Freie Universität Berlin
FB Mathematik und Informatik
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Optimierung I

Excercise Sheet 3

Submission: until 14:15 on Tuesday, Mai 16 2017

Exercise 3.1 **10 Points**

Show that the following definitions are equivalent:

Definition 1 *Let $C \subseteq \mathbb{R}^n$ be convex. A function $f : C \to \mathbb{R}$ is called* convex*, if*

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y) \quad \forall x, y \in C, \lambda \in [0,1].$$

Definition 2 *Let $C \subseteq \mathbb{R}^n$ be convex. A function $f : C \to \mathbb{R}$ is called* convex*, if*

$$f\left(\sum_{i=1}^{p} \lambda_i x_i\right) \leq \sum_{i=1}^{p} \lambda_i f(x_i)$$

for arbitrary points $x_1, \ldots, x_p \in C$ and $\lambda_1, \ldots, \lambda_p \geq 0$ with $\sum_{i=1}^{p} \lambda_i = 1$.

Exercise 3.2 **10 Points**

Let $C \subseteq \mathbb{R}^n$ convex, $f_1, f_2 : C \to \mathbb{R}$ convex, $\alpha > 0$.

For each of the following statements give either a proof of a counterexample:

a) αf_2 is convex

b) $f_1 + f_2$ is convex

c) $f_1 - f_2$ is convex

d) $f_1 \cdot f_2$ is convex

e) $\max\{f_1, f_2\}$ is convex

f) $\min\{f_1, f_2\}$ is convex

Exercise 3.3 **10 Points**

For the construction of a new bridge over the Tay a financing plan has to be established. Table 1 (left) gives the estimated cost over the 6 years of construction. The City of Dundee plans to raise the funds needed to pay these costs by issuing bonds. Such a bond is valid up to 6 years. It can be taken out every 1st of January

year	cost
1	20 Mio £
2	17 Mio £
3	23 Mio £
4	24 Mio £
5	25 Mio £
6	21 Mio £

length of validity	overall interest rate
1 years	7%
2 years	15%
3 years	23%
4 years	32%
5 years	41%
6 years	50%

Table 1: Construction costs each year and interest rates for bonds.

and is due on the 31st December of the year that it is due—the validity period is fixed beforehand. Of course, interest has to be paid on bonds when they are due, depending on how long they are valid, see Table 1 (right).

Money that is not used for construction can be invested at the Royal Bank of Scotland at an interest rate of 6.8% anually.

The problem is to find out how many bonds to which terms should be issued each year to keep the outstanding debts at the end as low as possible.

a) Set up a linear program that models the problem.

b) Use ZIMPL and SCIP to find an optimal solution.

c) Suppose that all bonds issued are due to the end of the 6th year. Adapt the above linear program to this special situation and find the optimal solution.

Homepage of the Lecture: `http://www.zib.de/ss17_Optimierung_I`
Questions?: `klug@zib.de`