Freie Universität Berlin
FB Mathematik und Informatik
Sommersemester 2017

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# Optimierung I

## Excercise Sheet 4

Submission: until 17:00 on Monday, Mai 22 2017

**Exercise 4.1** **8 Points**

Let $f : \mathbb{R} \to \mathbb{R}$ be convex and $a, b \in \mathbb{R}$ with $a < b$.

a) Show that for all $x \in [a, b]$,

$$f(x) \leq \frac{b-x}{b-a} f(a) + \frac{x-a}{b-a} f(b).$$

b) Show that for all $x \in (a, b)$,

$$\frac{f(x) - f(a)}{x - a} \leq \frac{f(b) - f(a)}{b - a} \leq \frac{f(b) - f(x)}{b - x}.$$

c) Make a sketch that illustrates the inequalities.

**Exercise 4.2** **8 Points**

Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function and $M \subset \mathbb{R}^n$ be compact.

Proof the following statements:

a)

$$f \text{ convex} \quad \Rightarrow \quad \mathcal{L}(f, \beta) := \{x \in M \mid f(x) \leq \beta\} \text{ convex } \forall \beta \in \mathbb{R}$$

b) The converse of a) does not hold.

c)

$$\max\{f(x) \mid x \in M\} = \max\{f(x) \mid x \in \mathsf{conv}\, M\}$$

**Exercise 4.3** **14 Points**

*Campus Liquids Inc.* (CLI) is engaged in the production and sale of two kinds of hard liquor. CLI purchases intermediate-stage products in bulk, purifies them by repeated distillation, mixes them, bottles the product under its own brand names and sells it. One product is a bourbon, the other a blended whiskey. The problem is to decide how many bottles of each should be produced in the next production period.

As the company's products are very popular on the market, the production capacity is inadequate to produce all that CLI might sell. The bourbon requires 3 machine hours per bottle, while the blended whiskey requires 4 hours of machine time per bottle. There are 20,000 machine hours available in the production period. The direct operating costs, which are mainly for labor and materials, are 3.00€per bottle of bourbon and 2.00€per bottle of blended whiskey. The working capital available to finance these costs is 4400€however, 40% of the sales revenues will be collected during the production period and made available to finance ongoing operations.

The selling price is 5€for a bottle of bourbon and 4.50 €for a bottle of blended whiskey.

a) Set up a linear program with two variables $x_1$ and $x_2$ that maximizes CLI's profit in the production period to come, subject to limitations on machine capacity and working capital.

b) Sketch the feasible set in the plane and give the coordinates of the vertices.

c) What is the optimal production mix to schedule and how large is the company's profit with this schedule?

d) Suppose CLI could spend some money to repair machinery and increase its available machine hours by 2000 hours (before production starts). Should the investment be made and if so, up to which price?

e) Model the problem using ZIMPL. Do not restrict, however, your model to a fixed number of products. This means the model should be able to handle an arbitrary number of products. The input data of the model should be given from the outside. Therefore, you find on the web page three data files (data1, data2, data3). An example is given below:

```
hours 20000
money 4400
reuse 40
bourbon 5   3 3
whiskey 4.5 2 4
```

The first line gives the total number of available machine hours. The second line states the available working capital. The third line gives the percentage of the sale revenues which are made available to finance the ongoing operations. The rest of the file gives for each product one line. In these lines the first column states the name of the product, column two the sales price, the third column the production cost, and the last column the required machine hours to produce this product. Send the ZIMPL model to `klug@zib.de`