

Optimization III

Exercise Sheet 3

Submission: Thursday, 09.05.2018, 14:00

Exercises:

Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two.

Homepage of the Lecture: http://www.zib.de/ws17_Optimierung_II

Questions?: maristany@zib.de

Exercise 3.1

10 Points

Let $D := (V, A)$ be a directed graph with capacities $u : A \rightarrow \mathbb{R}_+$, and let $b : V \rightarrow \mathbb{R}$ with $\sum_{v \in V} b(v) = 0$ be the balance function on the nodes. Prove that there exists a b -flow on D that respects u if and only if

$$\sum_{a \in \delta^+(X)} u(a) \geq \sum_{v \in X} b(v), \quad \forall X \subseteq V.$$

Exercise 3.2

10 Points

Show that the *Maximum Flow Problem* can be regarded as a special case of the *Minimum Cost Problem*.

Exercise 3.3

10 Points

Given a directed graph $D := (V, A)$, let (T, L, U) be a partition of A , and T a spanning tree of D .

- Prove that there exists a unique flow x on D such that $x_a = l_a$ for all $a \in L$ and $x_a = u_a$ for all $a \in U$.
- In the first part of the exercise, nothing is said about the flow value x_a for arcs $a \in T$. Is the unique flow x always feasible?

Exercise 3.4

10 Points

This week's modeling exercise will be the *Diet Problem*. We are trying to find a combination of foods and nutritional values that leads to a funny solution once the model is solved. The final version of the exercise will be posted on Friday, May 4th.

[Model the problem as an Integer Program.]