Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

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Optimization 2

Exercise Sheet 3

Submission: Wednesday, 22.11.2017, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in until 22.11 (in my office, room 3301 at ZIB) or after one of the lectures next week.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 3.1 **10 Points**

Consider following pair of primal/dual optimization problems:

$$(P): \ \min c^T x \qquad Ax = b \qquad x \geq 0$$

$$(D): \ \max y^T b \qquad y^T A + z^T = c^T \qquad z \geq 0$$

(a) Show that the KKT-Conditions for (P_μ) and (D_μ) (see Proposition 8.10(d)) are equivalent to

$$c - A^T y > 0$$

$$\mu \left(\sum_{j=1}^{n} \frac{a_{ij}}{c_j - A_{.j}^T y} \right) = b_i \text{ for } i = 1, \ldots, n.$$

for $\mu > 0$.

(b) Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $c^T = (1,1,1)$, $b^T = (t,1)$. Calculate the central paths $x(\mu), y(\mu), z(\mu)$ for $t = 0$ and $t = 1$. Draw the polyhedron of (D) and the central paths $y(\mu)$ for $t = 0$ and $t = 1$.

Exercise 3.2 **4 Points**

Consider the optimization problem $\max\{c^T x : Ax \geq b, \sum_{i=1}^n x_i \text{ is even }, x \in \mathbb{Z}_{\geq 0}\}$ where $A \in \mathbb{Z}^{m \times n}$, $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$. Write this problem as an integer linear program.

Exercise 3.3 **6 Points**

An assembly line consisting of a collection of work stations has to perform a series of jobs in order to assemble a product. Each work station can perform one job at a specific time step. After finishing this job completely it can start a new job. There are also some restrictions on the order in which the jobs can be done, called precedence relations. Consider a product with 5 jobs on an assembly line with 4 work stations (station $j+1$ directly starts after station j is finished). The following table specifies processing times and the predecessors for job i.

Job i	time for i in minutes	predecessors
1	6	-
2	5	-
3	7	-
4	6	3
5	6	2,4

For example, job 3 has to be finished before job 4 can start, this means that job 4 is either processed on the same station after job 3 or job 4 is processed on a higher numbered station.

The total time of jobs assigned to one station is at most 12.

Use the binary variables

$$x_{i,j} = \begin{cases} 1, & \text{job i is done at station j} \\ 0, & \text{else} \end{cases}$$

to formulate an integer linear program which minimizes the number of machines used.

(Hint: Define an auxiliary cost function such that it is cheaper to produce a job i on a station with lower number.)