Freie Universität Berlin
FB Mathematik und Informatik
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Optimization 2

Exercise Sheet 9

Submission: Wednesday, 10.01.2018, 12:00

Exercises:
Rigorous mathematical proofs/arguments are expected if not stated otherwise. You are allowed to work in groups of two.

Homepage of the Lecture: `http://www.zib.de/ws17_Optimierung_II`
Questions?: `beckenbach@zib.de`

Exercise 9.1 — 10 Points

Analyze the following algorithm (Alg. 12.6 in the lecture)

Input: $G = (V, E), c \in \mathbb{Q}^E, m := |E|$
Output: $T \in \operatorname{argmin}\{c(T) : T \subseteq E, \text{ T spanning tree}\}$ or "G is not connected"
Sort E such that $c(e_1) \leq \ldots \leq c(e_m)$;
$T \leftarrow \emptyset$;
for $i = 1$ *to* m **do**
  if $T \cup \{e_i\}$ *contains no cycle* **then**
    $T \leftarrow T \cup \{e_i\}$
  end
end
if $|T| = |V| - 1$ **then**
  output T
else
  output "G is not connected"
end

Algorithm 1: Greedy-Min

Show that Greedy-Min correctly calculates a minimum cost spanning tree by showing that if G is not connected the algorithm outputs "G is not connected" and otherwise it outputs a minimum cost spanning tree. Furthermore, show that Algorithm 1 can be implemented to run in $\mathcal{O}(|E||V|)$ time (Sorting $|E|$ numbers can be done in $\mathcal{O}(|E| \log |E|)$ time).

You can earn 10 extra points if you implement Greedy-Min. The input graph and output tree are given as a text file (you define how these files look like, think of a suitable data structure for your graph).

Exercise 9.2 **4 Points**

Let $D = (V, A)$ be a digraph with arc weights $c \in \mathbb{Q}^A$. The minimum spanning arborescence problem (MSA) is to find a minimum weight arborescence T spanning D or decide that D has no spanning arborescence. In the minimum spanning rooted arborescence problem (MSRA) a vertex $r \in V$ is given and one has to find a minimum weight spanning arborescence T of D such that $|\delta^-(r) \cap T| = 0$. Show that MSA and MRSA are equivalent by giving a polynomial time reduction of one to the other (MSA$\propto$MRSA and MRSA$\propto$MSA).

Exercise 9.3 **6 Points**

Solve the maximum weight branching problem on the following graph using Edmond's Branching Algorithm.