-saturating pushes. How do each of these operations affect the value of $\Phi$?

**Exercise 2.3** **10 Points**

A company has $n \in \mathbb{N}$ different locations that might be suitable to open a new facility $i$, $i \in \{1, \ldots, n\}$. The costs for opening facility $i$ are $f_i \in \mathbb{R}_+$. In addition, there are $m \in \mathbb{N}$ clients, whose demands have to be satisfied. For each client exactly one facility has to cover all of its demands. Letting facility $i \in \{1, \ldots, n\}$ cover the demands of client $j \in \{1, \ldots, m\}$ results in costs $c_{ij}$ for the company.

Of course, the goal of the company is to minimize their overall costs (for the opening of new facilities and for the covering of the customers' demands).

Model the problem as an Integer Program.