Freie Universität Berlin
FB Mathematik und Informatik
WS 2017/18

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Optimization 2

Exercise Sheet 6

Submission: Wednesday, 06.12.2017, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two. Please put your name(s) on your exercise sheet and hand them in until 29.11 (in my office, room 3301 at ZIB) or after one of the lectures next week.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 6.1 — 3+3+2 Points

Let $G = (V, E)$ be a graph. A spanning forest is a subgraph $H = (V, F)$ of G such that $F \subseteq E$, every vertex v is incident to at least one edge of F, H has no cycle, and if we add an edge $e \in E \subseteq F$ to H, then the resulting graph has a cycle. Prove the following claims.

(a) The connected components of G partition its vertex set.

(b) H is a spanning forest if and only if it consists of spanning trees in each connected component of G.

(c) Show that every spanning forest contains $|V| - k$ edges where k is the number of connected components of G.

Exercise 6.2 — 3+3 Points

Let $D = (V, A)$ be a strongly connected digraph. This means that for every $s, t \in V$ there exists a directed (s, t)-path.

(a) Suppose $F \subseteq A$ is a spanning arborescence, i.e., $V(F) = V$, F induces a connected subgraph, $|F \cap \delta^-(v)| \leq 1$ for all $v \in V$. Show that there exists exactly one vertex $r \in V$ with $|F \cap \delta^-(r)| = 0$. (We call r the root, and F a spanning arborescence rooted at r).

please turn over

(b) Let $r \in V$ be any vertex of D. Show that D has a spanning arborescence rooted at r.

Exercise 6.3 **6 Points**

Let $G = (V, E)$ be connected graph such that for every edge $e \in E$ there exists a perfect matching M_e of G containing e. Prove that G has no articulation node.

(Hint: Proof by contradiction. Assume G has an articulation node $v \in V$, then $G - v$ contains a component with an odd number of vertices. How does a perfect matching of G looks like at the connected components of G with an odd and an even number of vertices?)