Freie Universität Berlin
FB Mathematik und Informatik
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Optimierung I

Excercise Sheet 10

Submission: until 17:00 on Monday, July 3, 2017

Exercise 10.1 **10 Points**

Prove that if a standard form LP has an optimal solution but no optimal solutions that are degenerated, then the dual problem has a unique optimal solution.

Exercise 10.2 **10 Points**

$$\begin{array}{rrcrcl} \min & 2x_1 & + & x_2 & & \\ & 3x_1 & + & x_2 & \geq & 3 \\ & 4x_1 & + & 3x_2 & \geq & 6 \\ & x_1 & + & 2x_2 & \leq & 3 \\ & x_1 & , & x_2 & \geq & 0 \end{array}$$

Solve by the Dual Simplex method.

Exercise 10.3 **10 Points**

Suppose n people start at the same place at time 0 to travel to a destination that is D miles away. They can walk and besides that they have one bicycle at their disposal that carries one person at a time. The walking and bicycling speed of person j is given by w_j and b_j, respectively. The task is to find the earliest point in time after which all n persons can have arrived at the destination.

a) Show that the linear program

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & t - x_j^+ - x_j^- - y_j^+ - y_j^- \geq 0 && j = 1, \ldots, n && (1) \\ & t - \sum_{j=1}^{n} y_j^+ - \sum_{j=1}^{n} y_j^- \geq 0 && && (2) \\ & w_j x_j^+ - w_j x_j^- + b_j y_j^+ - b_j y_j^- = D && j = 1, \ldots, n && (3) \\ & \sum_{j=1}^{n} b_j y_j^+ - \sum_{j=1}^{n} b_j y_j^- \leq D && && (4) \\ & x_j^+, x_j^-, y_j^+, y_j^- \geq 0 && j = 1, \ldots, n && (5) \end{aligned}$$

provides a lower bound on the best possible arrival time of the last person. In other words, give an interpretation for the variables t, x_j^+, x_j^-, y_j^+, and y_j^- and explain why the constraints (1)–(5) are valid for the problem.

b) Write a ZIMPL file and use SOPLEX to find lower bounds for the arrival time for the following values. (You only need to hand in the optimal values for the three data sets, not the ZIMPL files themselves.)

	D	n	$w_1, \ldots, w_n$	$b_1, \ldots, b_n$
(i)	10	3	$2, 4, 2$	$12, 16, 12$
(ii)	45	9	$3, 2, 4, 3, 3, 5, 4, 2, 6$	$14, 11, 15, 13, 17, 17, 14, 10, 18$
(iii)	100	20	$w_j = (10 + j)/5$	$b_j = (20 + j)/2$

c) Give an example for which the optimum of the above linear program is a strictly smaller value than the arrival time of the last person.

Homepage of the Lecture: `http://www.zib.de/ss17_Optimierung_I`
Questions?: `klug@zib.de`