Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

Prof. Dr. Ralf Borndörfer
Isabel Beckenbach

Optimization 2

Exercise Sheet 3

Submission: Wednesday, 15.11.2017, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in until 15.11 (in my office, room 3301 at ZIB) or after one of the lectures next week.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 3.1 — 5 Points

Consider the primal dual barrier problem (P_μ) given by

$$\begin{aligned} \min x^T z - \mu \sum_{i=1}^n \log x_i - \mu \sum_{i=1}^n \log z_i \\ Ax = b, x > 0 \\ A^T y + z = c, z > 0 \end{aligned}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $\mu \in \mathbb{R}, \mu > 0$ are given, and we assume that (P_μ) has an optimal solution.

Show that there exist $x^0, z^0 \in \mathbb{R}^n$ such that (x^*, z^*) is an optimal solution to (P_μ) if and only if it is an optimal solution to

$$\begin{aligned} \min\ (z^0)^T x + (x^0)^T z - \mu \sum_{i=1}^n \log x_i - \mu \sum_{i=1}^n \log z_i \\ Ax = b, x > 0 \\ A^T y + z = c, z > 0. \end{aligned}$$

please turn over

Exercise 3.2 **5 Points**

Consider the primal problem (P) $= \min\{c^T x : Ax = b, x \geq 0\}$, its dual (D) $= \max\{y^T b : A^T y + z = c, z \geq 0\}$, and the non-linear primal dual problem (PD) $= \min\{x^T z : Ax = b, A^T y + z = c, x \geq 0, z \geq 0\}$. Show that (PD) is feasible if and only if (D) and (P) are feasible and bounded.

Exercise 3.3 **10 Points**

Implement Newton's Method for polynomial functions $f(x) = \sum_{i=0}^{n} a_i x^i$ in one variable. You are given the coefficients $a_i \in \mathbb{Z}$, a starting point $x^{(0)}$, and a tolerance $\epsilon > 0$ (stop when $f(x^{(k)}) < \epsilon$) as input. Test your implementation on some examples.

You can use Matlab, Mathematica, Java, Python, C or C++.