Numerical simulation of the behavior of suspension in high frequency vibrational field

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Abstract

The paper deals with the behavior of a suspension of rigid particles in a gas or liquid under the influence of high frequency vibrations. Two dimensional nonlinear problem on evolution of slightly inhomogeneous suspension in a square cavity subjected to vibrations with linear polarization is studied numerically on the base of averaged equations. The state with zero average velocity and particle distribution in the form of symmetrical cloud with maximal concentration in the cavity center is chosen as the initial state. It is found that at first the particle distribution evolves to the state with the symmetry C_{4v}. However, this state becomes unstable and a spontaneous breakdown of the symmetry occurs, the cross-over effect. As the result, the solution with C_{2v} symmetry establishes. Simulations performed for initial position of the cloud slightly shifted from the cavity center, and for essentially non-symmetric initial distribution of particles show that different quasi-equilibrium solutions are possible. Evidently, this is related to the existence of infinite number of conservation laws for the transport equation. Initial distributions of particles with "up-down" mirror symmetry with respect to the vibration direction also demonstrate in the course of evolution the cross-over effect: at first the cloud is flattened in the direction of vibrations, then the particles distribution becomes distorted and takes inversion symmetry.

Key words: multiphase flows, vibrations, average description.

AMS subject classifications: 76T05, 65C20.

1 Introduction

It is known, that vibrations are able to exert significant influence on the behaviour of hydrodynamic systems in the presence of inhomogeneity of either density or vibrations [1]. Wide class of heterogeneous systems are the systems with particles suspended in a gas or liquid. However, for the system where the particles are uniformly distributed over the liquid, one should expect the behaviour in a vibrational field similar to that of homogeneous liquid. Therefore, consideration of slightly inhomogeneous suspension is interesting.

The governing equations for the description of pulsation flows of liquid and solid phases are obtained in [2] on the basis of two-fluid theoretical approach. The difference in the inertial properties of phases, the phase-to-phase interactions according to the Stokes law and the effect of joined masses are taken into account. The averaged equations are formulated within the framework of single-fluid approximation. The set of averaged equations is simplified for the case of slightly inhomogeneous suspension.

The linear stability of quasi-equilibrium of slightly inhomogeneous suspension filling plane infinite layer in weigthlessness conditions is studied for the case of particles concentration gradient to be constant and parallel to the vibration axis. It is shown, that the oscillatory instability arises at any small intensity of vibrations; the wavelength of the most dangerous perturbations increases with the vibrational parameter growth.

This paper is devoted to the numerical simulation of the behavior of slightly inhomogeneous suspension in a high frequency vibrational field.

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2 Problem formulation. Governing equations

We consider the square cavity, filled with the particles suspension, and subjected to high frequency linear polarized vibrations in weightlessness. According to [2], the behavior of this system is described by the equations

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v} - Rv\vec{j}(\vec{j}\nabla\phi) = -\nabla p + \triangle\vec{v}, \tag{1}$$

$$\frac{\partial \phi}{\partial t} + (\vec{v}\nabla\phi) = 0, \tag{2}$$

$$div\vec{v} = 0, \tag{3}$$

where $\vec{v}$ is the average velocity, p is the pressure, ϕ is the volume concentration of solid phase, and $\vec{j}$ is the unit vector in the direction of vibration axis. The behavior of system is governed by the dimensionless parameter, which is analog of the vibrational Rayleigh number

$$Rv = \frac{(a\omega L)^2 B}{(\eta/\rho_m)^2}, \tag{4}$$

where a and ω are amplitude and frequency of vibrations respectively, L is the linear semi-dimension of the cavity side, η is the dynamic viscosity of fluid, ρ_m is the characteristic density of suspension, B is the complicated function of material parameters of the system and dimensionless frequency introduced as $\Omega = \rho_s\omega/\gamma$. Here ρ_s is the solid phase density, γ is the phase-to-phase friction coefficient.

Coefficient B tends to zero with $\Omega = 0$. At small values of Ω parameter B is exhibits parabolic behavior accurate to the terms of order $O(\Omega^4)$. Let us denote B at $\Omega \to \infty$ by B_∞. The expression for B_∞ can be simplified in limiting cases.

In particular, consider the case of dust suspended in air. In this case $\rho/\rho_s << 1$ (ρ is the density of liquid phase), $\phi << 1$ and mass concentration $\xi = \rho_s\phi$ is finite. The expression for B_∞ takes the form

$$B_\infty = \frac{1}{4}\frac{\rho}{\rho + \xi}. \tag{5}$$

In opposite case of small bubbles suspended in a liquid, when $\rho/\rho_s >> 1$, we have

$$B_\infty = \frac{(1-\phi)\rho}{(2\phi+1)^3}. \tag{6}$$

In our model we neglect the difference in average velocities of solid and liquid phases. Physically this means that the particles are frozen in average flow.

The governing equations (1)-(3) should be completed with the boundary conditions. Velocity field has to satisfy the no-slip condition at the boundary S of of the cavity

$$\vec{v}|_S = 0. \tag{7}$$

Due to impermeability conditions (7) no any boundary conditions for the concentration ϕ is needed.

Let us x and y axes are transversal and longitudinal to the vibration axis respectively. The origin of coordinate system is located in the cavity center. In this case we have $\vec{j} = (0, 1, 0)$.

3 Numerical method

Incompressibility condition (3) for average flow allows to use two-field method [3]. We rewrite the equations (1), (2) formulated in the primitive variables, velocity $\vec{v}$ and pressure p, in terms of stream function ψ and vorticity ω defined as

$$\vec{v}_x = \frac{\partial \psi}{\partial y}, \quad \vec{v}_y = -\frac{\partial \psi}{\partial x}, \tag{8}$$

$$\omega = curl_z\vec{v}, \tag{9}$$

Equations (1)-(3) being rewritten in terms of the stream function and vorticity take the form

$$\frac{\partial \omega}{\partial t} + \frac{\partial \omega}{\partial x}\frac{\partial \psi}{\partial y} - \frac{\partial \omega}{\partial y}\frac{\partial \psi}{\partial x} - Rv\frac{\partial^2 \phi}{\partial x \partial y} = \triangle \omega, \tag{10}$$

$$\triangle \psi + \omega = 0, \tag{11}$$

$$\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \psi}{\partial y} - \frac{\partial \phi}{\partial y}\frac{\partial \psi}{\partial x} = 0, \tag{12}$$

and the boundary conditions are:

$$\psi|_S = 0, \quad \left.\frac{\partial \psi}{\partial n}\right|_S = 0. \tag{13}$$

To evaluate the values of the vorticity at the rigid boundary we use Thom formulae [3]:

$$\omega|_S = -\frac{2}{h^2}\psi|_{S-1} + O(h), \tag{14}$$

where $\omega|_{S-1}$ is the value of stream function in internal point of cavity, distant normally to the boundary on the step of grid h.

We introduce uniform grid

$$x_i = (i - 1/2)h, \quad i = 1, \ldots, N+1, \tag{15}$$

$$y_j = (j - 1/2)h, \quad j = 1, \ldots, N+1, \tag{16}$$

where $h = 2L/N$ is the step of grid. Attribute the values of stream function ψ and vorticity ω to the nodes of this grid. We intend to solve transport equation for concentration (12) by means of finite volume method. Therefore it is necessary to have the values of concentration in centers of cells, formed by the grid nodes, i.e. in the points with the coordinates

$$x_i = ih, \quad i = 1, \ldots, N, \tag{17}$$

$$y_j = jh, \quad j = 1, \ldots, N. \tag{18}$$

Numerical procedure at each time step is the following. At first we solve the equation (10) for the vorticity - by using known values of ψ^k and ϕ^k at the previous time step k, we find the values of vorticity ω^{k+1} at the new, $k+1$, time step. Then, the new values of the stream function ψ^{k+1} are found from the Poisson equation (11). Finally, the new values of the concentration ϕ^{k+1} are found from the transport equation (12).

The equation for the vorticity (10) is solved using conventional explicit finite-difference scheme. Solution of the Poisson equation (11), at each time step, is found by the successive overrelaxation method. Solution of transfer equation is carried out by the explicit scheme based on the finite volume discretization. This scheme was proposed by R.J.LeVeque [4] in the framework of wave propagation method for the conservation laws for hyperbolic equations and has the second order in space.

As the initial state we use the state with zero average velocity and distribution of the particles in the form

$$\phi(x, y, t = 0) = (1 - x^2)^2(1 - y^2)^2 exp(-x^2 - y^2). \tag{19}$$

4 Numerical results

The calculations were carried out for different values of vibration parameter Rv. It is found that the behavior of suspension at different values of vibration parameter is qualitatively similar. The more intensive vibration influence on the system, the more intensive a flow, arising in a cavity under vibrations and the faster transition to the stationary state.

Consider evolution of the "cloud" of particles (19) in time with fixed value of the vibration parameter $Rv = 100$. At first, under the influence of vibrations, the cloud becomes more and more flat in the direction of vibrations. The

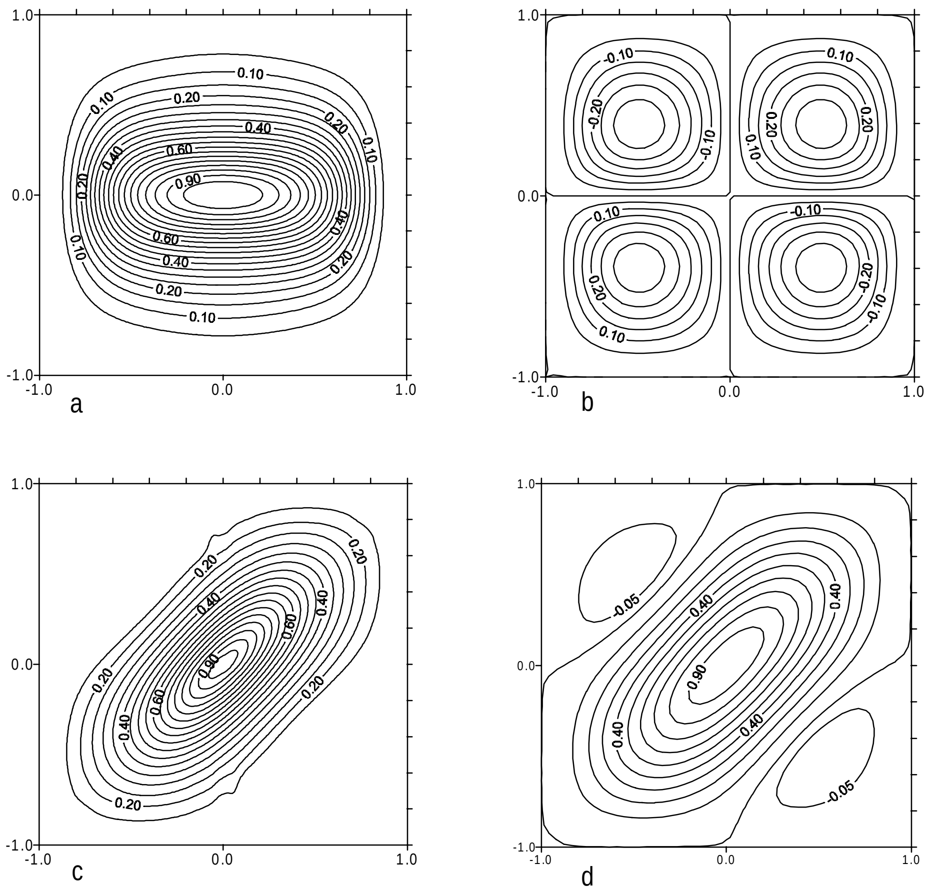

Figure 1: Isolines of the concentration and stream function for $Rv = 100$ respectively: a, b - solution at time t=0.15, c, d - final solution.

isolines of the concentration and stream function at time $t = 0.15$ are presented in fig. 1a, 1b. The distribution of the particles evolves to certain one, which has the mirror symmetry with respect to the straight lines, passing through the centers of sides and vertexes of square. This means the presence of symmetry group C_{4v}. Obviously, this symmetry group is allowed by the equations (10)-(12) and boundary conditions (13), (14).

However, this state turns out to be unstable to the symmetry-breaking perturbations. Spontaneous break-down of the symmetry is observed – the cross-over effect. As the result, the solution with inversion symmetry with respect to the cavity center and mirror symmetry with respect to the straight lines, passing through the vertex of the square is established. This means that the symmetry group C_{2v}, which is the subgroup of C_{4v}, takes place. Isolines of concentration and streamlines corresponding to this solution are presented in fig. 1c, 1d.

It follows from the structure of the transport equation that both average concentration and integral over the cavity of any function of concentration should be conserved in time. In numerical simulation we control the average value of the concentration over the cavity and its squared value. For example, changing of concentration up to the moment of setting of the solution with inversed symmetry was less than 10%. Then, this stationary solution gradually gets smeared by numerical diffusion. This naturally affects the integral characteristics.

There have been performed the simulations starting from some other initial distributions of particles, such that the cloud slightly shifted from the center and essentially non-symmetric initial particles distributions. It follows from these experiments, that different quasi-equilibrium solutions are possible. Evidently, this concerned with the existence of infinite numbers of conservation laws for transport equation. Initial distributions of particles with "up-down" mirror symmetry with respect to the direction of vibrations, evolve according to the scenario similar to that discussed above. At first such solutions become more and more flat in the direction of vibrations, then, due to cross-over effect, the distribution of the particles becomes distorted with the inversion symmetry.

5 Conclusion

Different types of solutions have been found for different inital states. The evolution from the initial state with zero average velocity and particle distribution in the form of symmetric cloud with maximal concentration in the cavity center demonstrates the cross-over effect. Initial distributions of particles with "up-down" mirror symmetry with respect to the vibration direction lead to the qualitatively similar results. The calculations using the initial distributions of particles in the form of cloud slightly shifted from the cavity center, and essentally non-symmetric initial distributions of particles show that different quasi-equilibrium solutions are possible.

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