

# Primal Heuristics in SCIP

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**DFG** Research Center MATHEON Mathematics for key technologies



Berlin, 10/11/2007







- Basics
- Integration Into SCIP

2 Available Heuristics

- Rounding Heuristics
- (Objective) Diving
- LNS & Others

### 3 Remarks & Results







- Basics
- Integration Into SCIP
- Available Heuristics
  Rounding Heuristics
  (Objective) Diving
  LNS & Others









- Basics
- Integration Into SCIP

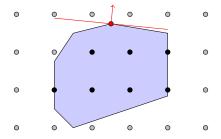
Available Heuristics
Rounding Heuristics
(Objective) Diving
LNS & Others





## Exact methods

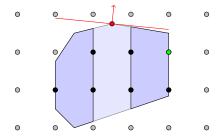
- Branch-And-Bound
- ▷ Cutting planes
- Branch-And-Cut





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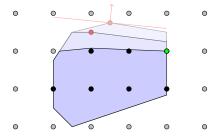
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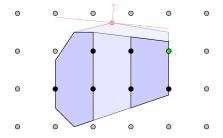
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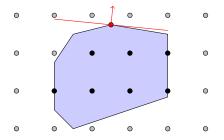




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# Heuristics

Often find good solutions

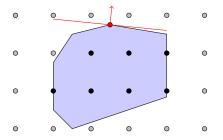




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# Heuristics

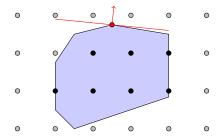
Often find good solutions
 in a reasonable time





- Branch-And-Bound
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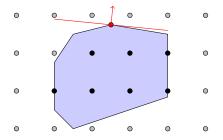
- $\triangleright~$  Often find good solutions
- in a reasonable time
- without any warranty!





- Branch-And-Bound
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- Often find good solutions
- in a reasonable time
- without any warranty!
- $\triangleright \rightsquigarrow$  Integrate into exact solver





# Why use heuristics inside an exact solver?

- ▷ Able to prove feasibility of the model
- Often nearly optimal solution suffices in practice
- Feasible solutions guide remaining search process

## Characteristics



## Why use heuristics inside an exact solver?

- ▷ Able to prove feasibility of the model
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- Feasible solutions guide remaining search process

# Characteristics

- Highest priority to feasibility
- Keep control of effort!
- Use as much information as you can get







- Basics
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- Available Heuristics
  Rounding Heuristics
  (Objective) Diving
  LNS & Others

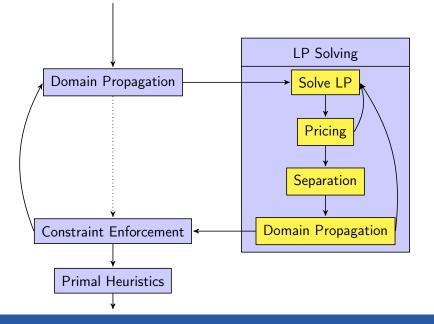




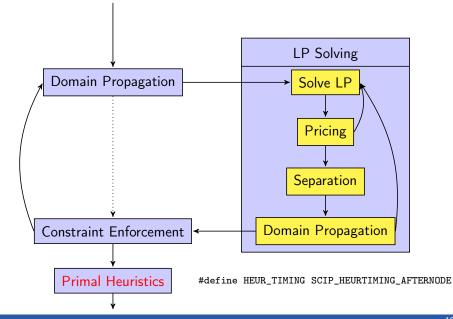
```
SCIPincludeHeur(
  scip,
  "Christofides",
  "Start_heuristic_for_TSP",
  'X',
  -15000,
  0,
  0,
  0,
  SCIP_HEURTIMING_BEFORENODE
);
```

```
// scip
// HEUR_NAME
// HEUR_DESC
// HEUR_DISPCHAR
// HEUR_PRIORITY
// HEUR_FREQ
// HEUR_FREQOFS
// HEUR_MAXDEPTH
// HEUR_TIMING
```

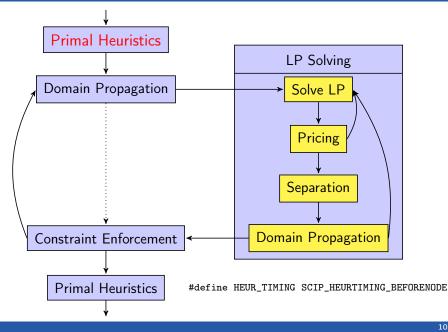




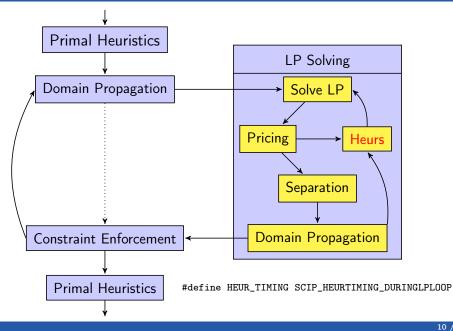
















#### Start heuristics

- Often already at root node
- Mostly start from LP optimum
- Improvement heuristics
  - Require feasible solution
  - Normally at most once for each incumbent





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#define HEUR\_FREQOFS 0





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- if( SCIPgetLPSolstat(scip) != SCIP\_LPSOLSTAT\_OPTIMAL )
   return SCIP\_OKAY;





#### Start heuristics

- Often already at root node
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- Improvement heuristics
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```
if( SCIPgetNSols(scip) <= 0 )
  return SCIP_OKAY;</pre>
```





#### Start heuristics

- Often already at root node
- Mostly start from LP optimum
- Improvement heuristics
  - Require feasible solution
  - Normally at most once for each incumbent

```
struct SCIP_HeurData
{
    SCIP_SOL* lastsol;
}
```



### Five main approaches

- Rounding assign integer values to fractional variables
- Diving: DFS in the Branch-And-Bound-tree
- Objective diving: manipulate objective function
- $\ \ \, \vdash \ \ L_{arge}N_{eighborhood}S_{earch}: \ \ solve \ \ some \ \ subMIP$
- Pivoting: manipulate simplex algorithm



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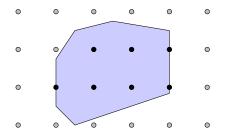


## Implemented into SCIP

- ▷ 5 Rounding heuristics
- 8 Diving heuristics
- ▷ 3 Objective divers
- > 4 LNS improvement heuristics
- $\triangleright$  3 Others

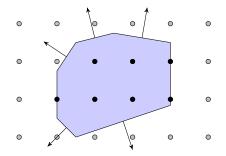


- Variables' locking numbers: Potentially violated rows
- Variables' pseudocosts: Average objective change
- ▷ Special points:
  - LP optimum at root node
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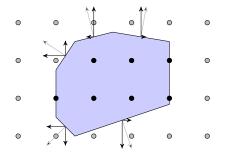


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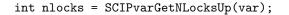


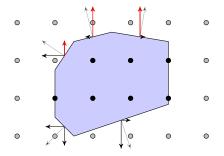
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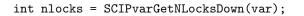
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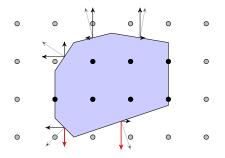






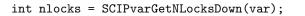
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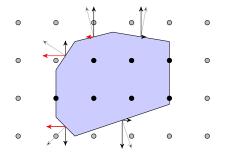






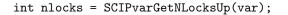
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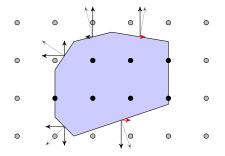






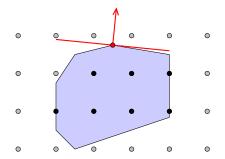
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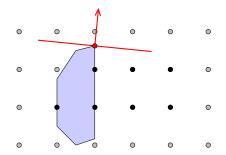
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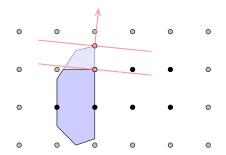
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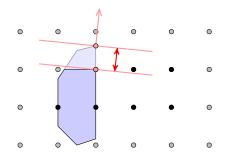
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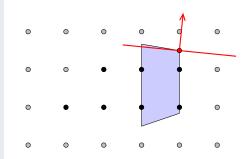
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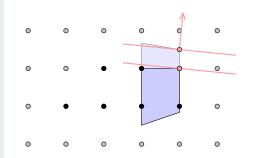
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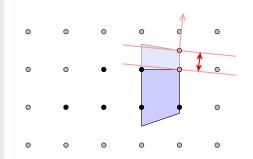
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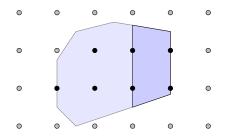
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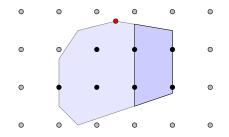
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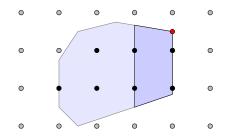


SCIP\_Real rootsolval = SCIPvarGetRootSol(var);



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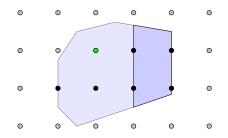


SCIP\_Real solval = SCIPgetSolVal(scip, NULL, var);



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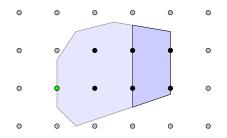


SCIP\_Sol\* bestsol = SCIPgetBestSol(scip); SCIP\_Real solval = SCIPgetSolVal(scip, bestsol, var);



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SCIP\_Sol\*\* sols = SCIPgetSols(scip); SCIP\_Real solval = SCIPgetSolVal(scip, sols[i], var);





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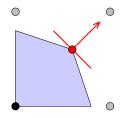


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- Rounding may violate constraints,
- Shifting may unfix integers,
- ▷ Integer Shifting finally solves an LP.



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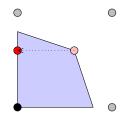
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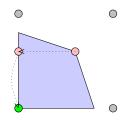
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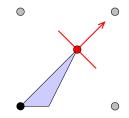
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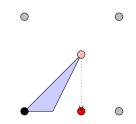
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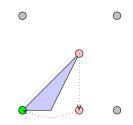
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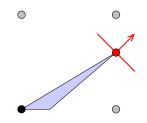
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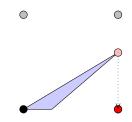
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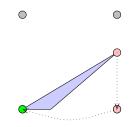
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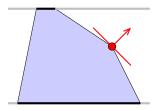
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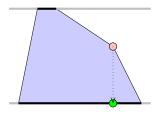


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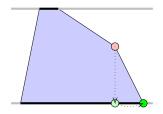


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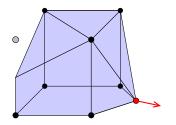




- 1.  $\bar{x} \leftarrow LP$  optimum;
- 2. Fix all integral variables:  $x_i := \bar{x}_i \quad \forall i : \bar{x}_i \in \mathbb{Z};$
- Reduce domain of fractional variables: x<sub>i</sub> ∈ { [x<sub>i</sub>]; [x<sub>i</sub>]};
- 4. Solve the resulting subMIP

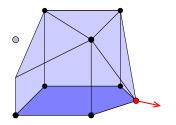


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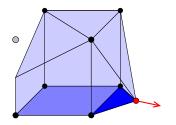


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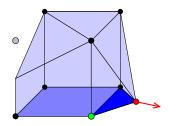


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## Observations

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- > Yields best possible rounding
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# Results



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# Results

- ▷ 82 of 129 test instances are roundable
- Rens finds a global optimum for 23 instances!
- Dominates all other rounding heuristics





#### Introduction

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### Idea

- Alternately solve the LP and round a variable
- Simulates DFS in Branch-And-Bound-tree
- Complementary target for branching
- Backtracking possible

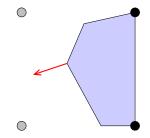


# Applied branching rules

- Fractional Diving: lowest fractionality
- Coefficient Diving: lowest locking number
- Linesearch Diving: highest increase since root
- Guided Diving: lowest difference to best known solution
- Pseudocost Diving: highest ratio of pseudocosts
- Vectorlength Diving: lowest ratio of objective change and number of rows containing the variable

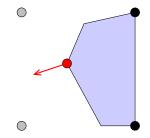


- 1. Solve LP;
- 2. Round LP optimum;
- 3. If feasible:
- 4. Stop!
- 5. Else:
- 6. Change objective;
- 7. Go to 1;



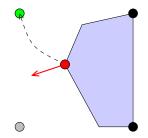


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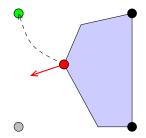


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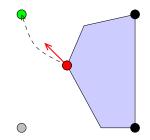
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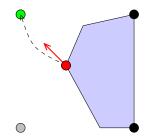
 $\Delta(x,\tilde{x}) = \sum |x_j - \tilde{x}_j|$ 





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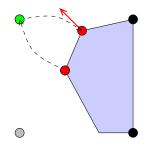
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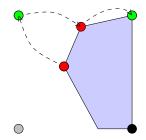
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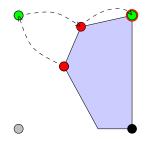


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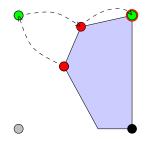


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# Objective Feasibility Pump (Achterberg & B.)

#### Improvements

- ▷ Objective  $c^T x$  regarded at each step:  $\tilde{\Delta} := (1 - \alpha)\Delta(x) + \alpha c^T x$ , with  $\alpha \in [0, 1]$
- Algorithm able to resolve from cycling
- Quality of solutions much better

# Results

# Objective Feasibility Pump (Achterberg & B.)

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### Results

- ▷ Finds a solution for 74% of the test instances
- ▷ On average 5.5 seconds running time
- $\triangleright$  Optimality gap decreased from 107% to 38%



# Other objective divers

- Objective Pseudocost Diving
  - analogon to Pseudocost Diving
  - Punishment by high objective coefficients
- Rootsolution Diving
  - analogon to Linesearch Diving
  - Objective function faded out





#### Introduction

- Basics
- Integration Into SCIP

#### 2 Available Heuristics

- Rounding Heuristics
- (Objective) Diving
- LNS & Others

#### 3 Remarks & Results



#### Idea: Create subMIP by fixing variables or adding constraints

Approaches

- ▷ Rins: fix variables equal in LP optimum and incumbent
- ▷ Crossover: fix variables equal in different feasible solutions
- Mutation: fix variables randomly
- ▷ Local Branching: add distance constraint wrt. incumbent

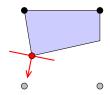


- ⊳ 1-Opt
  - Shifts value of integer variable
  - Solves LP afterwards



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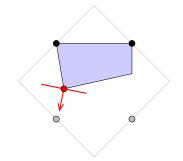
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- Ray shooting algorithm





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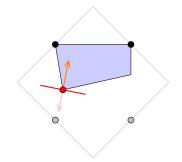
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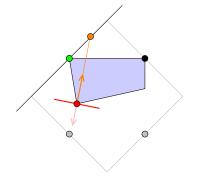


# Other Heuristics

# Combinatorial heuristics

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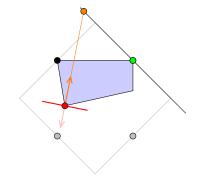


# Other Heuristics

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#### Some tips and tricks

- ▷ Use limits which respect the problem size
- LNS: stalling node limit
- Diving: try simple rounding on the fly
- Favor binaries over general integers
- Avoid cycling without randomness



# Some tips and tricks

- ▷ Use limits which respect the problem size
- LNS: stalling node limit
- Diving: try simple rounding on the fly
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```
int nlpiterations = SCIPgetNNodeLPIterations(scip);
int maxlpiterations = heurdata→maxlpiterquot * nlpiterations;
```



### Some tips and tricks

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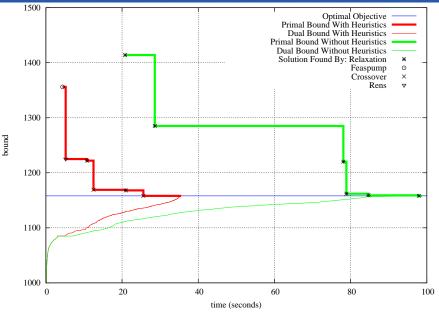
```
SCIP_CALL(
```

```
SCIPsetLongintParam(subscip,"limits/stallnodes",nstallnodes)
```

);



# An Example





# Remarks & Conclusions

# Single heuristics

- ▷ Deactivating a single heuristic yields 1%-6% degradation
- No heuristic dominates the others
- Coordination important

Overall effect (SCIP 0.82b)



# Remarks & Conclusions

# Single heuristics

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# Overall effect (SCIP 0.82b)

- Better pruning, earlier fixing
- $\triangleright$  7% less instances without any solution
- $\triangleright$  5% more instances solved within one hour
- only half of the branch-and-bound-nodes
- only half of the solving time



# Remarks & Conclusions

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- $\triangleright$  only half of the branch-and-bound-nodes  $\rightsquigarrow$  Not this much
- only half of the solving time

 $\rightsquigarrow$  in SCIP 1.00



#### To be implemented

- DINS heuristic
- $\triangleright \ \text{k-opt for } k>1$
- probing heuristics

#### Known problems

- Coordination could be strengthened
- Often poor for pure combinatorial problems



# Primal Heuristics in SCIP

Timo Berthold Zuse Institute Berlin

**DFG** Research Center MATHEON Mathematics for key technologies



Berlin, 10/11/2007