Paths and Complexity

Ralf Borndörfer

2015 Workshop on
Combinatorial Optimization with Applications in
Transportation and Logistics

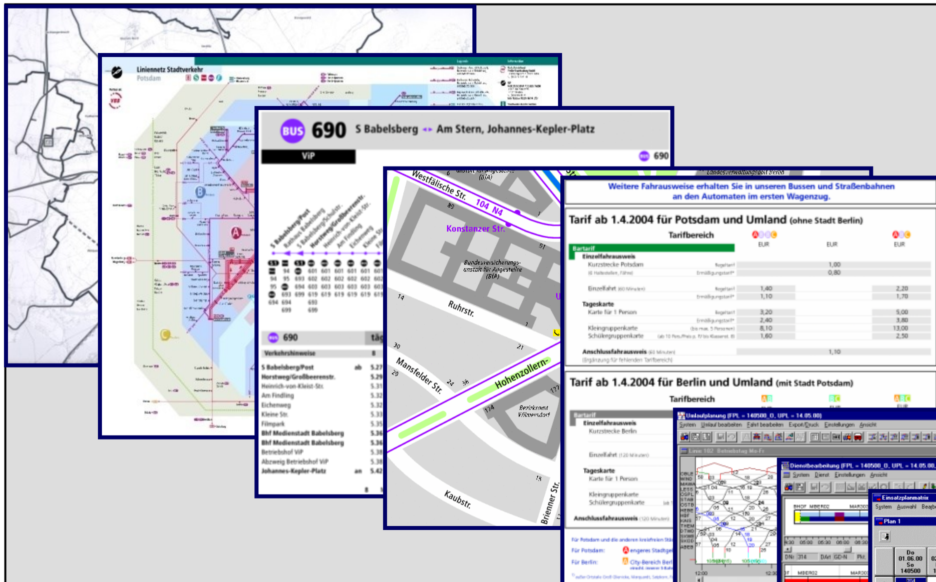
Beijing, 28.07.2015

Outline

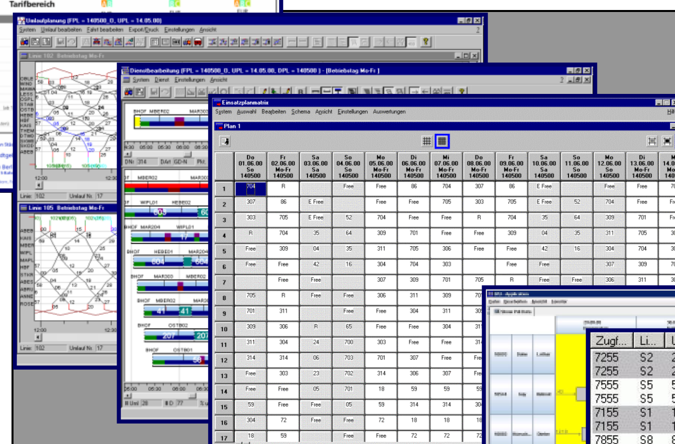
- Traffic Optimization
- Bridges of Königsberg
- Travelling Salesmen
- S-Bahn Challenge
- Shortest Paths
- Fuel Efficient Aircraft Trajectories

Planning Problems in Public Transit

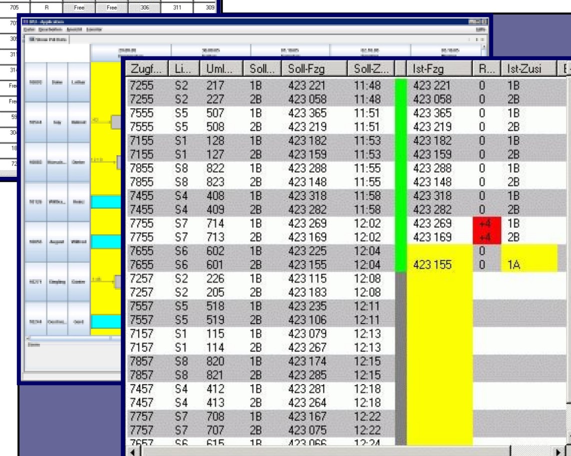
Service Design



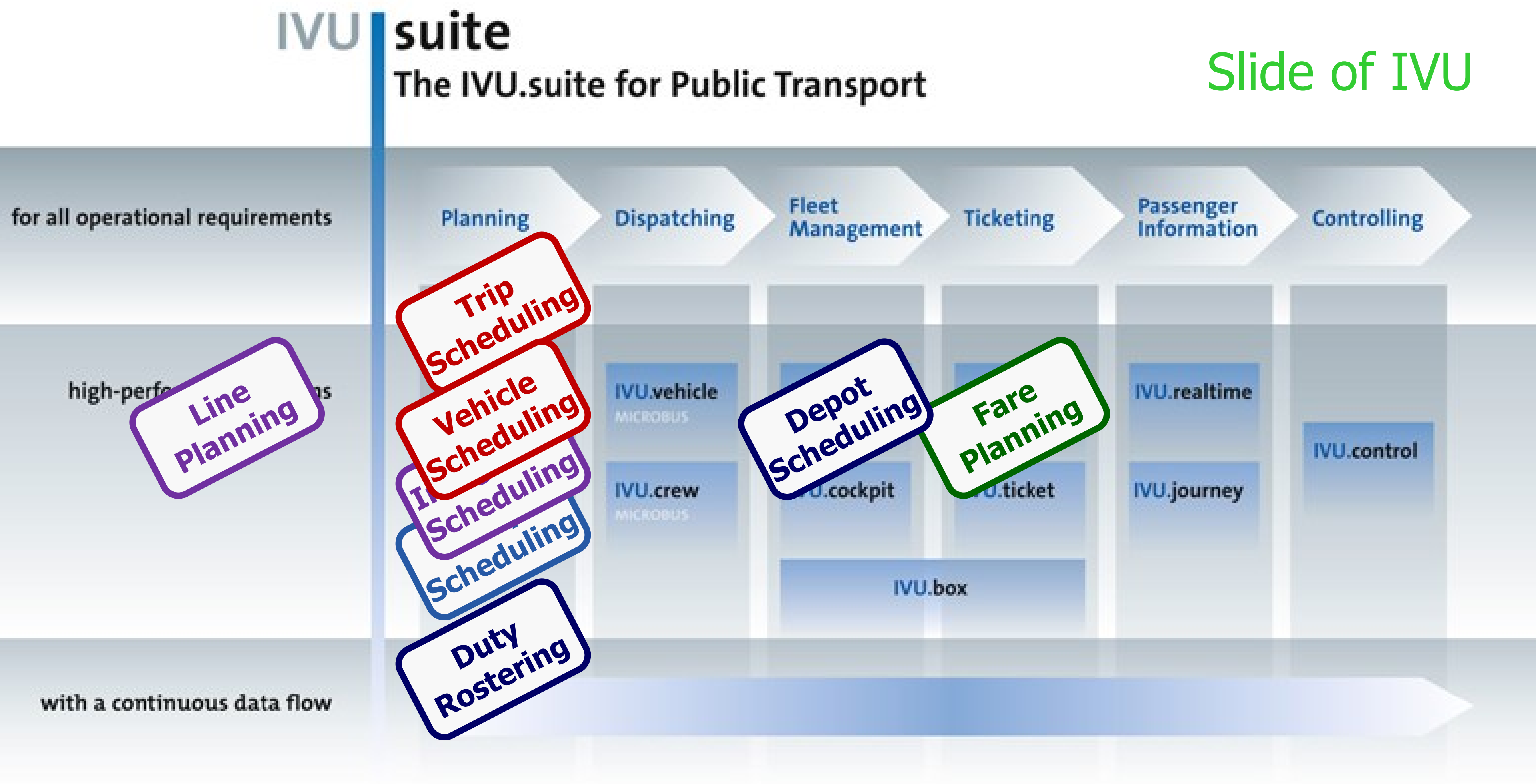
Operational Planning



Operations Control



Optimization in Public Transit



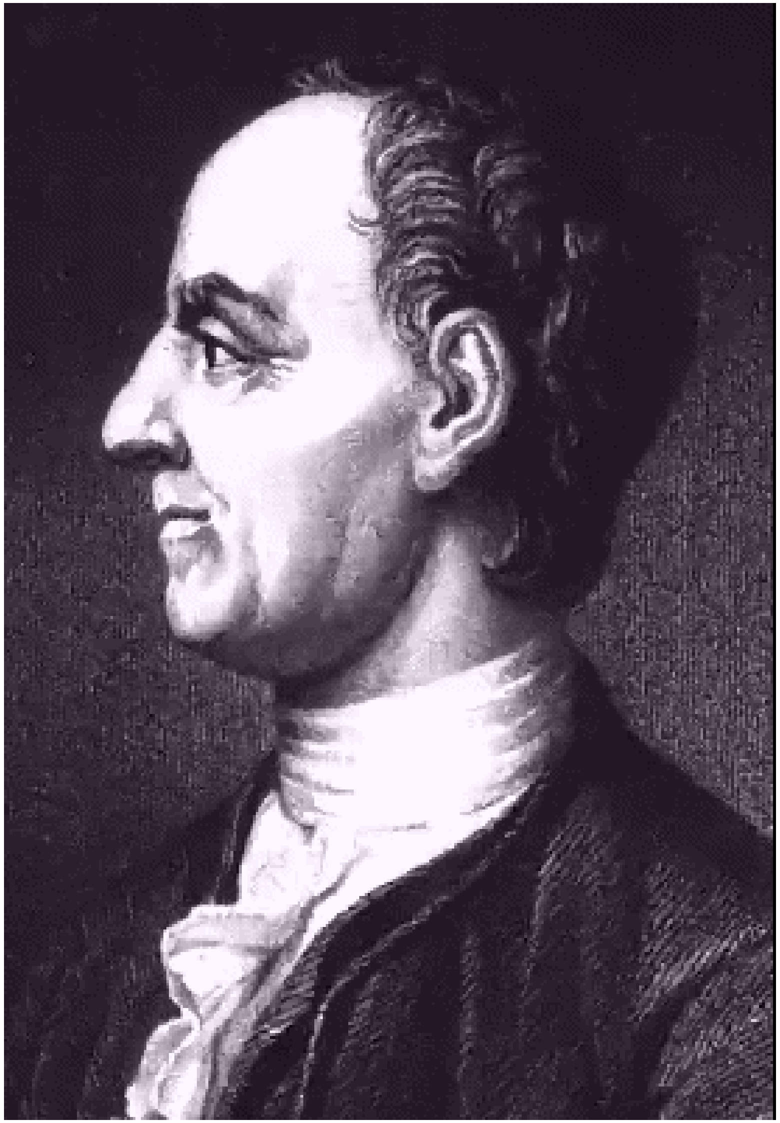
Multicom. Flow | **Set Partitioning** | **Integrated Scheduling** | **Other**

Overview

Freie Universität Berlin

1. Paths and Complexity
2. Vehicle Scheduling and Multicommodity Flows
3. Crew Scheduling and Column Generation
4. Track Allocation and Configurations
5. Vehicle Rotation Planning and Hyperassignments
6. Line Planning and Path Connectivity

Leonhard Euler (1707-1783)



e

π

i

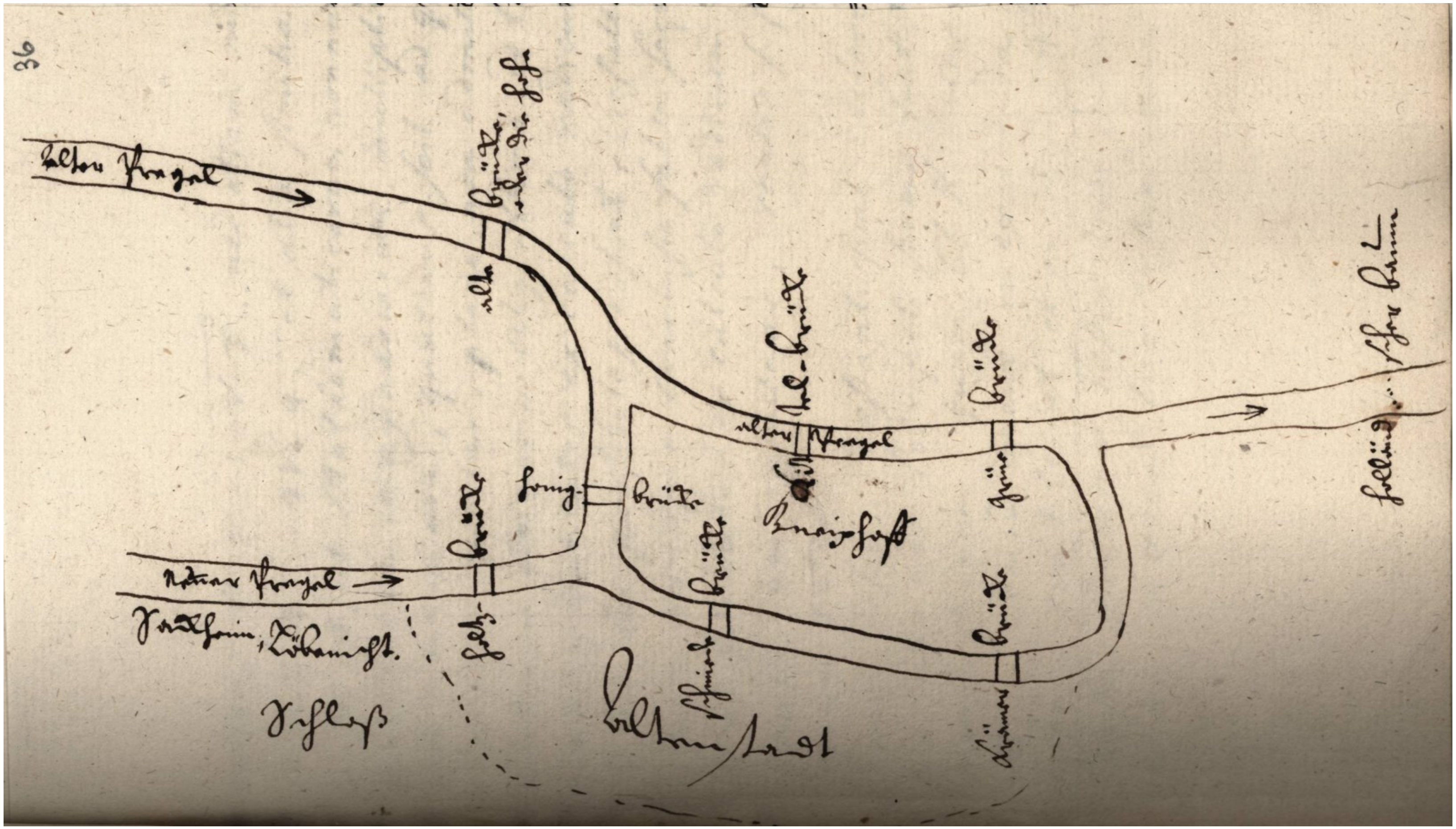
sin

cos

Σ

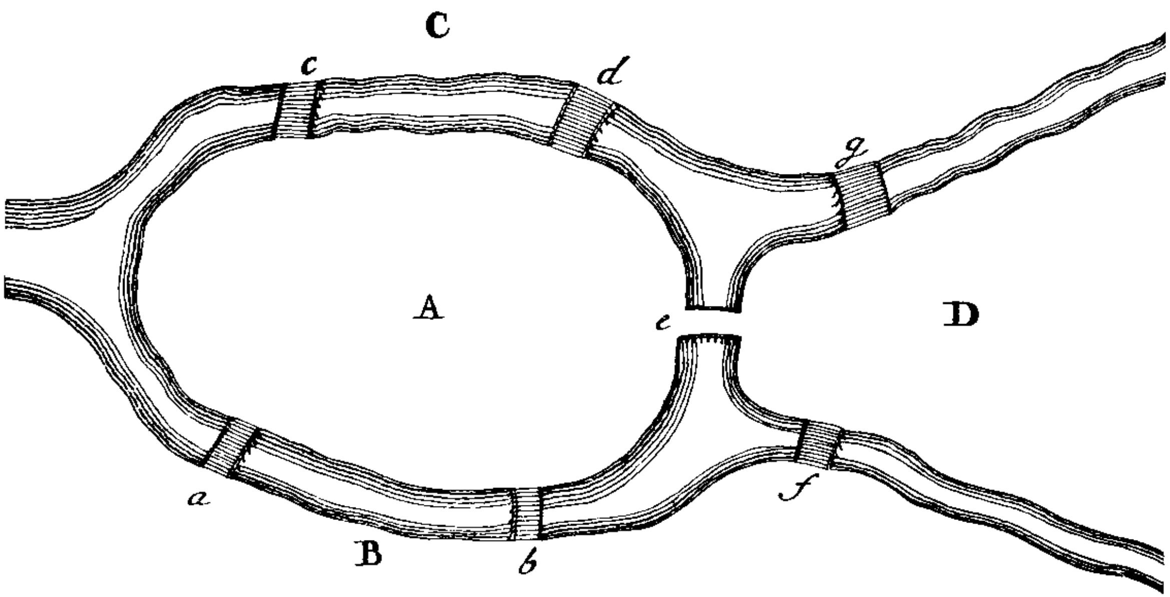
f(x)

… receives a letter of Mayor Carl Leonhard Ehler



Scan by Wladimir Velminski for his thesis in the History of Arts supervised by Prof. Bredekamp (HU Berlin, ~2008)

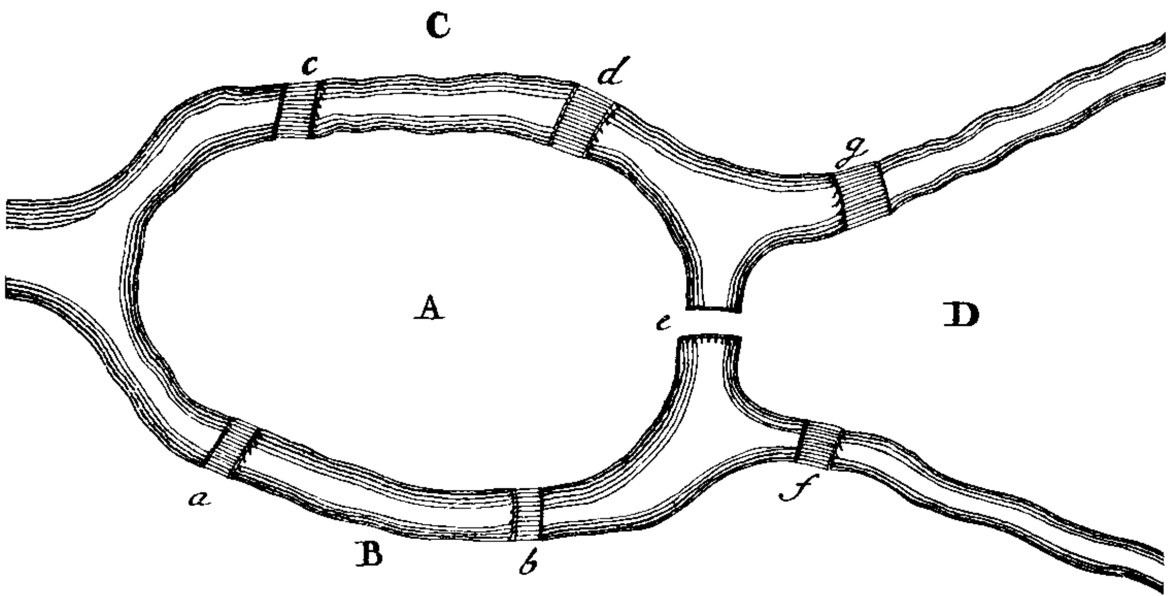
The Problem of the Königsberg Bridges (1736)



a Grüne Brücke
b Köttelbrücke
c Krämerbrücke
d Schmiedebrücke
e Honigbrücke
g Holzbrücke
f Hohe Brücke

„The problem, which I am told is widely known, is as follows: in Königsberg in Prussia, there is an island A, called „the Kneiphof"; the river which surrounds it is divided into two branches, as can be seen in Fig. 1, and these branches are crossed by seven bridges, a, b, c, d, e, f und g. Concerning these bridges, it was asked whether anyone could arrange a route in such a way that he would cross each bridge once and only once. I was told that some people asserted that this impossible, while others were in doubt; but nobody would actually assert that it could be done. From this, I formulated the general problem: whatever be the arrangement and division of the river into branches, and however many bridges there be, can one find out whether or not it is possible to cross each bridge exactly once?"

The Problem of the Königsberg Bridges (1736)



a Grüne Brücke
b Köttelbrücke
c Krämerbrücke
d Schmiedebrücke
e Honigbrücke
g Holzbrücke
f Hohe Brücke

„As far as the problem of the seven bridges of Königsberg is concerned, it can be solved by making an exhaustive list of all possible routes, and then finding whether or not any route satisfies the conditions of the problem. Because of the number of possibilities, this method of solution would be too difficult and laborious, and in other problems with more bridges it would be impossible. Moreover, if this method is followed to its conclusion, many irrelevant routes will be found, which is the reason for the difficulty of this method. Hence I rejected it, and looked for another method concerned only with the problem of whether or not the specified route could be found.; I considered that such a method would be much simpler.“

Combinatorial Problem

Input: A finite (implicitly given) set N={1,...,n}, a predicate f: N $\rightarrow$ {true, false}.

Question: Is there an element i s.t. f(i) = true?

Combinatorial Explosion

Number of routes (worst case)

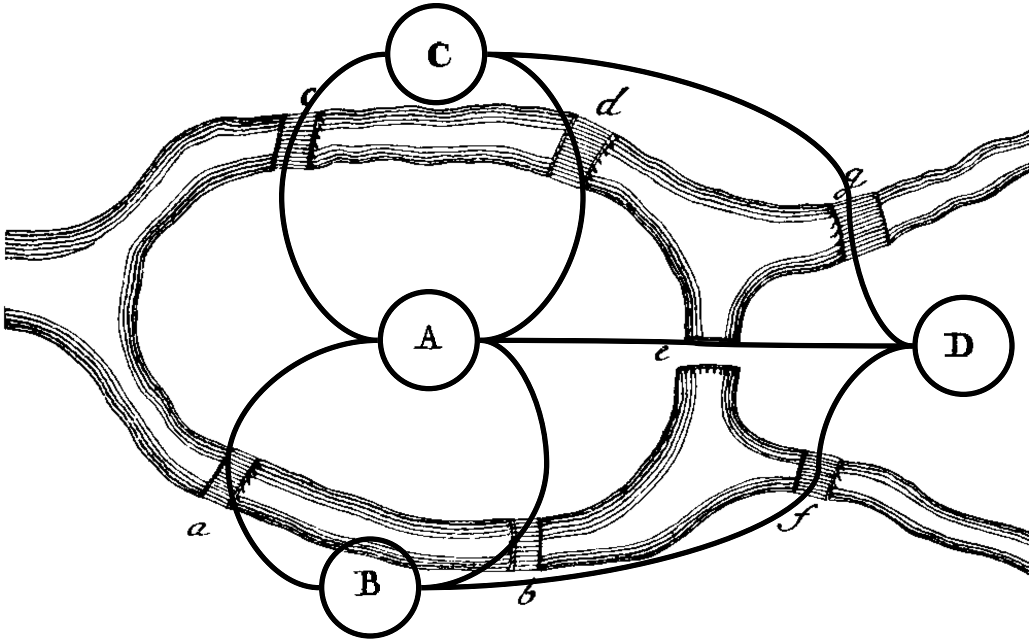
Here: $7!/2 = 7*6*5*4*3 = 2.520$

In general: $= O(n^n)$

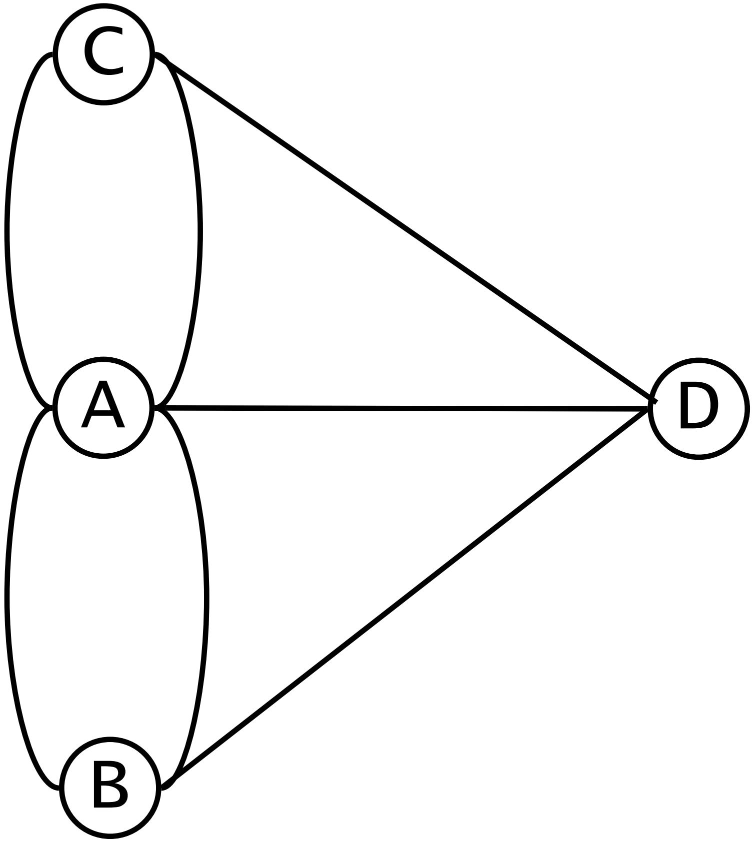
Stirling formula: $n!/2 \approx n^n e^{-n} \sqrt{2\pi n} / 2$

a		b		c	
ab	ac	ba	bc	ca	cb
abc	acb	bac	bca	cab	cba

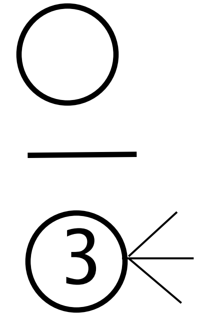
Geometria situs (Graph Theory)



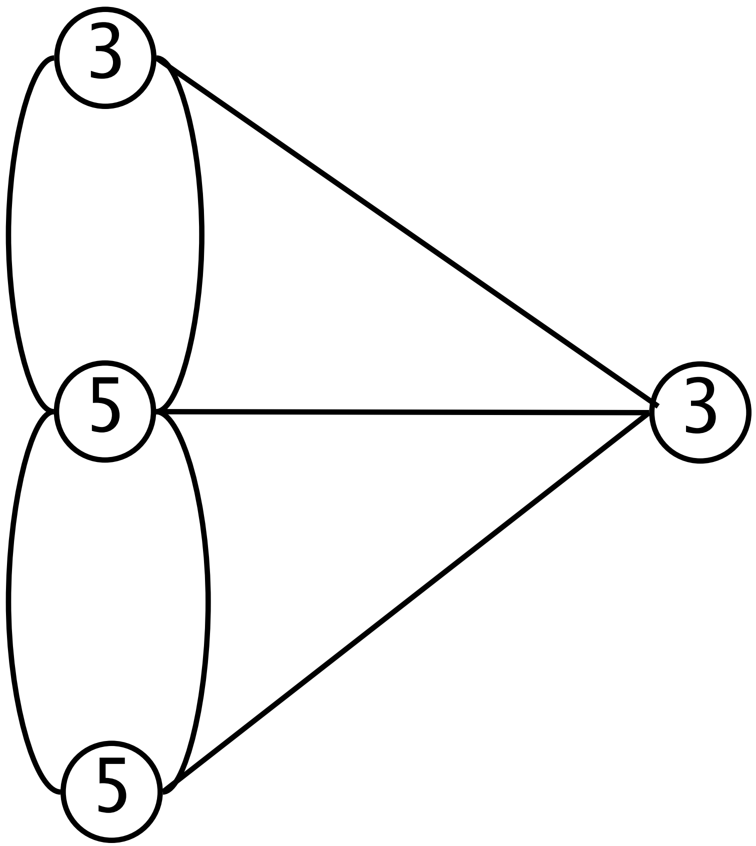
Geometria situs (Graph Theory)



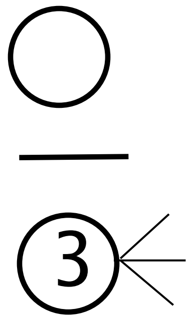
- Node
- Edge
- Degree



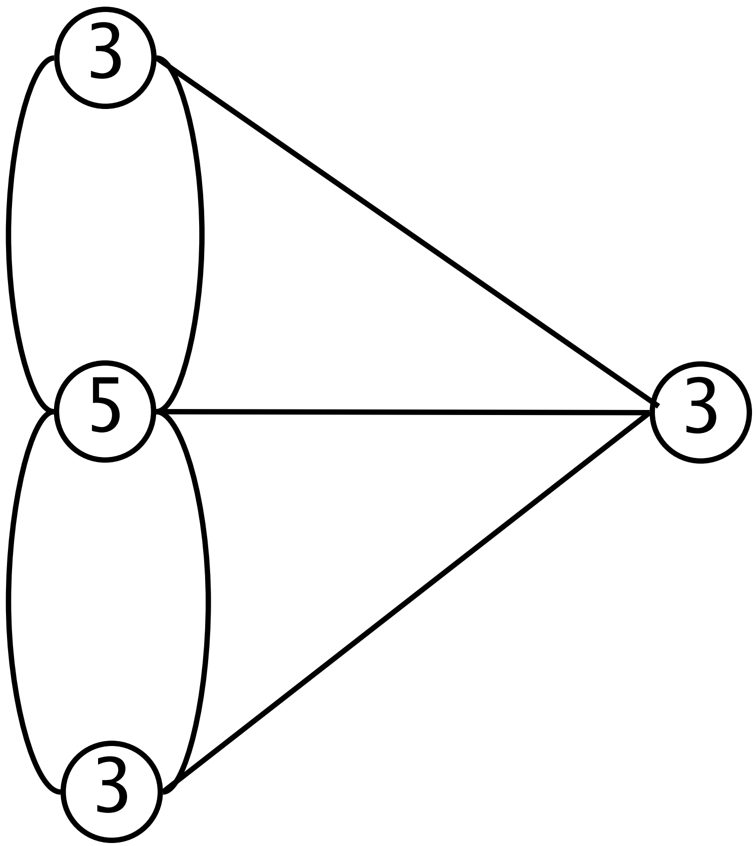
Geometria situs (Graph Theory)



- Node
- Edge
- Degree



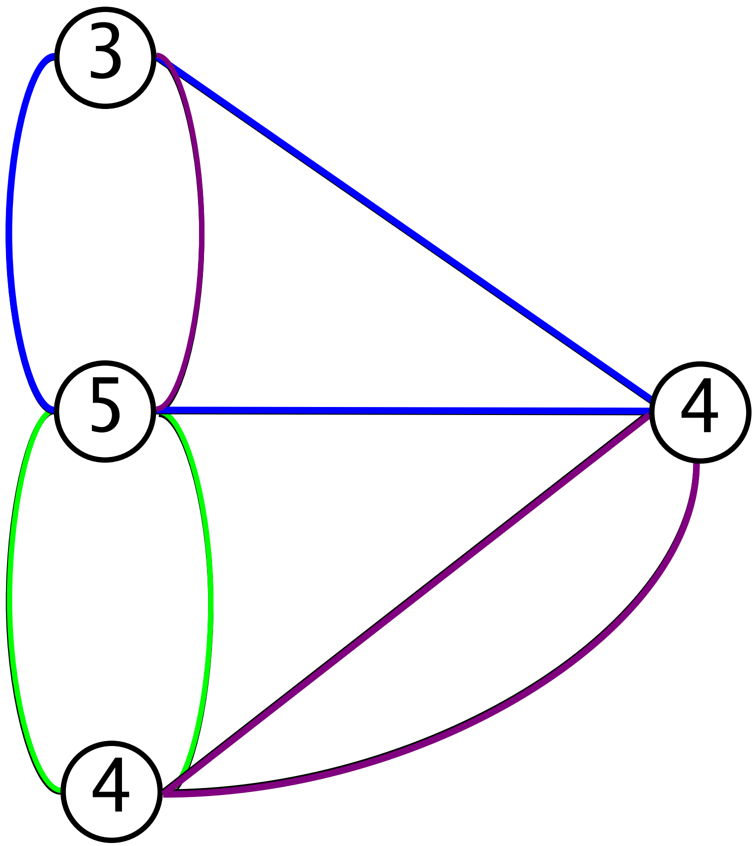
Geometria situs (Graph Theory)



Theorem: An Euler tour can only exist if at most 2 nodes have odd degree.

Proof. Inner nodes are even.

Geometria situs (Graph Theory)



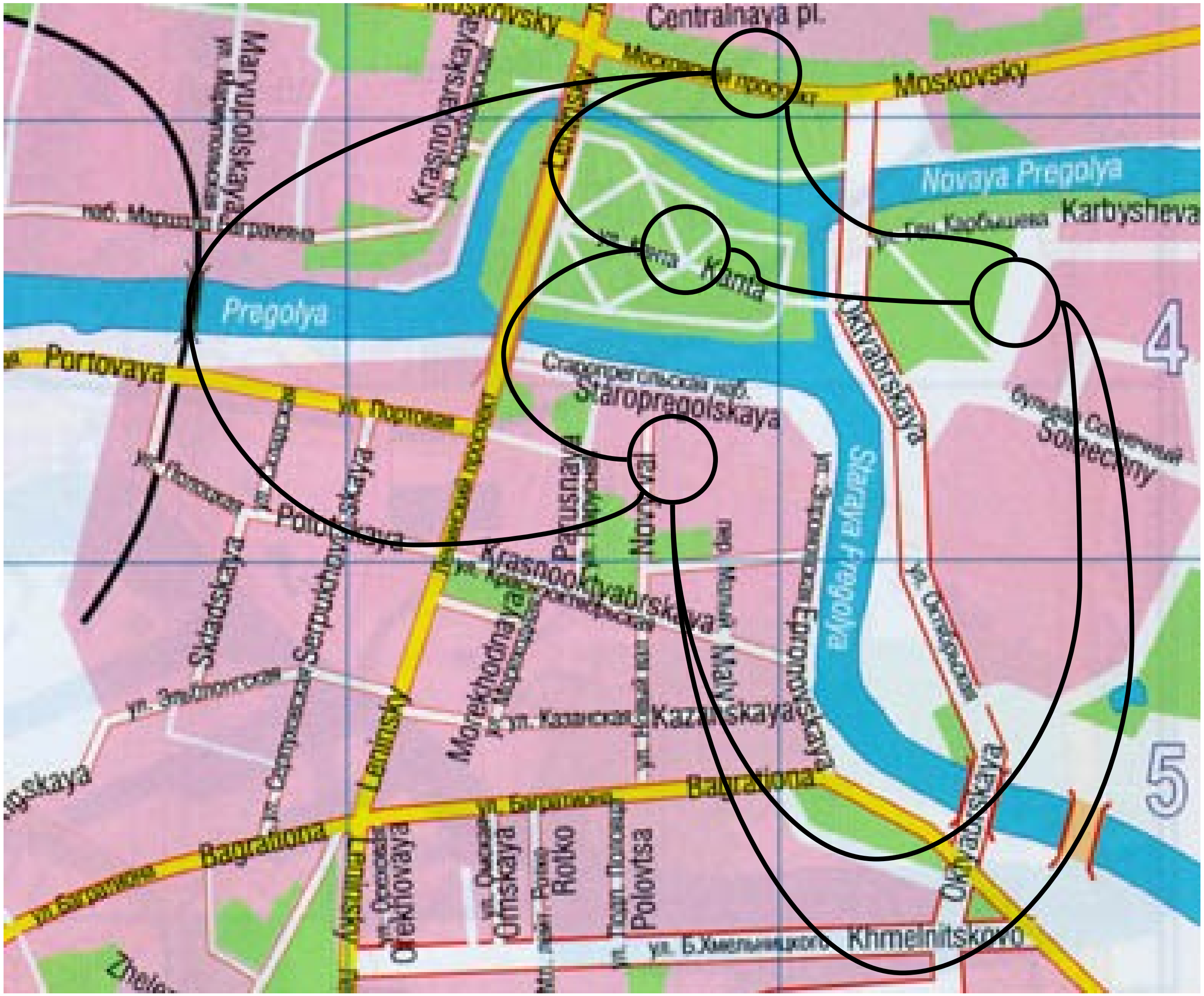
Theorem: An Euler tour exists if and only of at most 2 nodes have odd degree.

Proof

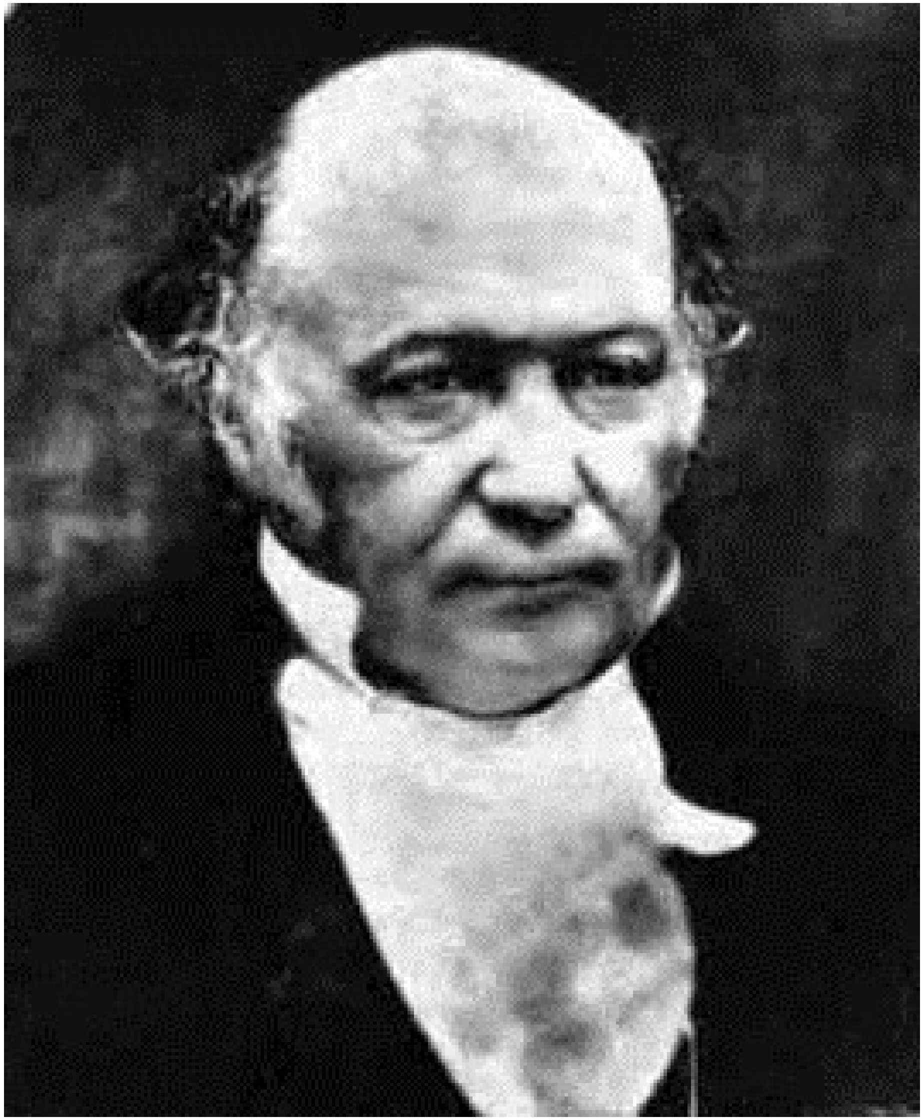
$\Rightarrow$: inner nodes even

$\Leftarrow$: path + cycles

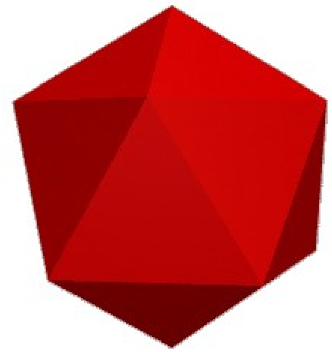
Kaliningrad

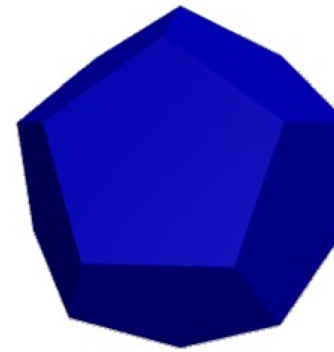


Sir William Rowan Hamilton (1805-1865)

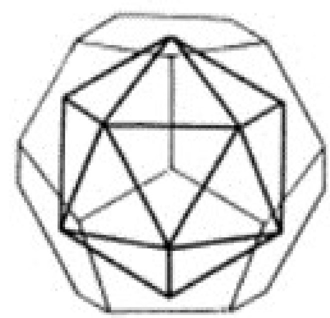


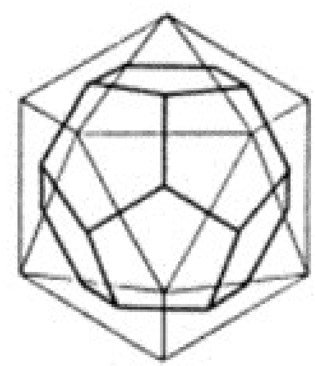
The Icosian Game (1856)



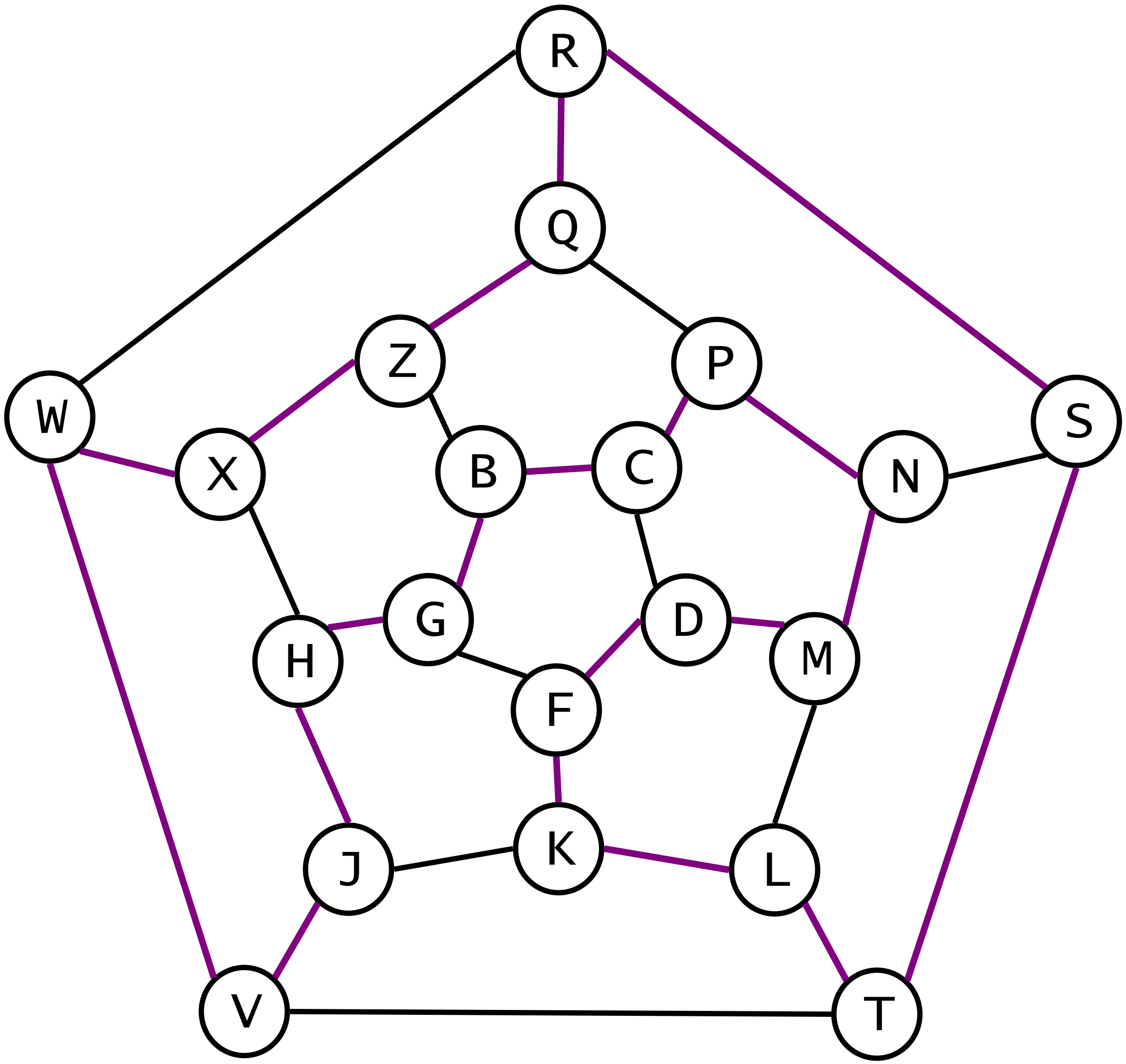


Icosahedron (20) Dodecahedron (12)





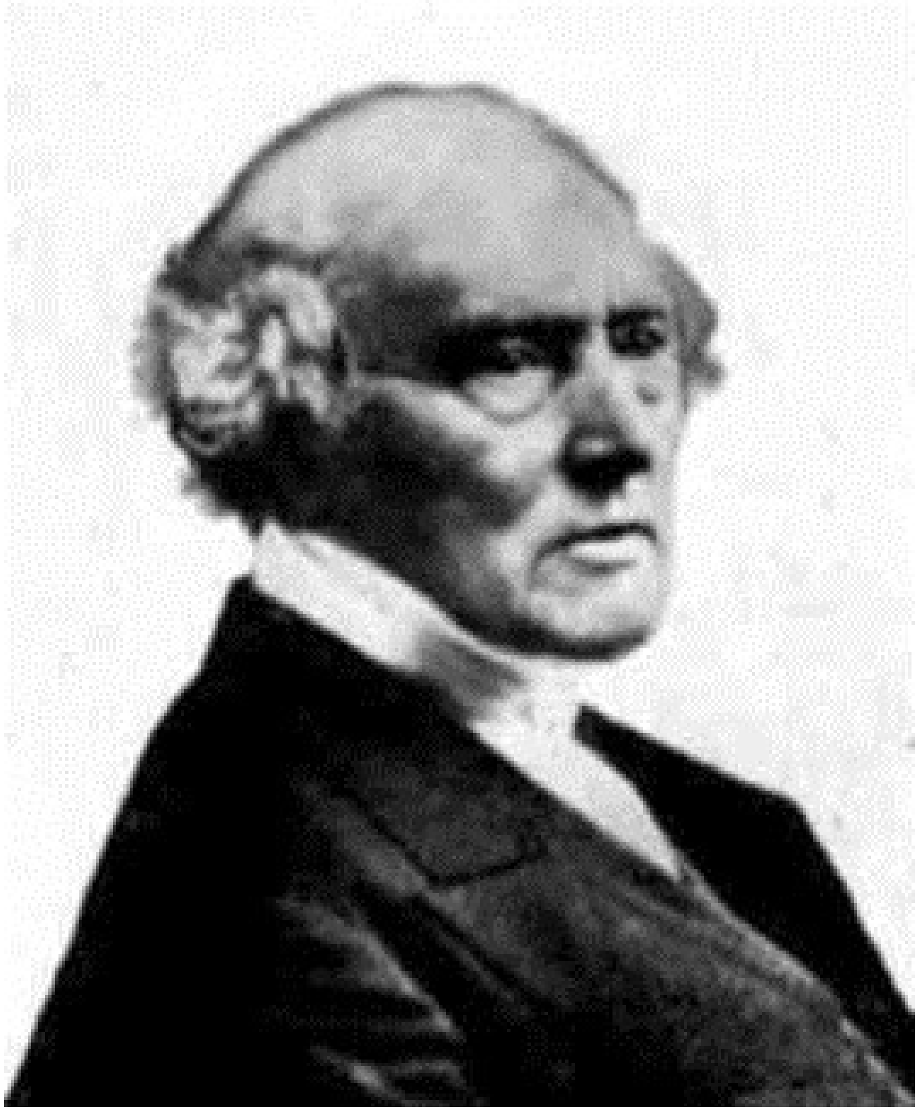
Is there a closed roundtrip (Hamiltonian cycle)?



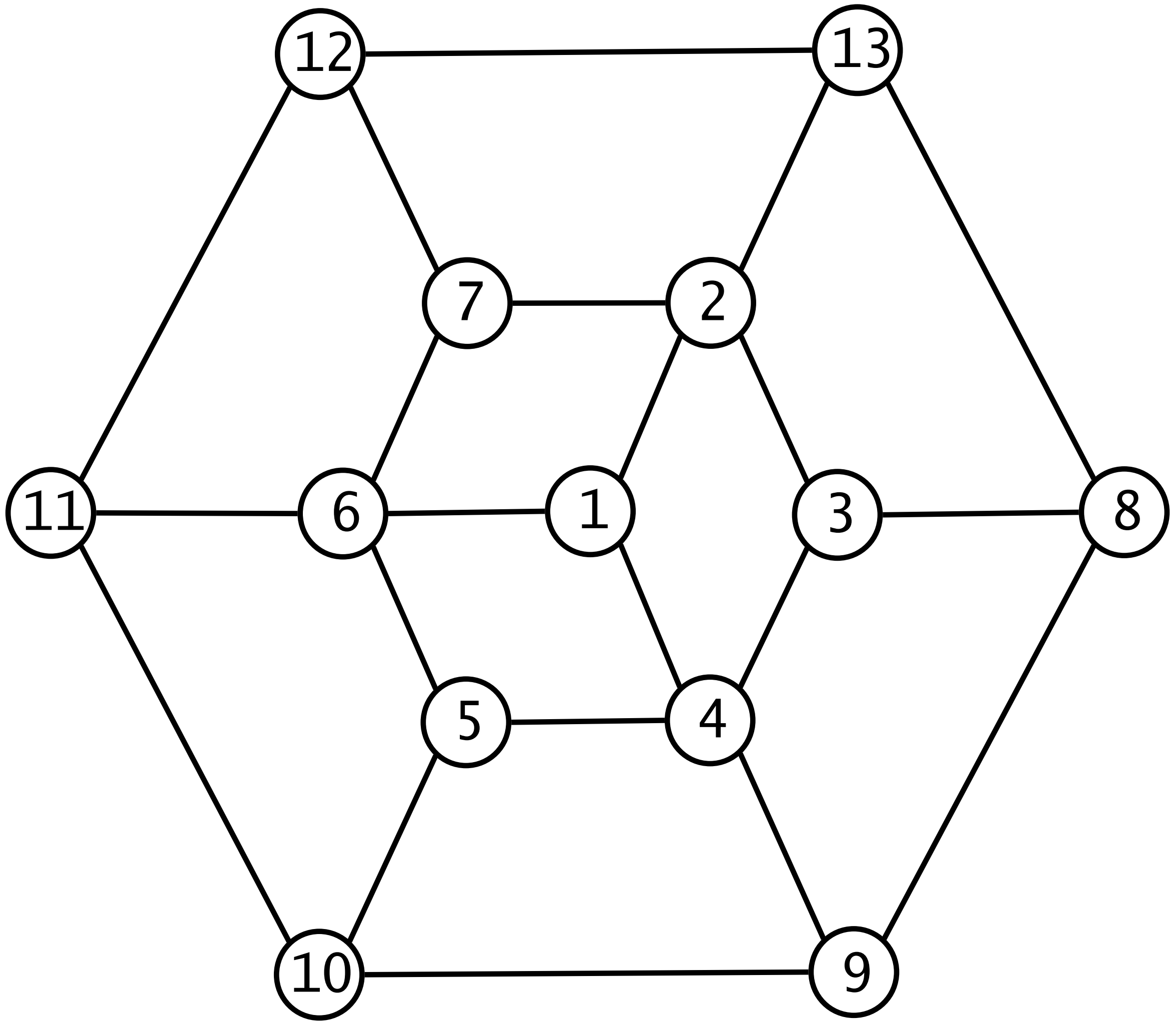
B C P N M D F K L T S R Q Z X W V J H G

B C P N M D F G H X W V J K L T S R Q Z

Thomas Penyngton Kirkman (1806-1895)



The Cell of the Bee



o e o e … o e

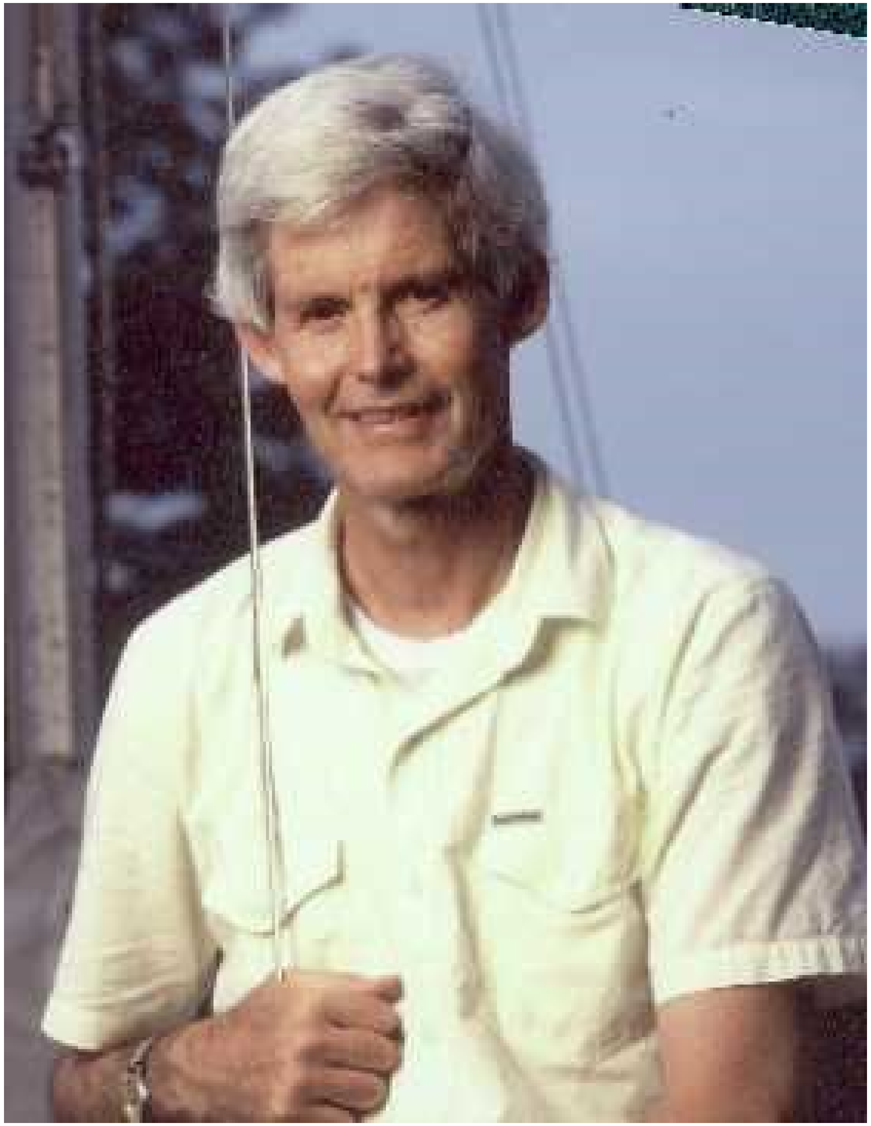
Enumeration?

- Number of Hamiltonian cycles (worst case)
  n cities: $n!/2 \approx n^n e^{-n} \sqrt{2\pi n}/2$ (Stirling formula)
- Exponential effort: f(n) = 2^n, n^n, etc.
  Polynomial effort: f(n) = p(n) = n, 1.000n, n^3, n^5, etc.

linear	quadratic	cubic	exponential	doubly exp.
n	n^2	n^3	2^n	n^n
10	100	1.000	1.024	10^{10}
100	10.000	10^6	10^{30}	10^{200}
1.000	10^6	10^9	10^{300}	10^{3000}
10.000	10^8	10^{12}	10^{3000}	10^{50000}

- Is there a polynomial method?

Stephen Cook



The Satisfiability Problem

Input: A set $U=\{x_1,\ldots,x_n\}$ of variables and a set $C=\{c_1,\ldots,c_m\}$ of conjunctions (product of ANDs) of variables from U.

Question: Is there a satisfying truth assignment for C (assignment of values true or false to the variables in U such that in each clause at least one variable is true)?

Example:

$(x1 \vee \neg x2 \vee x3) \wedge (\neg x1 \vee x2 \vee x3) \wedge (\neg x1 \vee x2 \vee \neg x3)$

The Classes NP, NPC, and NPH

Non-deterministic polynomial time algorithm A

Input: Instance I of problem P of size n bits

Algorithm: Guess solution L and check in time poly(n) that L solves I.

NP: Class of decision problems (answer yes or no) for which such an algorithm exists.

NPC: Class of decision problems to which NP can be reduced.

NPH: Class of optimization problems to which NPC can be reduced.

Theorem: SAT $\in$ NPC.

Theorem: HC $\in$ NPC.

Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven *Prize Problems*. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each. During the Millennium Meeting held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled *The Importance of Mathematics*, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

One hundred years earlier, on August 8, 1900, David Hilbert delivered his famous lecture about open mathematical problems at the second International Congress of Mathematicians in Paris. This influenced our decision to announce the millennium problems as the central theme of a Paris meeting.

The rules for the award of the prize have the endorsement of the CMI Scientific Advisory Board and the approval of the Directors. The members of these boards have the responsibility to preserve the nature, the integrity, and the spirit of this prize.

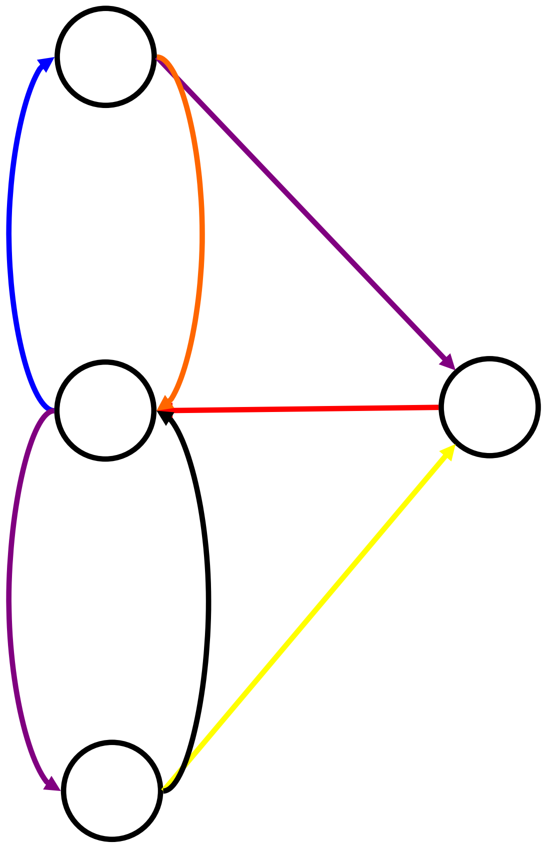
Paris, May 24, 2000

Please send inquiries regarding the Millennium Prize Problems to prize.problems@claymath.org.

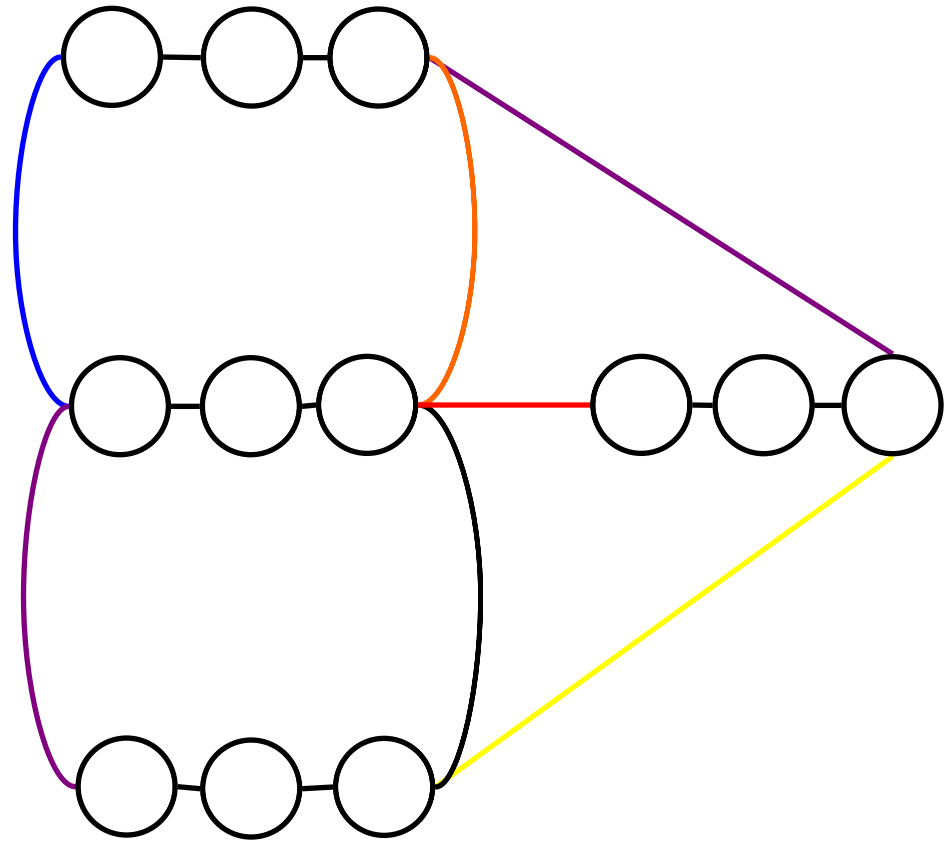
- Birch and Swinnerton-Dyer Conjecture
- Hodge Conjecture
- Navier-Stokes Equations
- P vs NP
- Poincaré Conjecture
- Riemann Hypothesis
- Yang-Mills Theory

- Rules
- Millennium Meeting Videos

Directed Hamiltonian Cycle

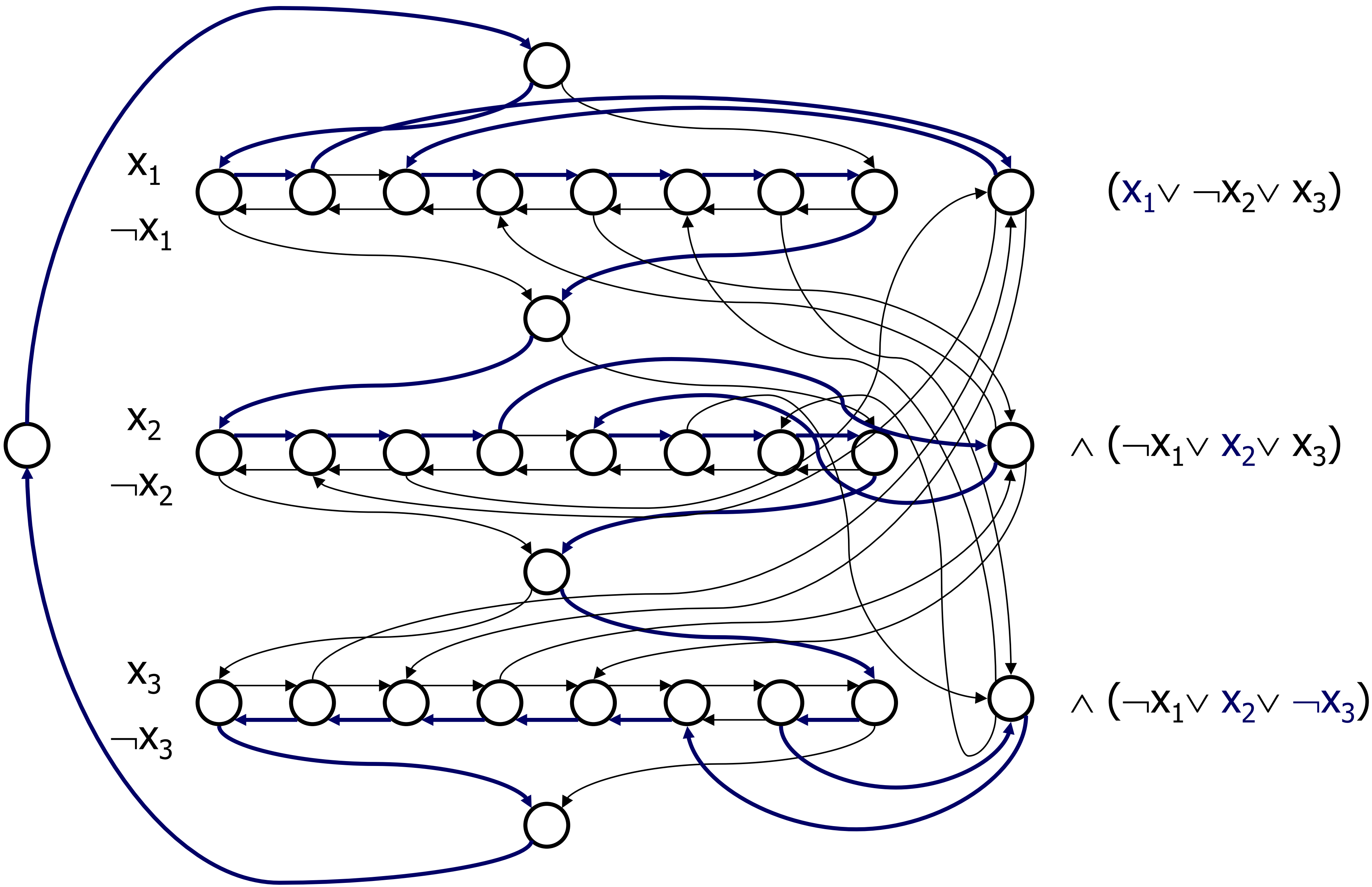


Directed
Hamiltonian Cycle
Problem

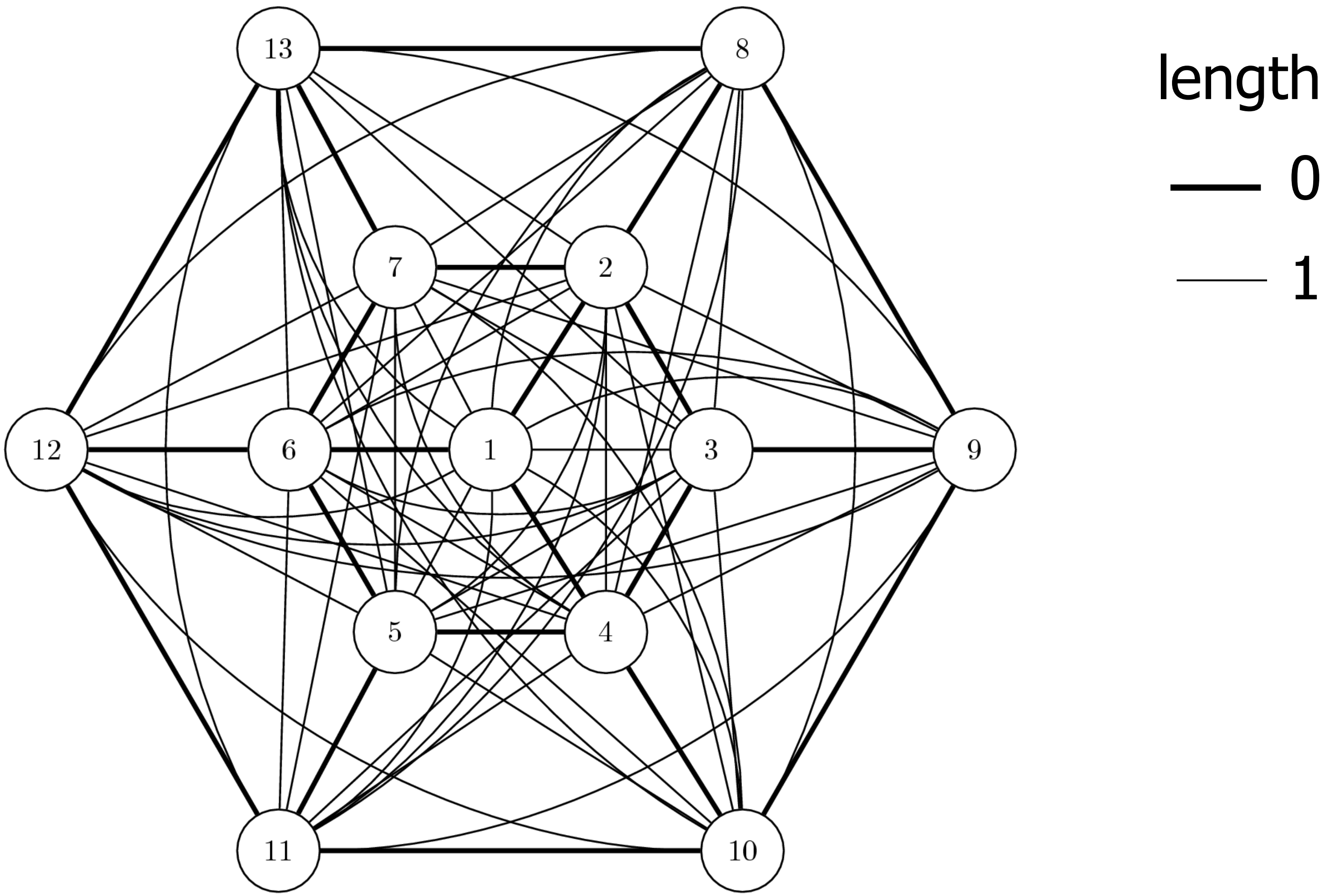


(Undirected)
Hamiltonian Cycle
Problem

The Satisfiability Problem



Theorem: HC $\in$ NPH.



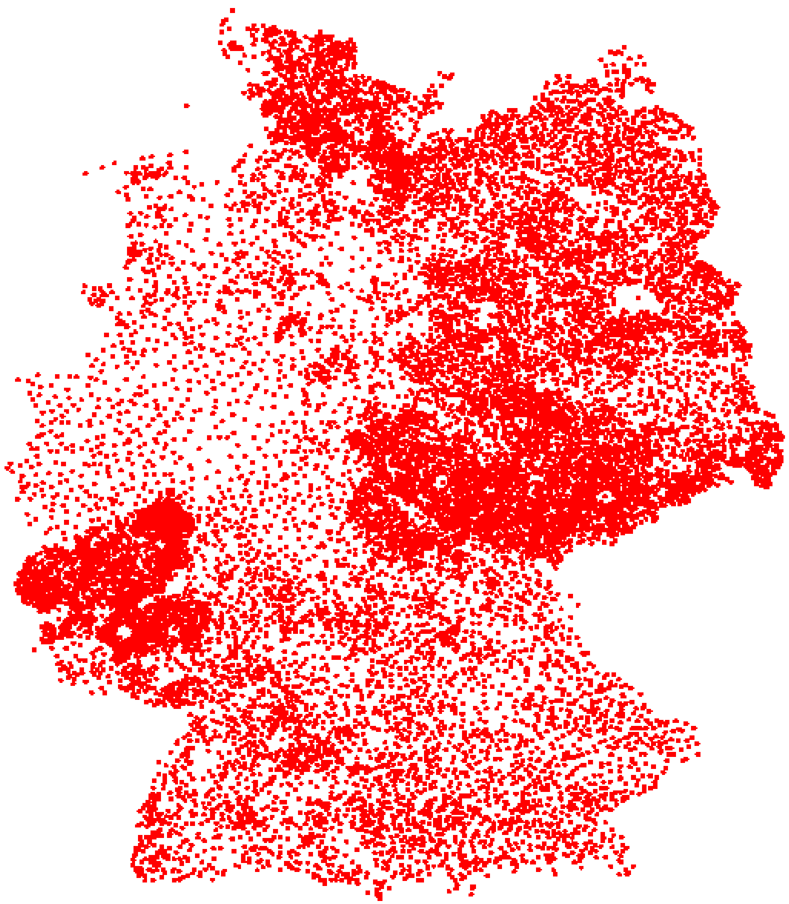
Combinatorial Optimization Problem

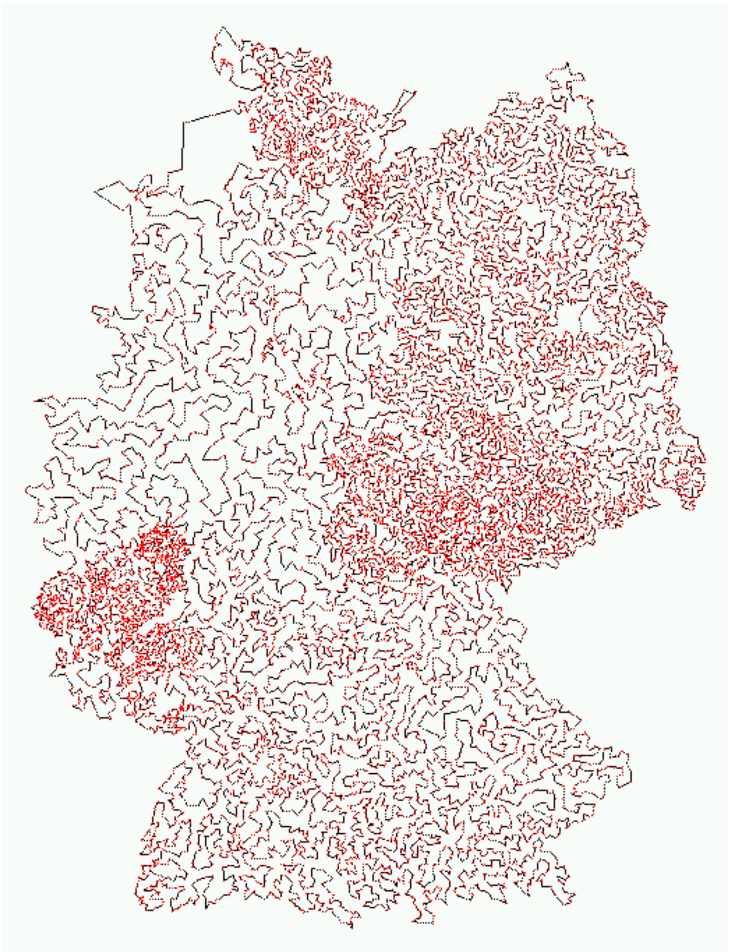
Input: A finite (implicitly given) set N={1,...,n}, an objective function f: N $\rightarrow$ IR.

Question: What is the minimum of f over N?

$$\boxed{\min f(x),\ x \in N}$$

The Travelling Salesman Problem





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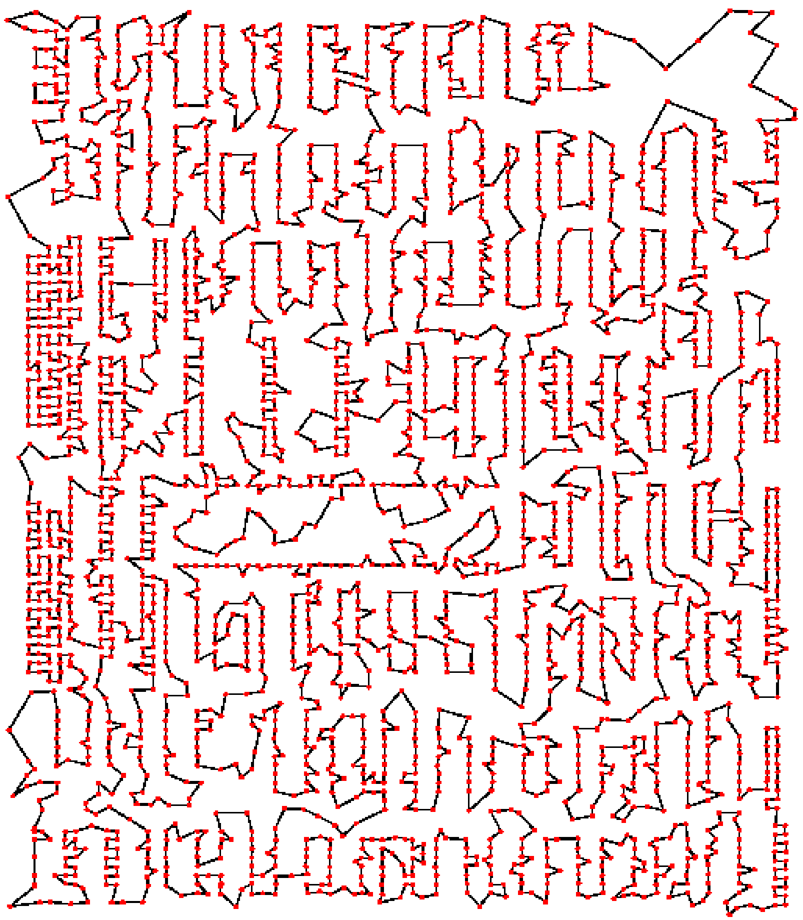
The Travelling Salesman Problem



PCB3038

TSP World Records

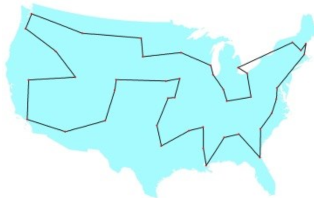
1954: dantzig42

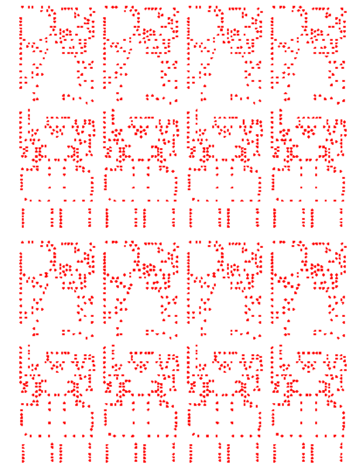
1977: gr120

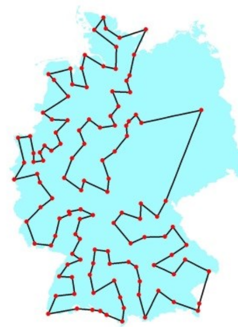
1987: gr666, pr2392

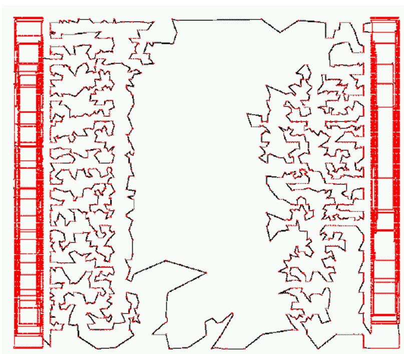
1994: pla7397

2001: d15112

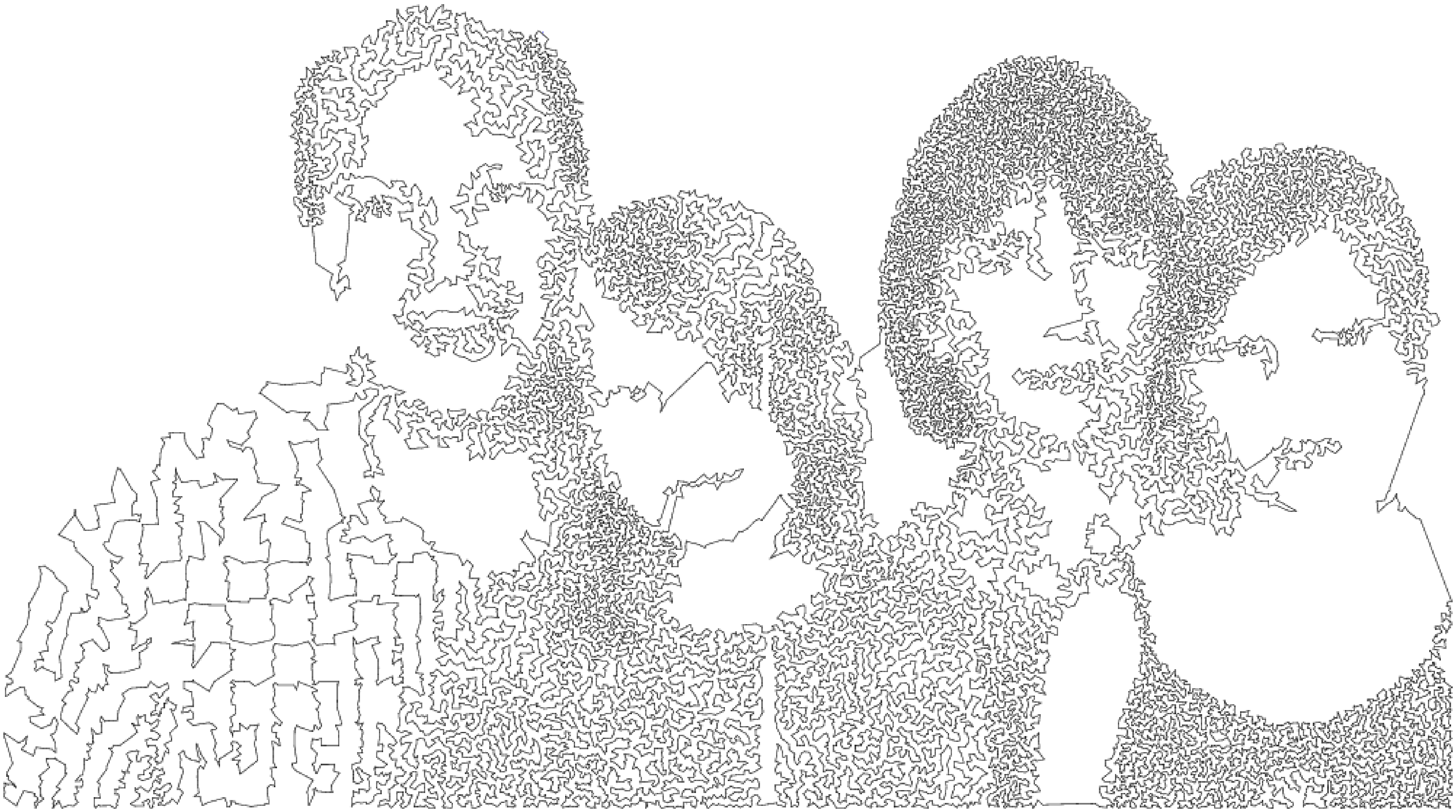








TSP Art



Solving TSPs with Concorde

(http://www.math.uwaterloo.ca/tsp/concorde.html)

TSP

Concorde Home

Home

> Concorde Home

Windows GUI

Benchmarks

Documentation

Downloads

Contact Info

Concorde TSP Solver

Concorde is a computer code for the symmetric traveling salesman problem (TSP) and some related network optimization problems. The code is written in the ANSI C programming language and it is available for academic research use; for other uses, contact William Cook for licensing options.

Concorde's TSP solver has been used to obtain the optimal solutions to 106 of the 110 TSPLIB instances; the largest having 85,900 cities.

The Concorde callable library includes over 700 functions permitting users to create specialized codes for TSP-like problems. All Concorde functions are thread-safe for programming in shared-memory parallel environments; the main TSP solver includes code for running over networks of UNIX workstations.

Concorde now supports the QSopt linear programming solver. Executable versions of concorde with qsopt for Linux and Solaris are now available

Hans Mittelmann has created a NEOS Server for Concorde, allowing users to solve TSP instances online.

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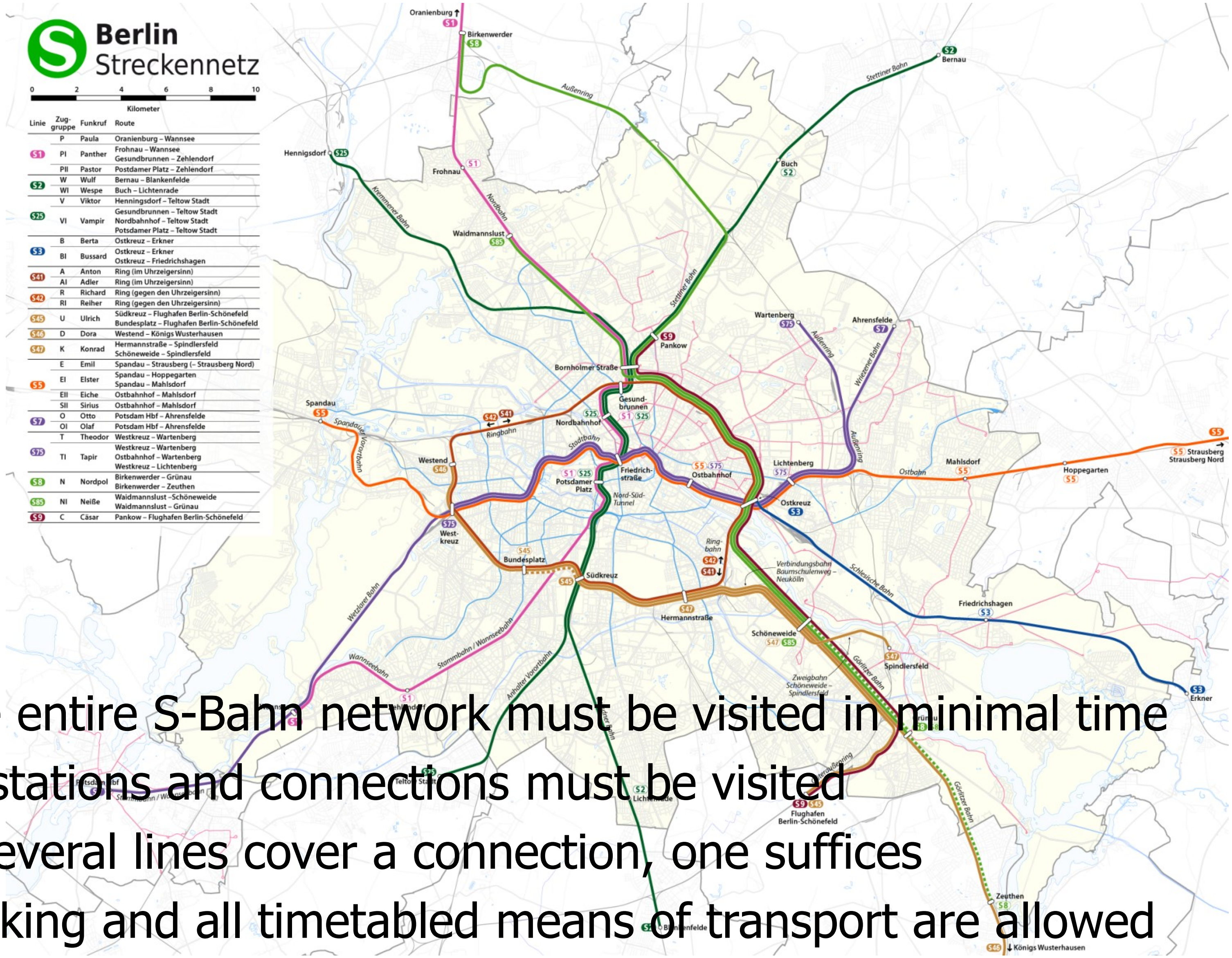
Last Updated: December 2011

The S-Bahn Challenge

Freie Universität Berlin



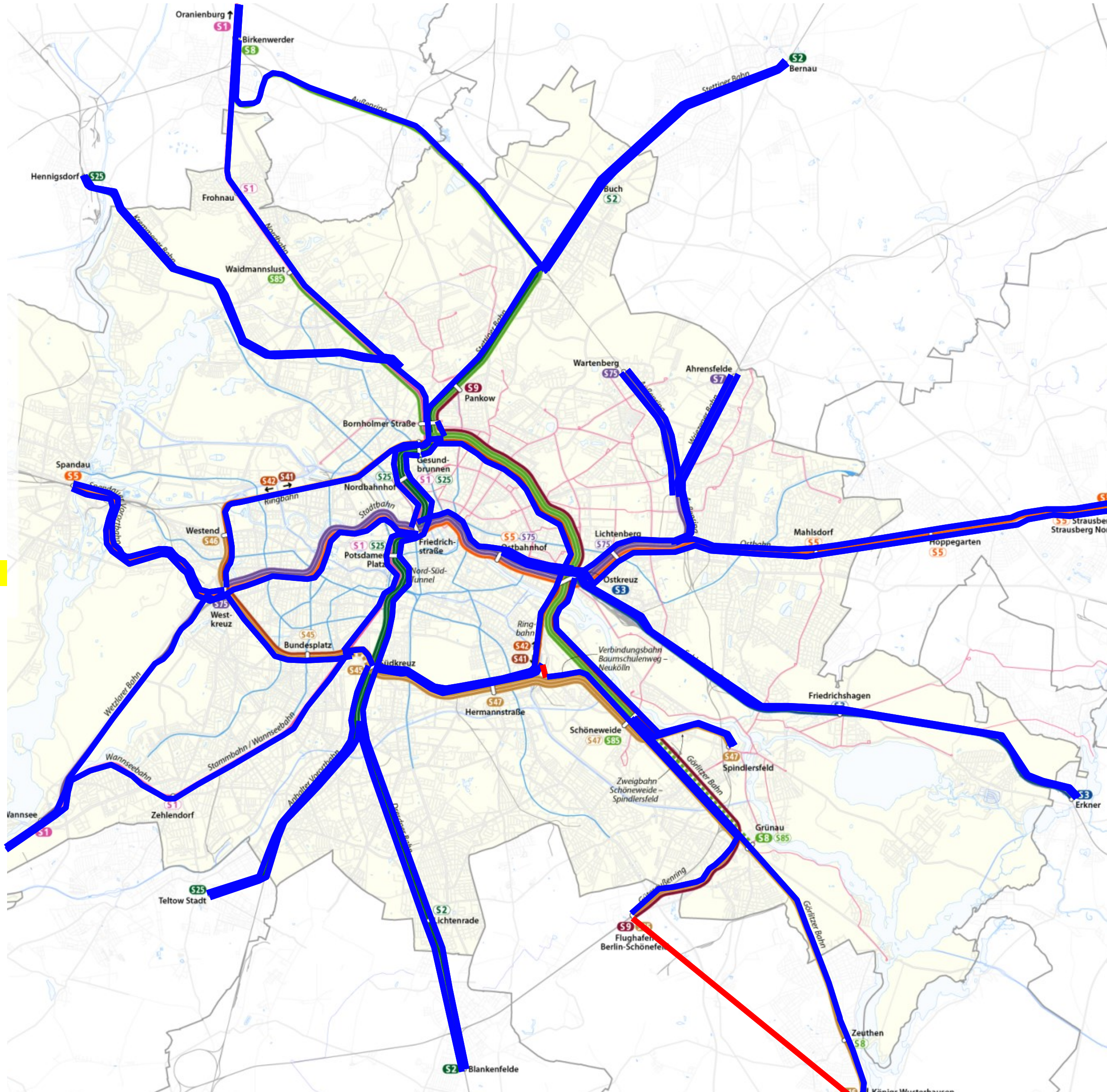
- The entire S-Bahn network must be visited in minimal time
- All stations and connections must be visited
- If several lines cover a connection, one suffices
- Walking and all timetabled means of transport are allowed

David's Solution ...

Station	**board line**	Departure	Arrival	Changing time	**Total**	**Remarks**
Bornholmer Strasse	25	05:24	05:47	0:02		On weekends the trains go all night every 30 min, so we can start basically anytime
Henningsdorf	25	05:49	06:07	0:15		
Schönholz	1	06:22	06:50	0:01		
Oranienburg	1	06:51	07:00	0:04	1:36	
Birkenwerder	8	07:04	07:24	0:01		
Blankenburg	2	07:25	07:52	0:17		
Bernau	2	08:09	08:22	0:05	2:58	
Gesundbrunnen	42	08:27	08:37	0:04		here we are done with the Northern network branches
Westkreuz	5	08:41	08:55	0:03		
Spandau	5	08:58	09:11	0:01		
Westkreuz	7	09:12	09:28	0:01		
Wannsee	1	09:29	09:35	0:05	4:11	
Potsdam	1	09:40	10:15	0:03		
Schöneberg	41	10:18	10:28	0:04		here we are done with the Southwestern network branches
Westkreuz	5 / 7	10:32	10:47	0:02		From here...
Friedrichstrasse	1 / 2	10:49	11:00	0:04		
Yorckstrasse	1	11:04	11:07	0:02	5:43	
Schöneberg	42	11:09	11:10	0:04		... to here we move in the city centre
Südkreuz	25	11:14	11:32	0:04		From here...
Teltow Stadt	25	11:36	11:50	0:02		
Priesterweg	2	11:52	12:12	0:14	6:48	
Blankenfelde	2	12:26	13:07	0:08		
Gesundbrunnen	41	13:15	13:45	0:06		
Südkreuz	42	13:51	14:00	0:04		... to here we do the Southern branches
Sonnenallee	M41	14:04	14:07	0:03	8:43	From here...
Köllnische Heide	M41 / S42 / S9	14:10	14:42	0:22		
Flughafen Berlin-Schönefeld	Bus EV or RB19	15:04	15:19	0:00		
Königs Wusterhausen	46	15:19	15:44	0:11		
Schöneweide	8	15:55	16:01	0:03		... to here we do the Southeastern branches
Spindlersfeld	47	16:04	16:28	0:07		
Ostkreuz	3	16:35	17:10	0:52	11:46	Here we do the Eastern branches. From here...
Erkner / Erkner ZOB	3	18:02	18:42	0:17	13:18	
Ostbahnhof / Strausberg	75	18:59	19:09	0:06		
Strausberg Nord	5	19:15	19:55	0:06		
Friedrichsfelde Ost	7	20:01	20:14	0:06	14:50	
Ahrensfelde	7	20:20	20:29	0:01		
Springpfuhl	75	20:30	20:39	0:07		
Wartenberg	75	20:46	21:09	0:11		
Ostkreuz	5 / 7	21:20	21:22	0:05		... to here we are flexible and can actually take whatever comes first
Friedrichstrasse	2	21:27	21:35	0:00	16:11	City Centre once more
Gesundbrunnen	41	21:35	21:37	0:04		
Schönhauser Allee	9	21:41	**21:44**		**16:20**	
Bornholmer Strasse	DONE					

… took 17:01 –

Station	in Linie	Ab	An	Um	Total
Bornholmer Strasse	25	05:24	05:47	0:02	
Henningsdorf	25	05:49	06:07	0:15	
Schönholz	1	06:22	06:50	0:01	
Oranienburg	1	06:51	07:00	0:04	1:36
Birkenwerder	8	07:04	07:24	0:01	
Blankenburg	2	07:25	07:52	0:17	
Bernau	2	08:09	08:22	0:05	2:58
Gesundbrunnen	42	08:27	08:37	0:04	
Westkreuz	5	08:41	08:55	0:03	
Spandau	5	08:58	09:11	0:01	
Westkreuz	7	09:12	09:28	0:01	
Wannsee	1	09:29	09:35	0:05	4:11
Potsdam	1	09:40	10:15	0:03	
Schöneberg	41	10:18	10:28	0:04	
Westkreuz	5 / 7	10:32	10:47	0:02	
Friedrichstrasse	1 / 2	10:49	11:00	0:04	
Yorckstrasse	1	11:04	11:07	0:02	5:43
Schöneberg	42	11:09	11:10	0:04	
Südkreuz	25	11:14	11:32	0:04	
Teltow Stadt	25	11:36	11:50	0:02	
Priesterweg	2	11:52	12:12	0:14	6:48
Blankenfelde	2	12:26	13:07	0:08	
Gesundbrunnen	41	13:15	13:45	0:06	
Südkreuz	42	13:51	14:00	0:04	
Sonnenallee	M41	14:04	14:07	0:03	8:43
Köllnische Heide	M41 / S42 / S9	14:10	14:42	0:22	
Flughafen Berlin-Schönefeld	Bus EV or RB19	15:04	15:19	0:00	
Königs Wusterhausen	46	15:19	15:44	0:11	
Schöneweide	8	15:55	16:01	0:03	
Spindlersfeld	47	16:04	16:28	0:07	
Ostkreuz	3	16:35	17:10	0:52	11:46
Erkner / Erkner ZOB	3	18:02	18:42	0:17	13:18
Ostbahnhof / Strausberg	75	18:59	19:09	0:06	
Strausberg Nord	5	19:15	19:55	0:06	
Friedrichsfelde Ost	7	20:01	20:14	0:06	14:50
Ahrensfelde	7	20:20	20:29	0:01	
Springpfuhl	75	20:30	20:39	0:07	
Wartenberg	75	20:46	21:09	0:11	
Ostkreuz	5 / 7	21:20	21:22	0:05	
Friedrichstrasse	2	21:27	21:35	0:00	16:11
Gesundbrunnen	41	21:35	21:37	0:04	
Schönhauser Allee	9	21:41	**21:44**		**16:20**
Bornholmer Strasse	DONE				



The Guinness Book of Records

- **Fastest time to travel to all the Berlin U-Bahn metro stations**
  The fastest time to travel to all the Berlin U-Bahn metro stations is 7 hr 33 min 15 sec and was achieved by Oliver Ziemek, Henning Colsman-Freyberger, Michael Wurm and Rudolf von Grot (all Germany) at Hönow station, Berlin, Germany on 2 May 2014.
- **Fastest time to travel to all London Underground stations**
  The fastest time to travel to all London Underground network stations is 16 hr 14 min 10 sec, and was achieved by Clive Burgess and Ronan McDonald (both UK) in London, UK, on 19 February 2015. Clive and Ronan's record breaking journey began at Chesham and ended at Heathrow Terminal 5.
- **Fastest time to travel to all New York City Subway stations**
  The fastest time to travel the entire New York City Subway is 22 hr 26 min 02 sec and was achieved by Andi James, Steve Wilson, Peter Smyth, Martin Hazel, Glen Bryant and Adham Fisher (all UK) between 18 and 19 November 2013. Andi James, Steve Wilson and Martin Hazel are previous record holders of the record for the 'Fastest time to travel to all London Underground stations.'

The History of the Transit-Challenge

Date	Record Holder(s)	Stations	Time
30.05.1940	Herman Rinke	All stations	25:00:00~
01.06.1966	Michael Feldman and James Brown	All stations	23:16:xx
12/13.12.1988	Rich Temple, Phil Vanner and Tom Murphy	All stations	29:47:12
25/26.10.1989	Kevin Foster	All stations	26:21:08
28/29.12.2006	Bill Amarosa Jr., Michael Boyle, Brian Brockmeyer, Stefan Karpinski, Jason Laska and Andrew Weir	All stations	24:54:03
22/23.01.2010	Matt Ferrisi and Chris Solarz	All stations	22:52:36
18/19.11.2013	Andi James, Steve Wilson, Martin Hazel, Glen Bryant, Peter Smyth and Adham Fisher	All stations	22:26:02

1. Ride that requires a rider to traverse every line, but not necessarily the entire line. (Class A)
2. Full-system ride that requires a rider to stop at each station. (Class B)
3. Skip-stop ride that only requires a rider to pass through each station. (Class C)

The Rules of the Guinness Book

TRAVELLING THE NEW YORK CITY SUBWAY IN THE SHORTEST TIME

The following act as a guide to the specific considerations and undertakings, in addition to the general requirements as detailed in the General Rules of the Record Breakers' Pack, for any potential attempt on the above record.

They should be read and understood by all concerned – organisers, participants and witnesses – prior to the event.

Please note that, as detailed in the Agreement Regarding Record Attempts, these guidelines in no way provide any kind of safety advice or can be construed as providing any comfort that the record is free from risk.

GUIDELINES

This record is for travelling the entire MTA New York City Subway system in the least amount of time.

1 - All of the stations served by the subway system must be visited. To `visit' a station, the challenger must arrive and/or depart by a subway train in normal public service. It is necessary for a train to stop at the station for the visit to count, although the challenger does not need to leave the train at that station. If a station is normally open only at certain times of the day, this must be taken into account during planning. Only if a station is temporarily closed (e.g. for rebuilding, or in an emergency) will a non-stop pass through a station be acceptable.

2 - It is only necessary to visit all the stations on the network, not to travel every stretch of line. Thus, if a station is served by more than one line it is not necessary to visit that station on each line.

3 - Challengers may travel the same stretch of track (and visit the same station) more than once if necessary.

4 - Attempts on this record must be continuous (i.e. any breaks or stops that are taken must be included in the final time).

5 - Transfers between subway lines must be made by scheduled public transport or on foot. The use of private motor vehicles, taxis or any other form of privately arranged transport (bicycles, skateboards etc) is not allowed.

AUTHENTICATION

For the purposes of verifying any claim, the following must be provided: -

Witness Book

Any attempt must take place in view of the public, wherever possible, and a book made available for independent witnesses to sign. The book should be set up so that the following details can be included for each potential witness:

Date & Time	Location	Name	Signature

For solo and unsupported attempts, we appreciate that it might not be possible to gain an unbroken line of witnesses for the attempt, but one should try to obtain as many as possible. For an attempt, which is supported by a backup team, we would expect it to be possible to gain sufficient numbers of independent witnesses to enable verification for the entire duration of the attempt. Where possible, local dignitaries and police should be sought to sign the book.

Log Book

A logbook detailing every stage of the journey, i.e. the time of arrival and departure from each station, line changes, commutes between lines and stations, etc. must be maintained. This book should illustrate clearly the route followed.

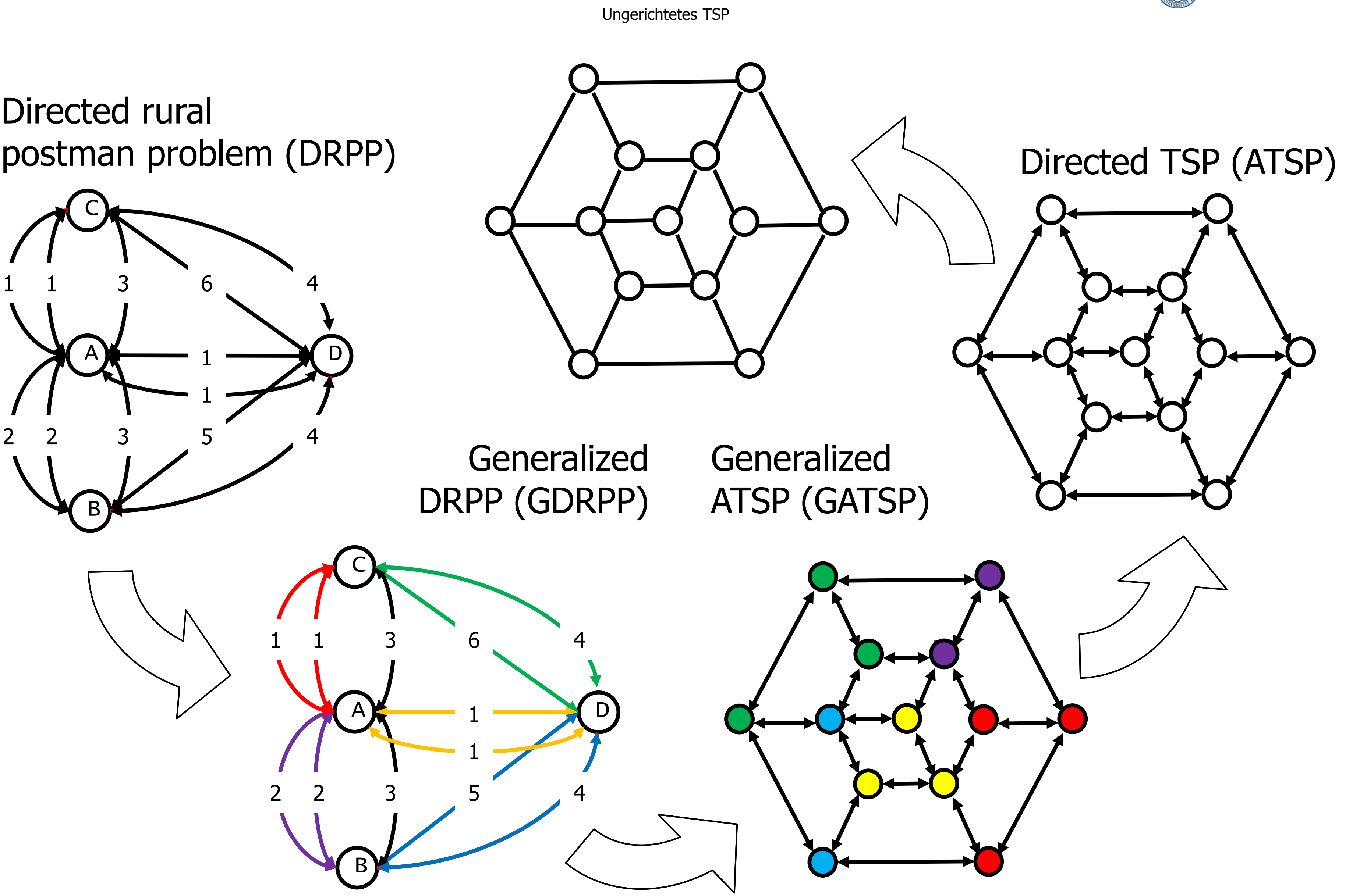
All rest breaks or stoppages for whatever reason must also be fully detailed in the log.

To attest to the validity and genuineness of the claim, we require signed statements of authentication by two independent persons of some standing, one of whom should have attended the beginning of the event, and if possible the end.

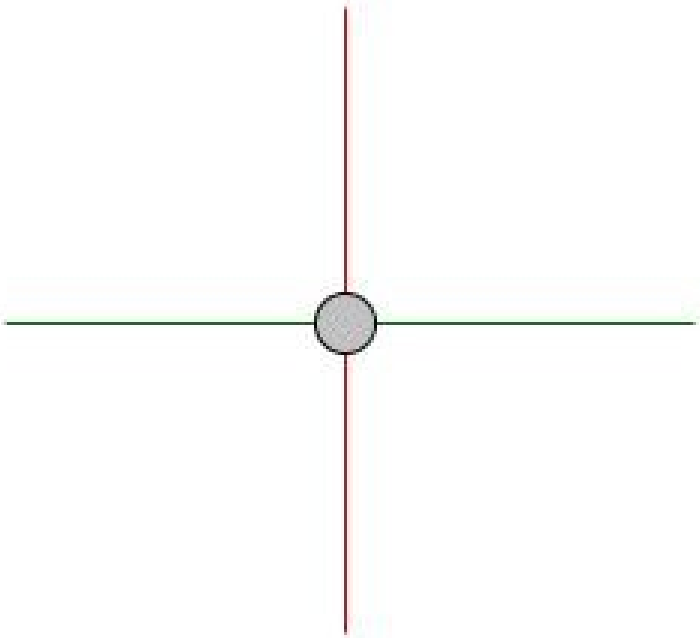
These statements should originate directly from the witnesses (in their own hand) and be submitted where possible on their own headed notepaper and include full contact details

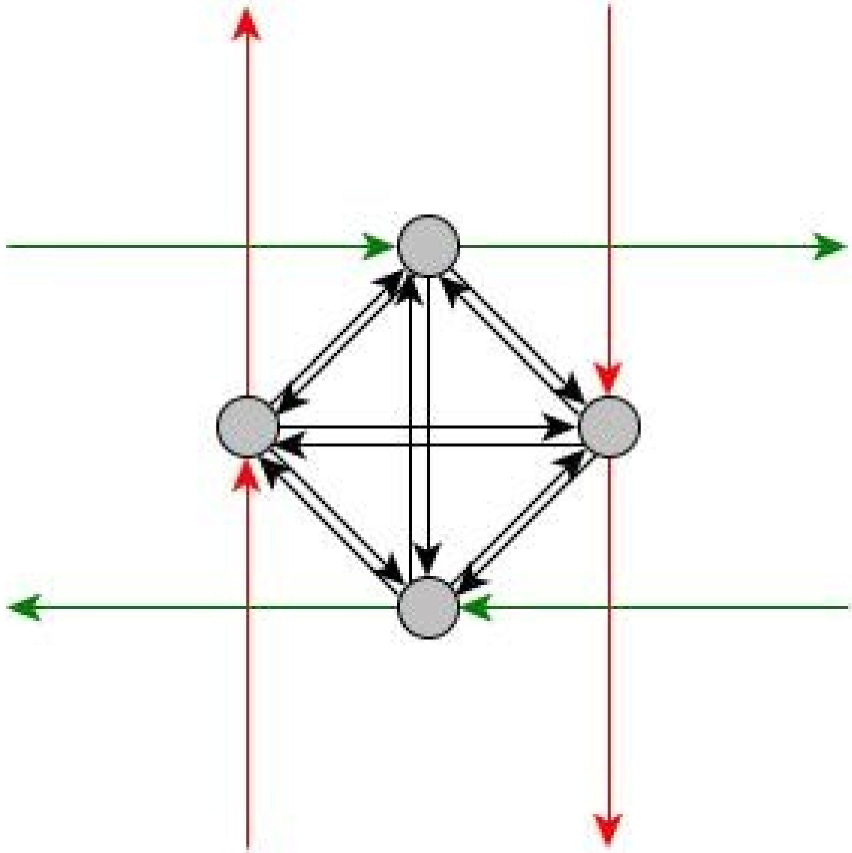
Statements should not take the form of documents pre-prepared by those involved in the record attempt.

Quattro Transformationi



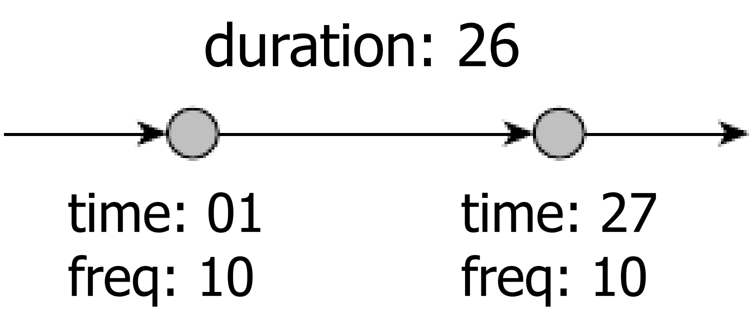
Transfers …

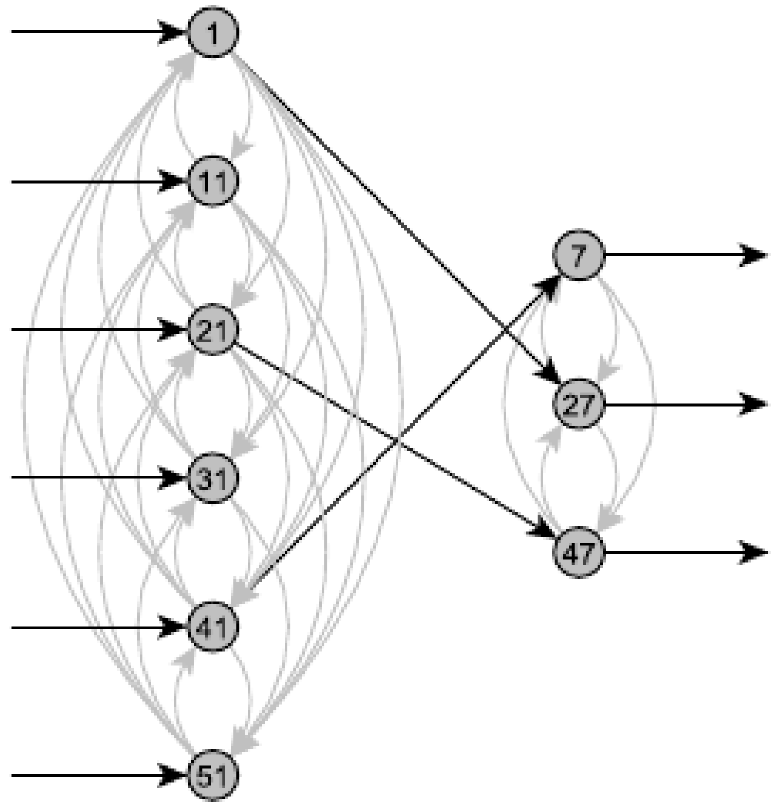




… in a Frequency Timetable modulo 60

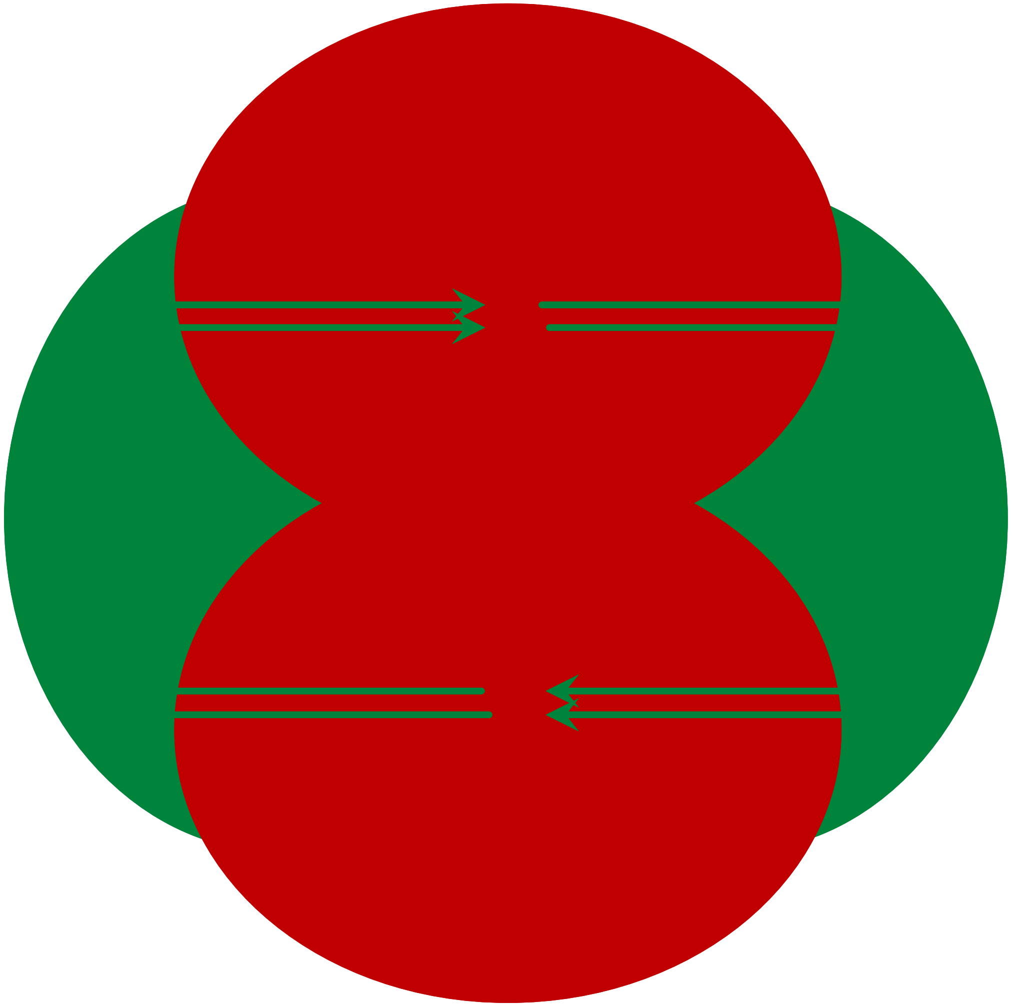
- Most lines runs every 10 or 20 mins
- One line runs every 40 mins, regional trains every 60 mins
- One 40 and one 60 mins connection can be shifted into a 20 mins solution





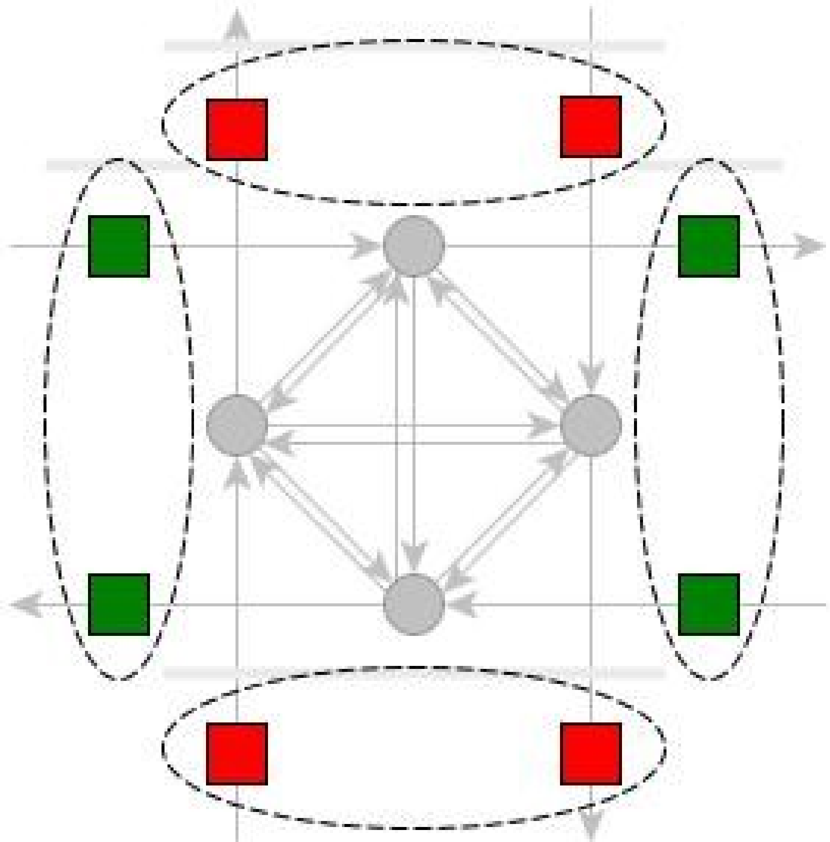
The Generalized Direct Rural Postman Problem

- Visit every group of connections at least once



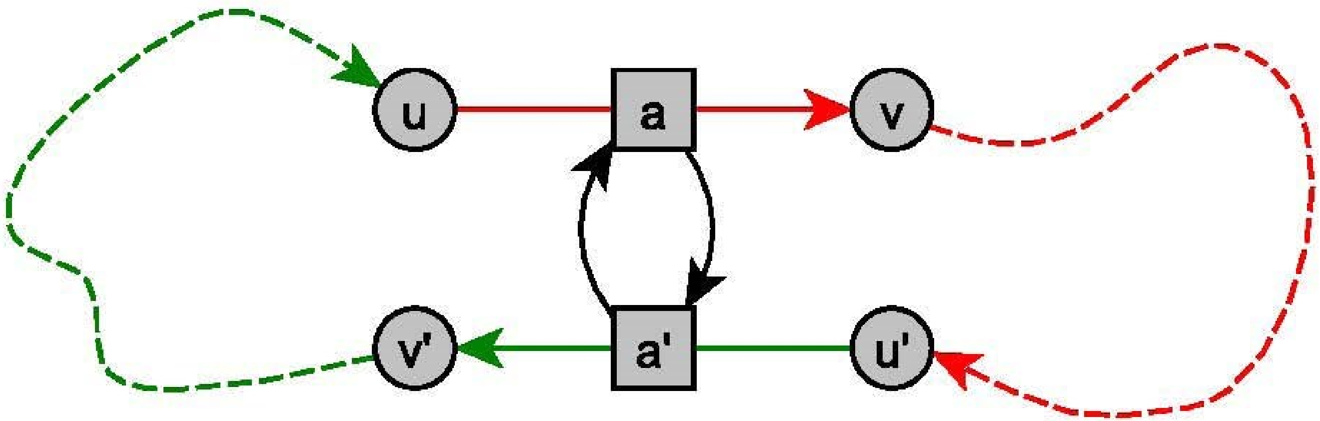
From the GDRPP to the Generalized directed TSP (1)

- Turn edges into nodes and connect succeeding ones
- Visit every group of nodes at least once



From the GDRPP to the Generalized directed TSP (2)

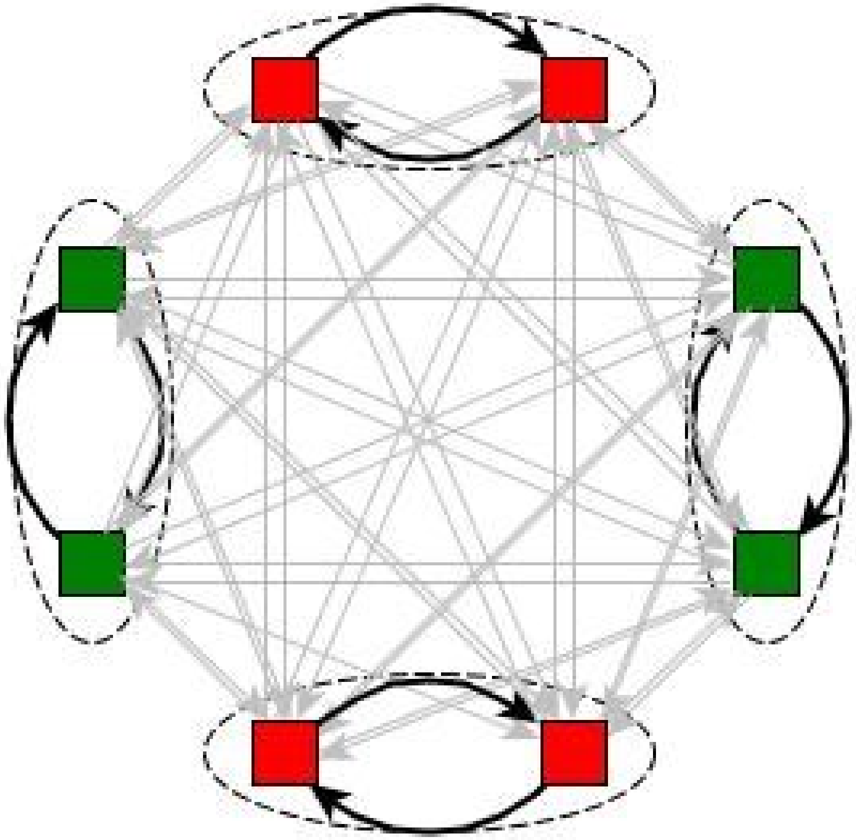
- Connect all nodes from **different** groups using two arcs, namely, nodes a and a' using arcs (u, v) and (u', v') s.t.
  - arc (a, a') with length $l(u, v)$ + length of shortest (v, u')-path
  - arc (a', a) with length $l(u', v')$ + length of shortest (v', u)-path



length of arc (a, a') length of arc (a', a)

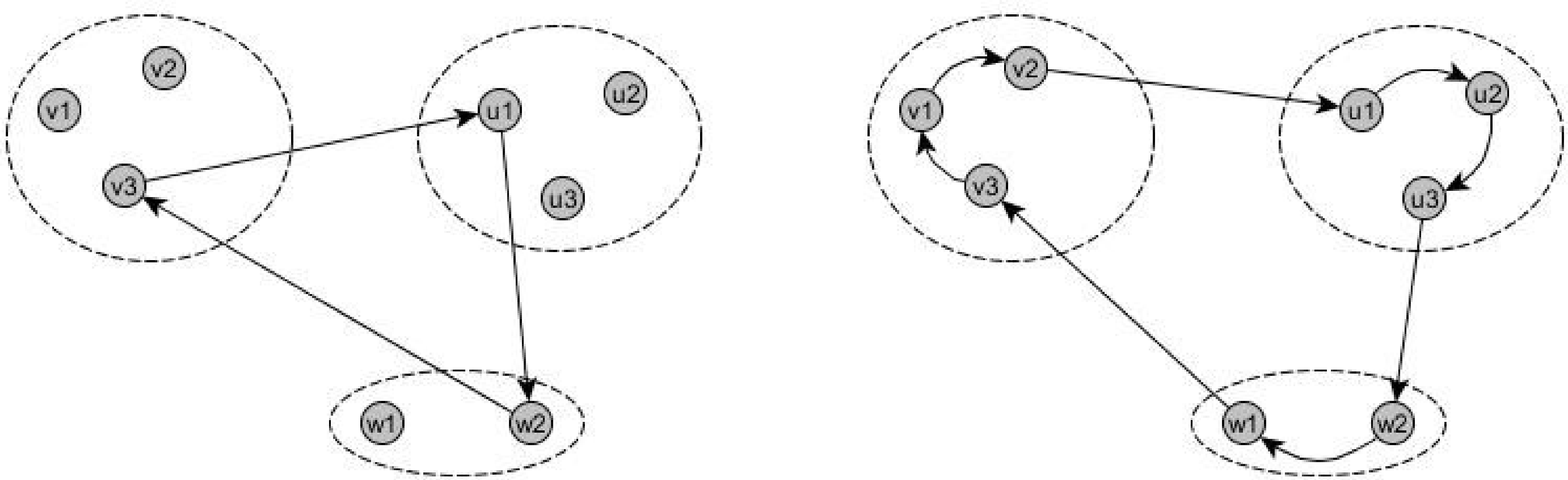
From the GATSP to the ATSP (1)

- Visit every group of nodes exactly once $\rightarrow$ visit every node exactly once: connect every group of nodes using a directed cycle of length 0



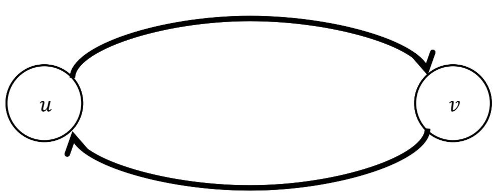
From the GATSP to the ATSP (2)

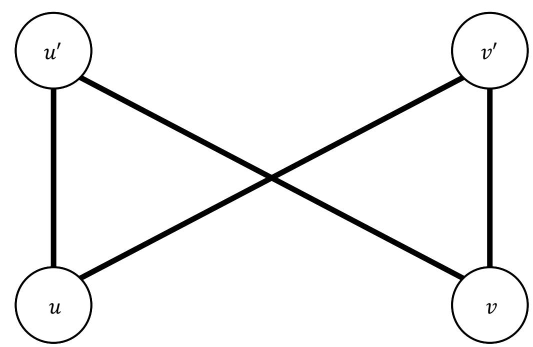
- Replace arcs between different groups of nodes by new arcs
  - New start node is the one that preceeds in the cycle
  - Increase weight by a large constant M_1 that ensures that every group is visited exactly once, i.e., for a cycle $(u_1, \ldots, u_i, u_{i+1}, \ldots)$ set $w'(u_i, v_j) = w(u_{i+1}, v_j) + M_1$



From the ATSP to the TSP

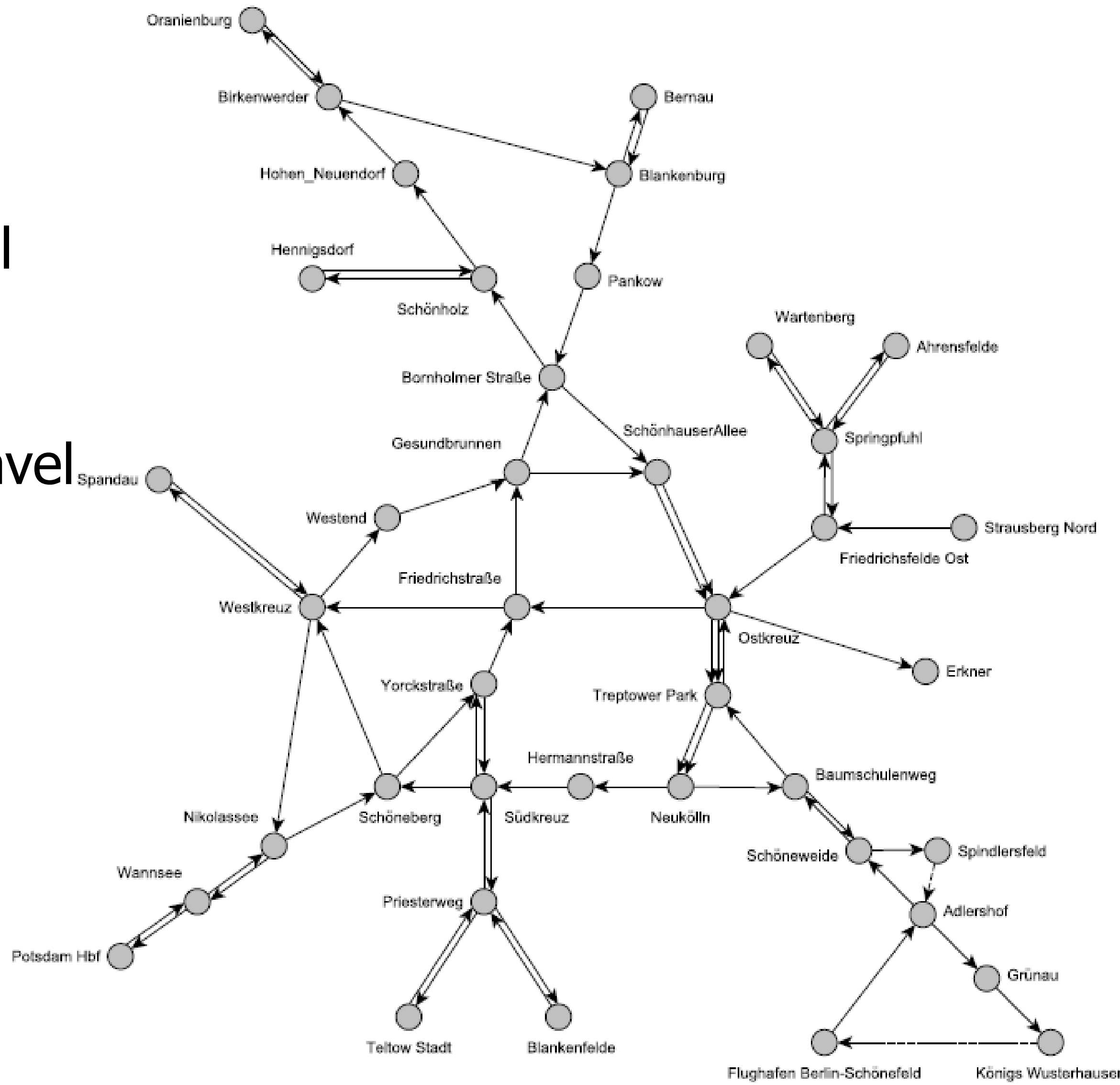
- Make two copies v und v' of every node v
  - Connect v and v' via undirected edges with weight $-M_2$
  - Connect u and v' via undirected edge with weight $w(u,v)$
- Open tour with given start node
  - Add additional node 0 for tour start and end
  - Add suitable weights 0 and ∞





Preprocessing

- Only start, end, or transfer stations: 166 $\rightarrow$ 113 stations
- No transfer on parallel lines: 113 $\rightarrow$ 42 stations
- Assumption: equal travel and transfer times



Program TOPTraC

(http://www.zib.de/projects/s-bahn-challenge-berlin)

Freie Universität Berlin

- Open or closed tours
- Start, end station, or both
- Timetable or constant transfer times
- Exceptions for minimum transfer times at large stations
- TSP mode (for visiting only stations)

Name	Änderungsdatum	Typ	Größe
Illustration Network.pdf	20.01.2015 14:52	Adobe Acrobat D...	38 KB
Itinerary.xlsx	20.01.2015 15:26	Microsoft Excel-Ar...	11 KB
main.cpp	06.03.2015 13:06	CPP-Datei	45 KB
Network_arcs.lgf	03.12.2014 09:37	LGF-Datei	1 KB
Network_nodes.lgf	04.03.2015 14:38	LGF-Datei	2 KB
outputconcorde.txt	06.03.2015 10:53	Textdokument	1 KB
Short User Manual.txt	06.03.2015 12:58	Textdokument	2 KB
toptrac	06.03.2015 13:06	Datei	764 KB

Program TOPTraC
(http://www.zib.de/projects/s-bahn-challenge-berlin)

Input
- File with all edge and travel times
- File with all stations, departure times, lines, and frequencies
- Timetable period

Output
- TSP file

Solution
- Via Concorde on NEOS server

Result
- List of all edges in optimal solution (unsorted)

The Solution

Best found solution: 13:17

- Contains some 1-minute transfers at large stations, which are not feasible according to the BVG trip planner
- The last connection Strausberg-Strausberg Nord on the S5 is operated at a 40 mins frequency

More realistic solution: 13:44

- Start at Strausberg Nord
- Lower bounds on transfers at large stations
- Feasible according to BVG trip planner

With luck: 13:24

- If some infeasible transfers are caught

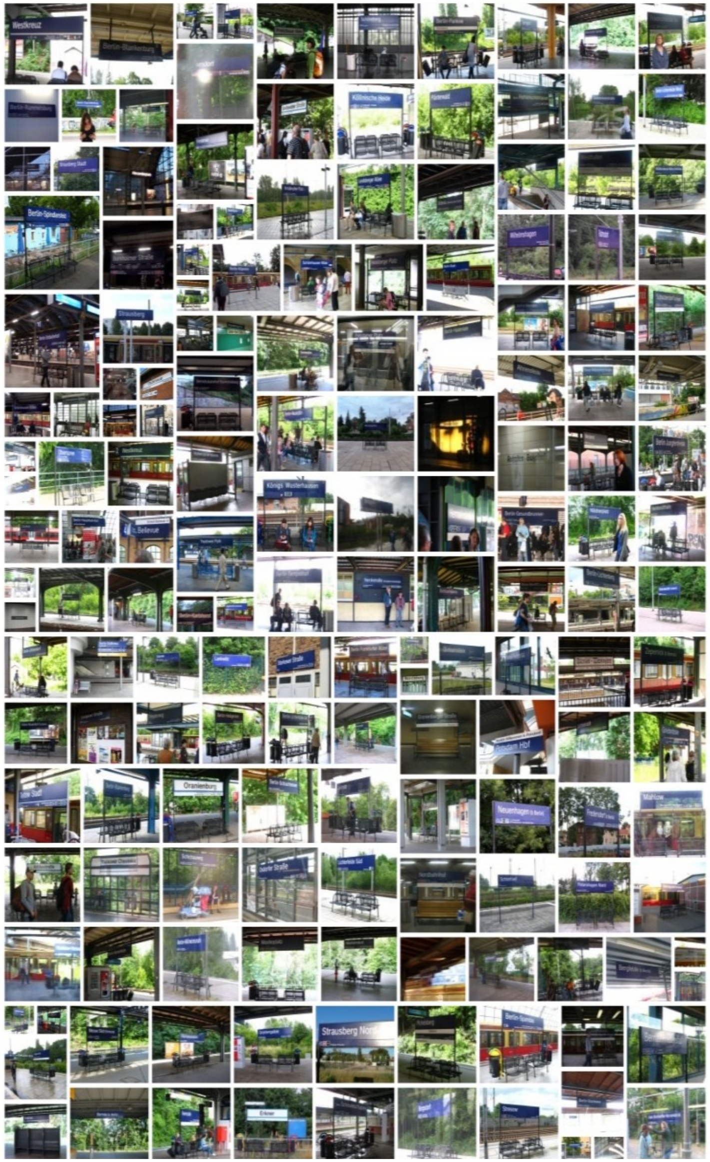
The Realistic Solution: 13:44

The Record Attempt: 10.01.2015, 09:55

- The first 7 hours worked according to the plan
- One infeasible transfer was caught: 20 mins ahead of schedule!

The Record Attempt: 10.01.2015, 09:55

- The first 7 hours went according to the plan
- One infeasible transfer was caught
- Thunderstorm Felix
- Severe service disruptions
- Finally 15:04 instead of 13:44
- 80 mins delay
- 2 hours faster than before
- Not yet (?) in Guinness Book
- 2 stations were visited by regional trains
- Legal or not?



In the Press

Freie Universität Berlin

Berliner Morgenpost

5. Mai 2015, 20:17
Berlin 27° | 11°

Jobs | Immobilien | Archiv | E-Paper | Apps | Abo-Service | Login
Suchen

Home Berlin Bezirke Politik Wirtschaft Sport Aus aller Welt Kultur Wissen Reise Lifestyle Gesundheit Abo Specials Märkte

In den Nachrichten: Top-News über WhatsApp | Hertha BSC | Bahnstreik | Berliner Foto-Quiz | Im Westen Berlins

Sie befinden sich hier: Home » Studentin fährt alle S-Bahnstationen in 15 Stunden ab

Ausdrucken Bookmarken Versenden Twittern g+1 Gefällt mir Teilen

21.02.15 REKORD IN BERLIN

Studentin fährt alle S-Bahnstationen in 15 Stunden ab

Eine niederländische Mathematik-Studentin fährt die 166 Haltestellen der Berliner S-Bahn ab – in Rekordzeit. Was nach einer verrückten Kneipenwette klingt, ist eine echte fachliche Herausforderung.

Von Thomas Fülling

Foto: University of Twente

NEWS-TICKER

20:37 – Was Sie zum Bahnstreik wissen müssen

20:16 – Wenn sich die Netzgemeinde zum Netzwerken in Berli...

20:06 – DEB-Team verliert trotz Steigerung gegen die Schwe...

Alle Meldungen »

TOP-THEMA

Die besten Berlin-Videos

Das sind die Youtube-Favoriten der Redaktion

MEISTGELESENE ARTIKEL

1. NOTFAHRPLAN
   Was Sie zum Bahnstreik wissen müssen
2. FEHLER UND FACHWÖRTE
   Berlins Bildungssenatorin und die Rechtschreibung
3. PANNEN-FLUGHAFEN
   Südbahn-Betrieb gibt Vorgeschmack auf den BER-Lärm
4. WETTER
   Nach der Wärme kommen jetzt die Gewitter nach Berlin
5. NOTFAHRPLAN
   Liveticker - So kommen Sie durch den Bahnstreik in Berlin

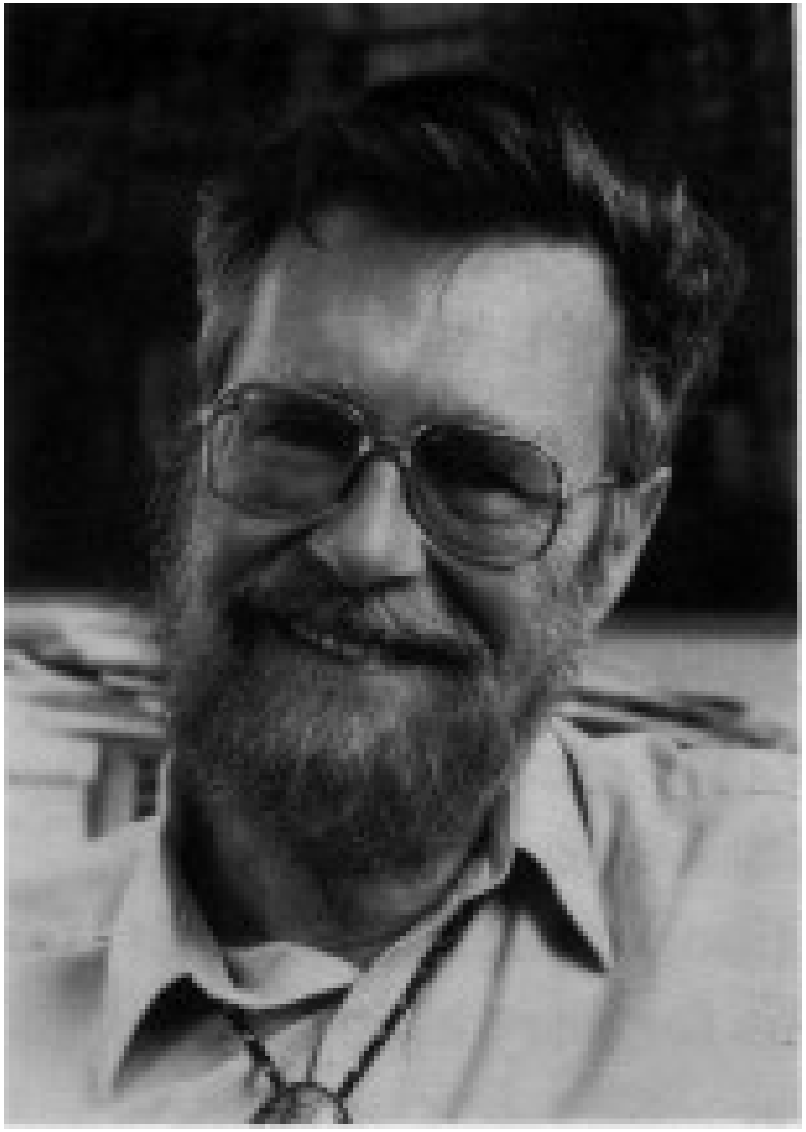
FLUGROUTEN RADAR

Wie flugbelastet ist Ihr Standort?

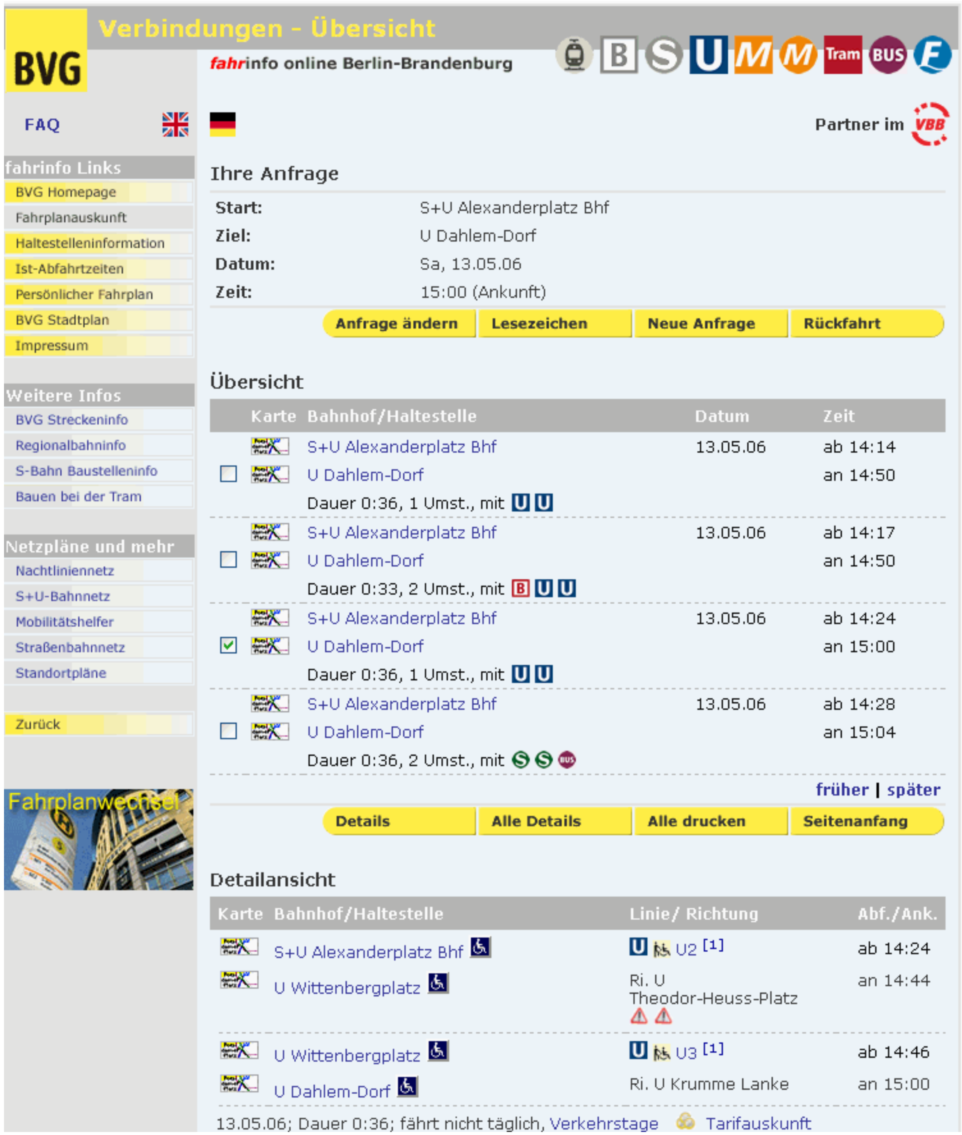
Bitte geben Sie Ihre Adresse e

zum Flugrouten-Radar »

Edsger Wybe Dijkstra (1930-2002)



Passenger Information


BVG
Verbindungen - Übersicht
fahrinfo online Berlin-Brandenburg
FAQ
Partner im VBB

fahrinfo Links
BVG Homepage
Fahrplanauskunft
Haltestelleninformation
Ist-Abfahrtzeiten
Persönlicher Fahrplan
BVG Stadtplan
Impressum

Weitere Infos
BVG Streckeninfo
Regionalbahninfo
S-Bahn Baustelleninfo
Bauen bei der Tram

Netzpläne und mehr
Nachtliniennetz
S+U-Bahnnetz
Mobilitätshelfer
Straßenbahnnetz
Standortpläne

Zurück

Fahrplanwechsel

Ihre Anfrage

Start:	S+U Alexanderplatz Bhf
Ziel:	U Dahlem-Dorf
Datum:	Sa, 13.05.06
Zeit:	15:00 (Ankunft)

Anfrage ändern | Lesezeichen | Neue Anfrage | Rückfahrt

Übersicht

Karte	Bahnhof/Haltestelle	Datum	Zeit
	S+U Alexanderplatz Bhf	13.05.06	ab 14:14
	U Dahlem-Dorf		an 14:50
	Dauer 0:36, 1 Umst., mit U U		
	S+U Alexanderplatz Bhf	13.05.06	ab 14:17
	U Dahlem-Dorf		an 14:50
	Dauer 0:33, 2 Umst., mit B U U		
	S+U Alexanderplatz Bhf	13.05.06	ab 14:24
	U Dahlem-Dorf		an 15:00
	Dauer 0:36, 1 Umst., mit U U		
	S+U Alexanderplatz Bhf	13.05.06	ab 14:28
	U Dahlem-Dorf		an 15:04
	Dauer 0:36, 2 Umst., mit S S BUS		

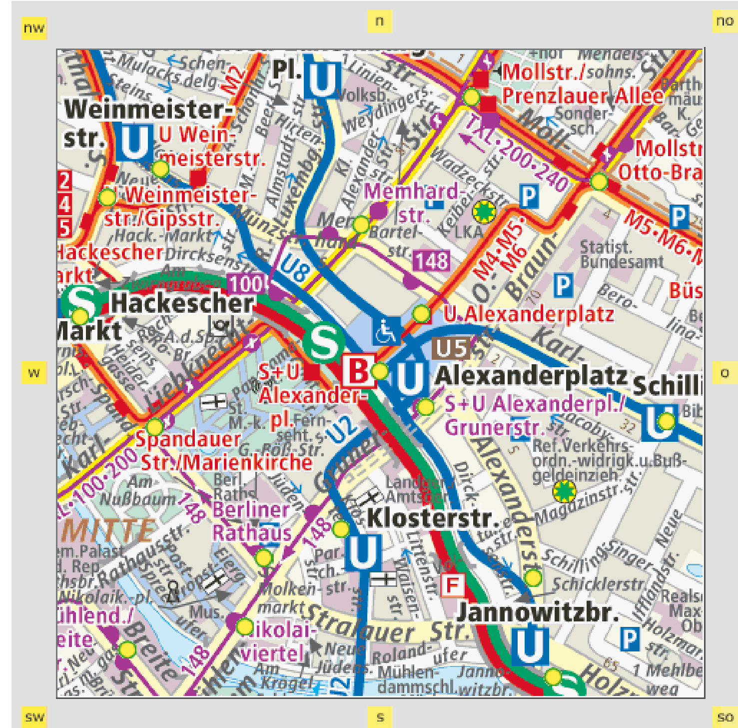
früher | später

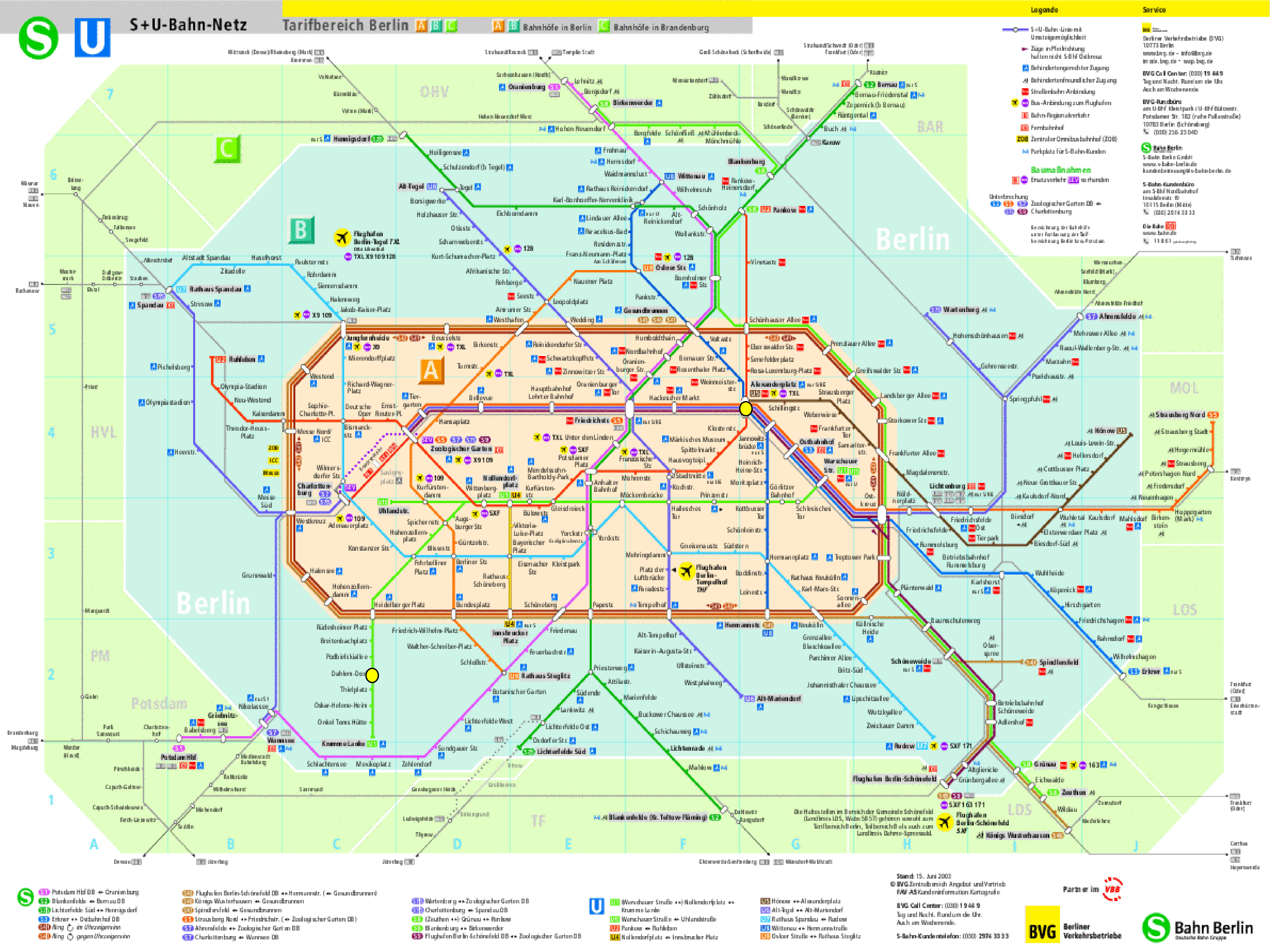
Details | Alle Details | Alle drucken | Seitenanfang

Detailansicht

Karte	Bahnhof/Haltestelle	Linie/ Richtung	Abf./Ank.
	S+U Alexanderplatz Bhf	U U2 [1]	ab 14:24
	U Wittenbergplatz	Ri. U Theodor-Heuss-Platz	an 14:44
	U Wittenbergplatz	U U3 [1]	ab 14:46
	U Dahlem-Dorf	Ri. U Krumme Lanke	an 15:00

13.05.06; Dauer 0:36; fährt nicht täglich, Verkehrstage Tarifauskunft

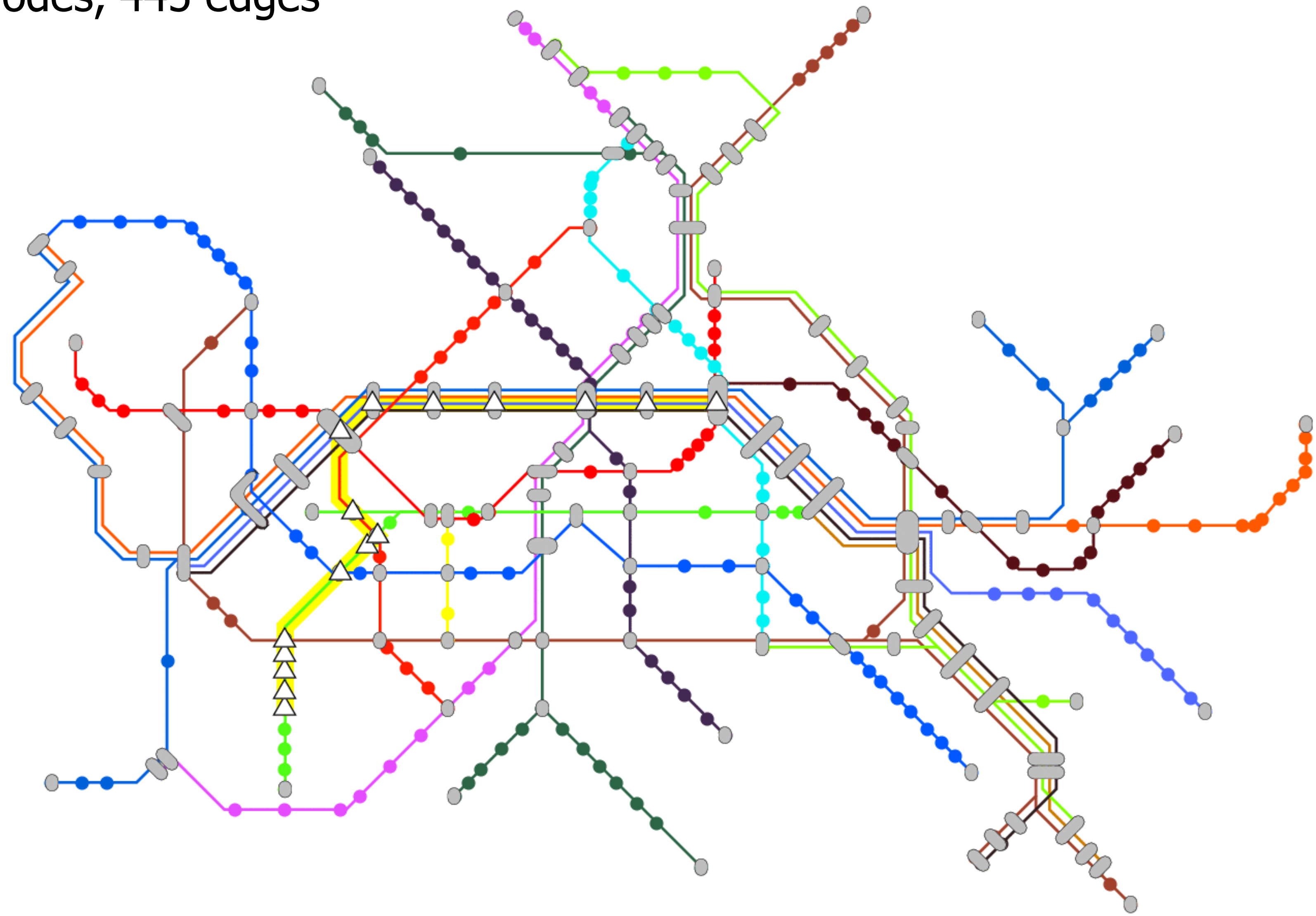




S+U-Bahn-Netz
Tarifbereich Berlin A B C
A B Bahnhöfe in Berlin
C Bahnhöfe in Brandenburg
Legende
Service
Baumaßnahmen
Berlin
Berlin
OHV
BAR
MOL
LOS
LDS
TF
PM
HVL
Potsdam
A
B
C
Oranienburg
Birkenwerder
Hennigsdorf
Bernau
Blankenburg
Pankow
Wittenau
Alt-Tegel
Flughafen Berlin-Tegel TXL
Rathaus Spandau
Spandau
Ruhleben
Jungfernheide
Westend
Charlottenburg
Zoologischer Garten
Friedrichstr.
Alexanderplatz
Ostbahnhof
Warschauer Str.
Lichtenberg
Wartenberg
Ahrensfelde
Strausberg Nord
Gesundbrunnen
Osloer Str.
Westkreuz
Uhlandstr.
Hermannstr.
Flughafen Berlin-Tempelhof THF
Rathaus Steglitz
Krumme Lanke
Wannsee
Potsdam Hbf
Lichterfelde Süd
Blankenfelde (Kr. Teltow-Fläming)
Alt-Mariendorf
Rudow
Flughafen Berlin-Schönefeld
Königs Wusterhausen
Grünau
Zeuthen
Erkner
Spindlersfeld
Schöneweide
Hönow
Rathaus Spandau
Stand: 15. Juni 2003
Partner im VBB
BVG Berliner Verkehrsbetriebe
S-Bahn Berlin

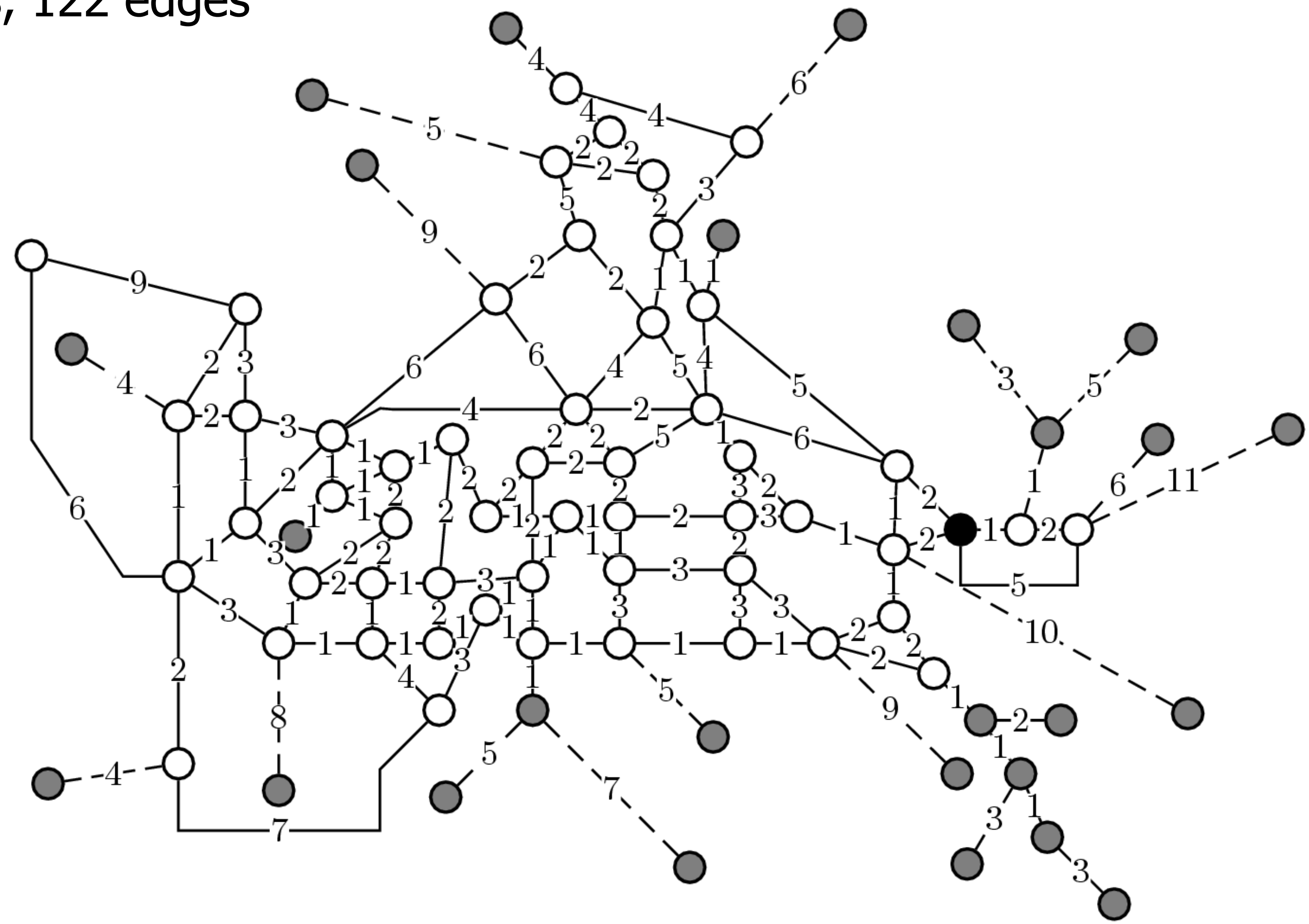
Graph Theoretic Model

306 nodes, 445 edges

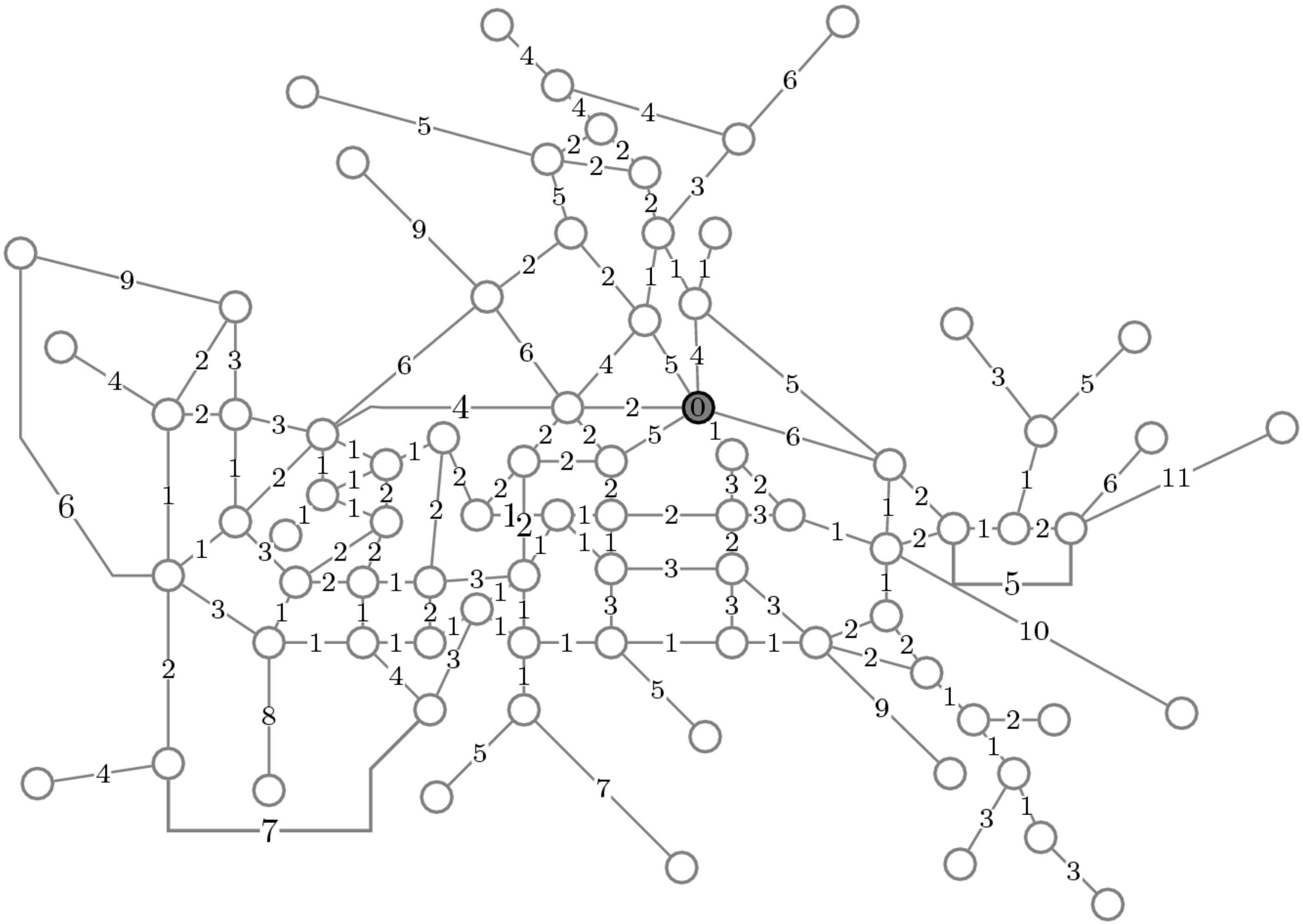


Preprocessing

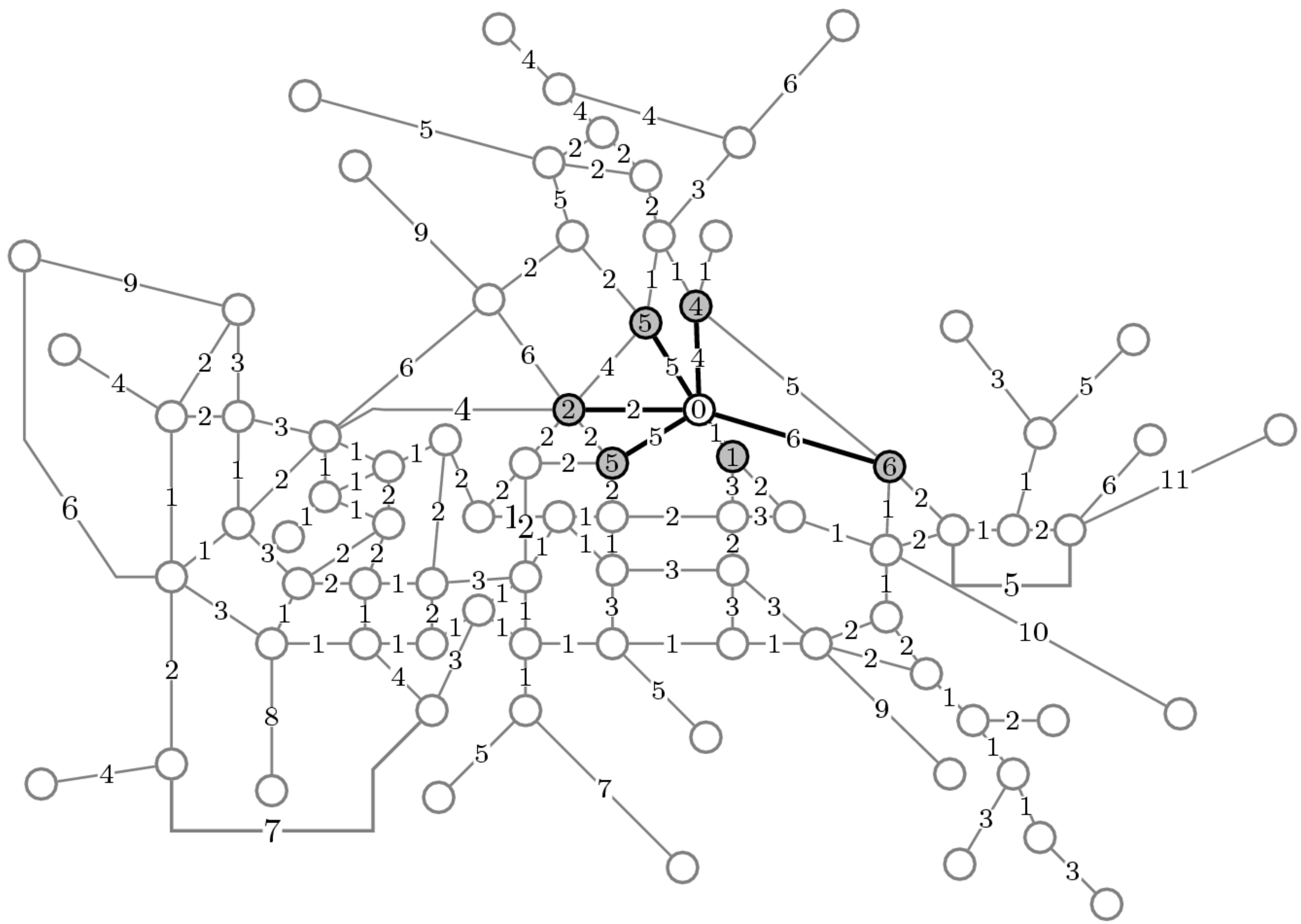
80 nodes, 122 edges



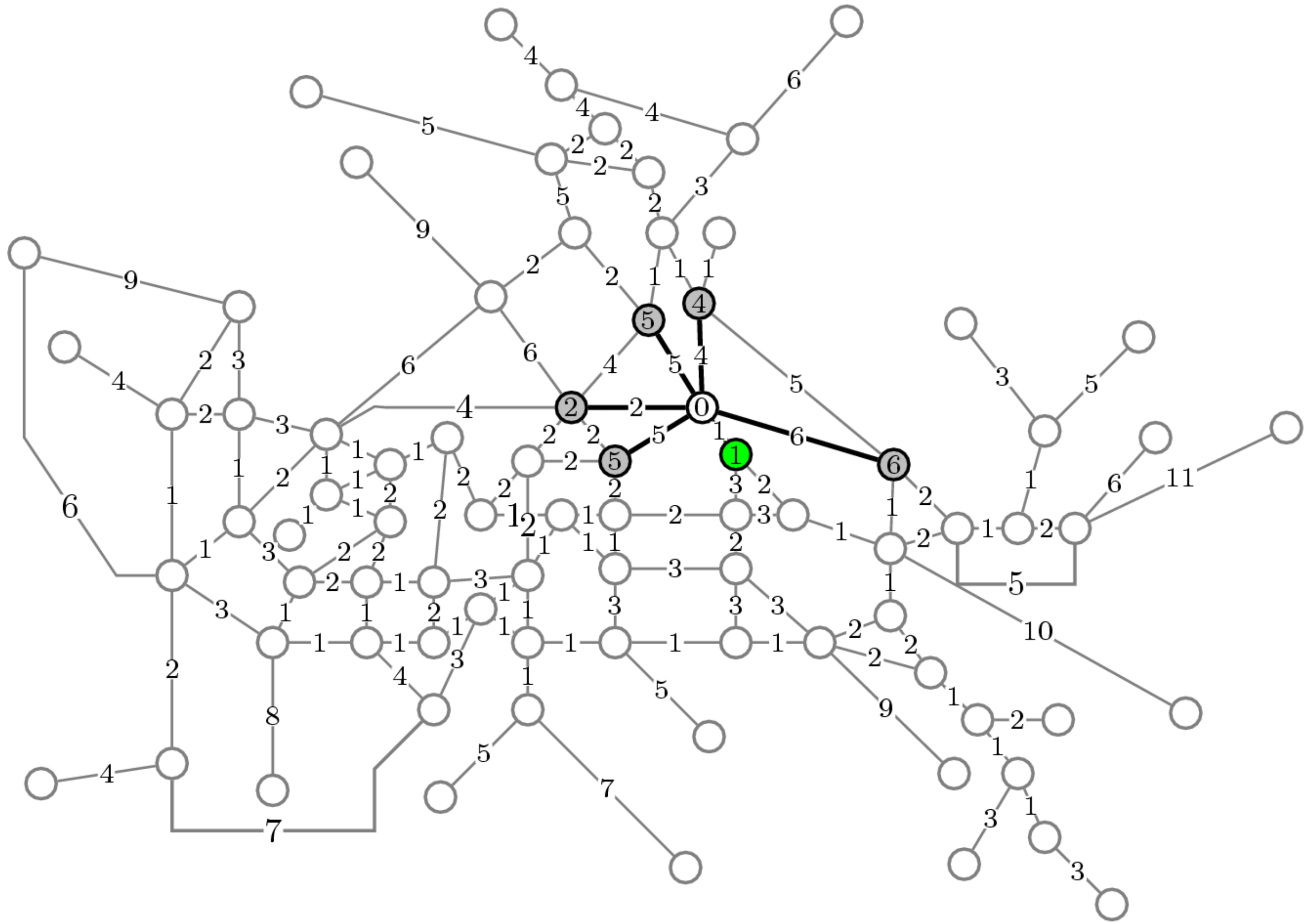
Dijkstra's Algorithm (0)



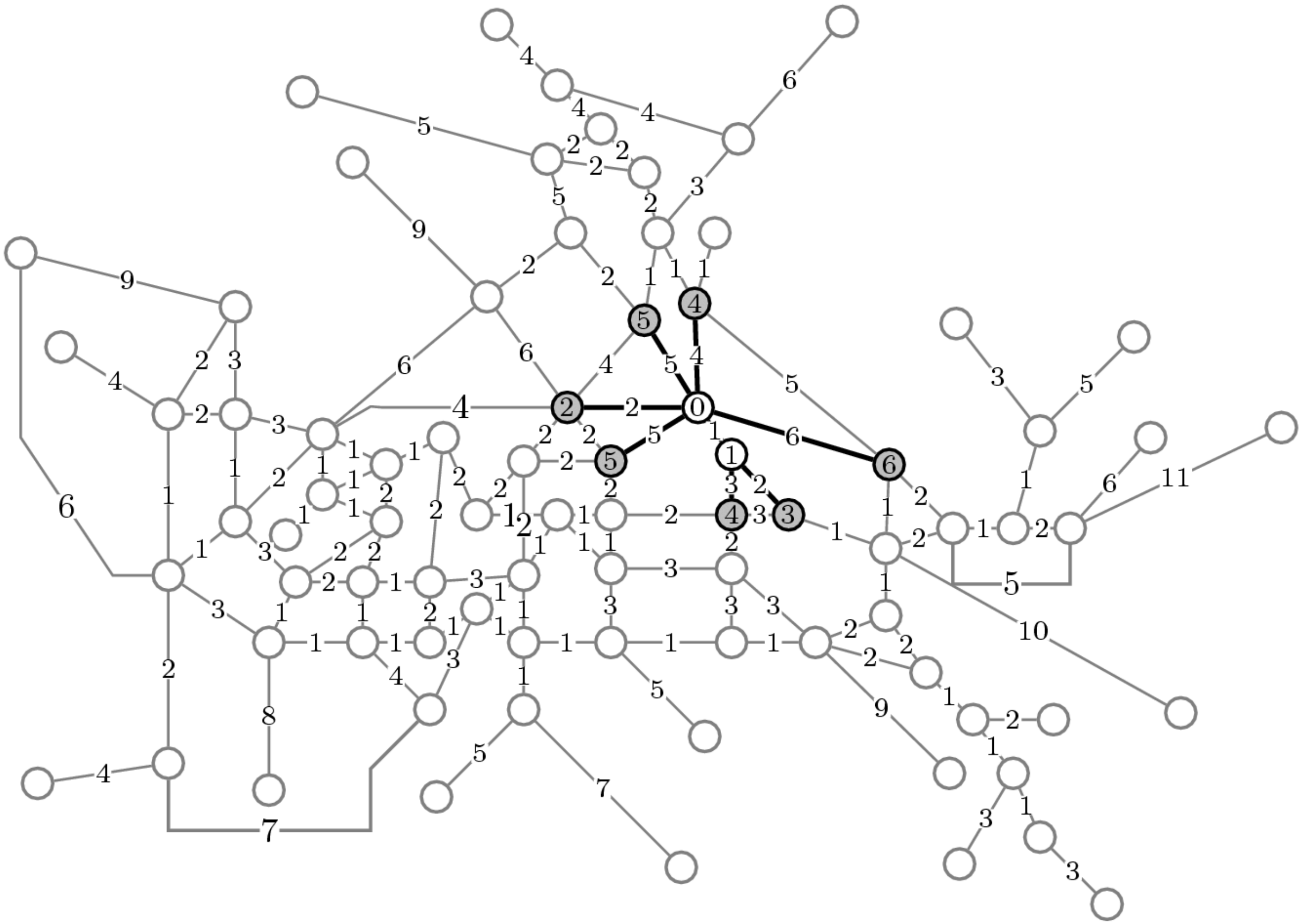
Dijkstra's Algorithm (1)



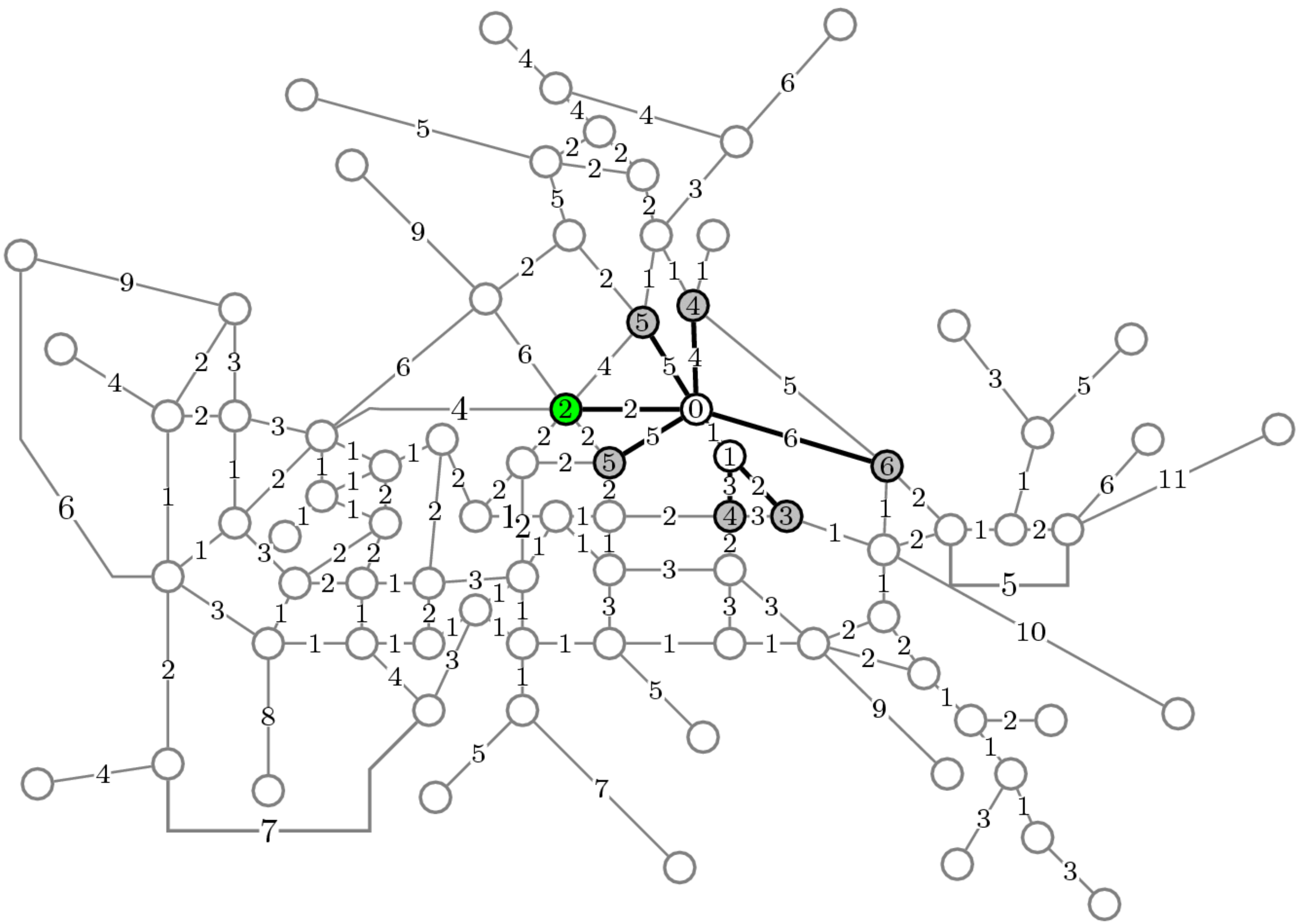
Dijkstra's Algorithm (2)



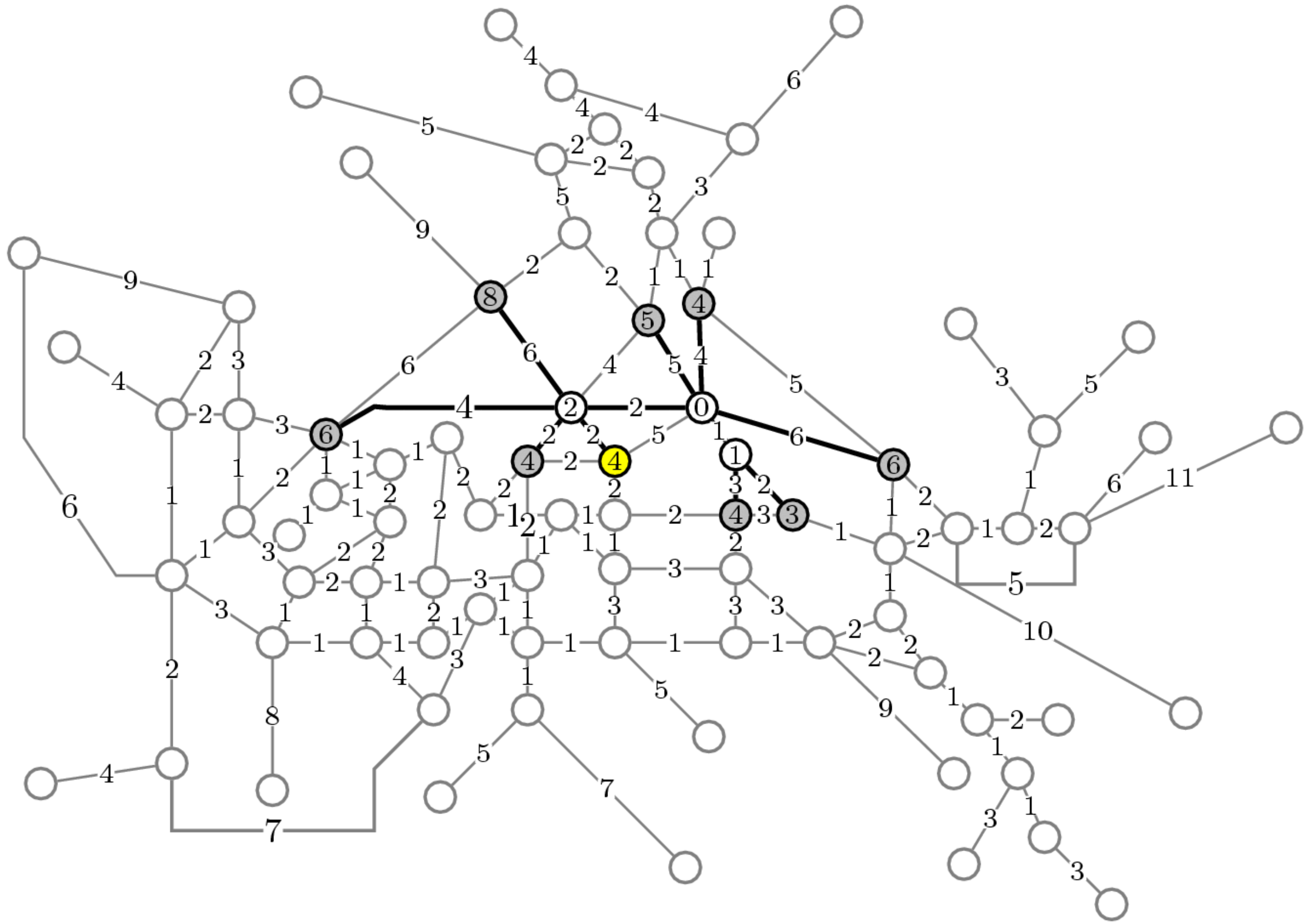
Dijkstra's Algorithm (3)



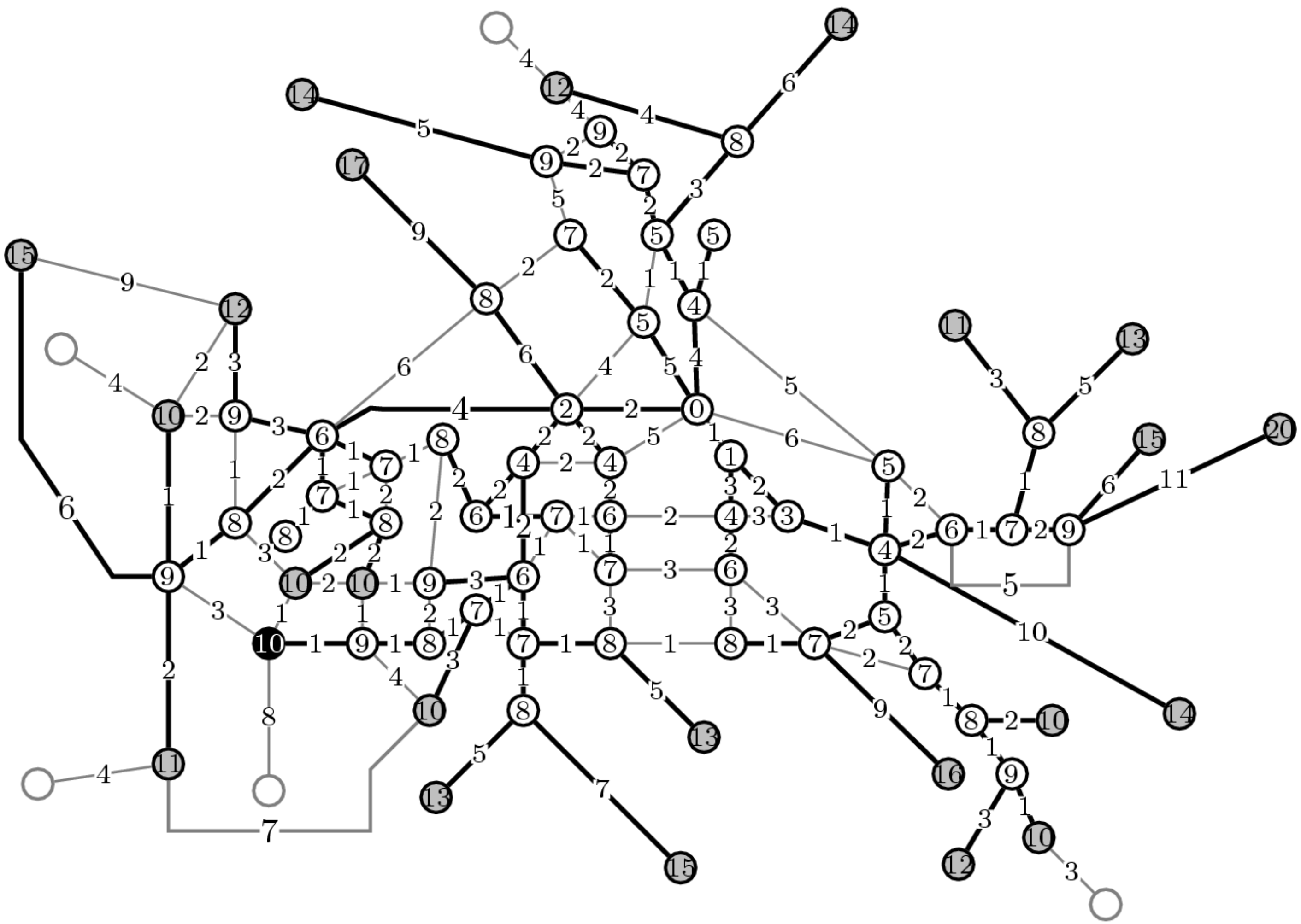
Dijkstra's Algorithm (4)



Dijkstra's Algorithm (5)



Shortest Path Tree



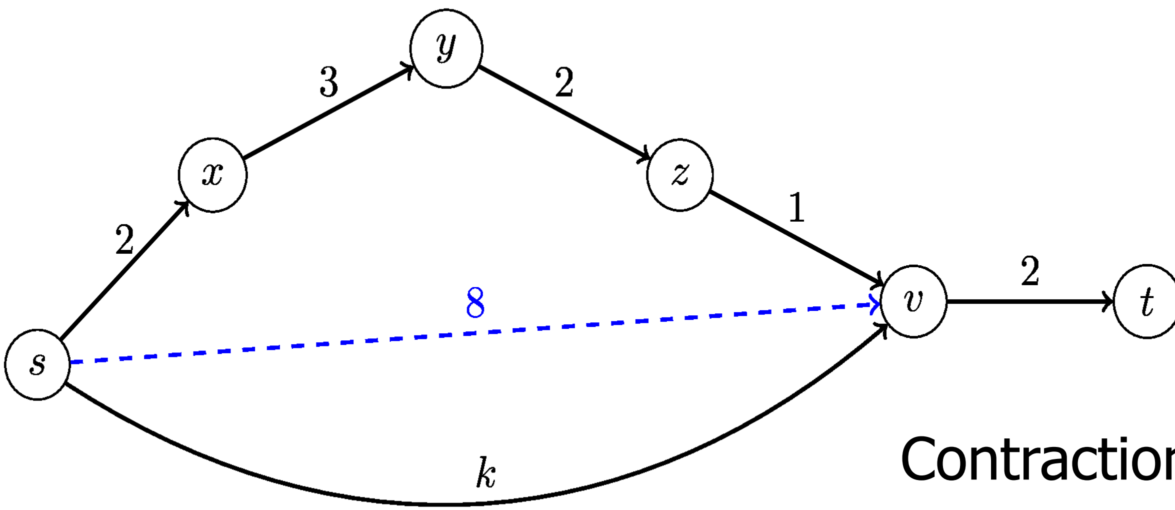
Running Time of Dijkstra's Algorithm

- Set all node labels = 0,
  distances = ∞, predecessors = none — O(n)
- Set distance at start node = 0, pred. = start — O(1)
- Repeat — n *
  - Find unlabeled node with minimum distance or stop, done! — O(n)
  - Label it — O(1)
  - For all outbound edges — O(n)
    - Update distance and predecessor labels

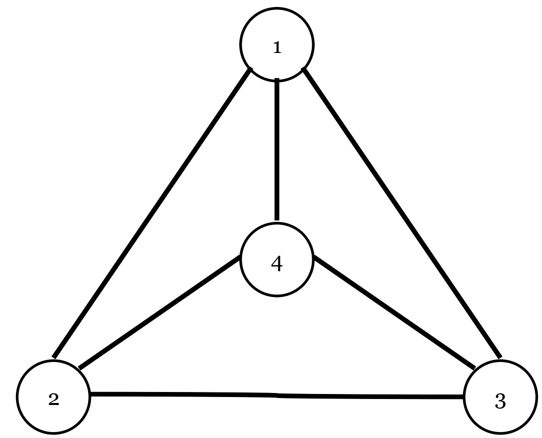
$O(n^2)$

Theorem: Dijstra's algorithm runs in polynomial time.

Contraction (Kolman & Pangrac [2009])



Contraction



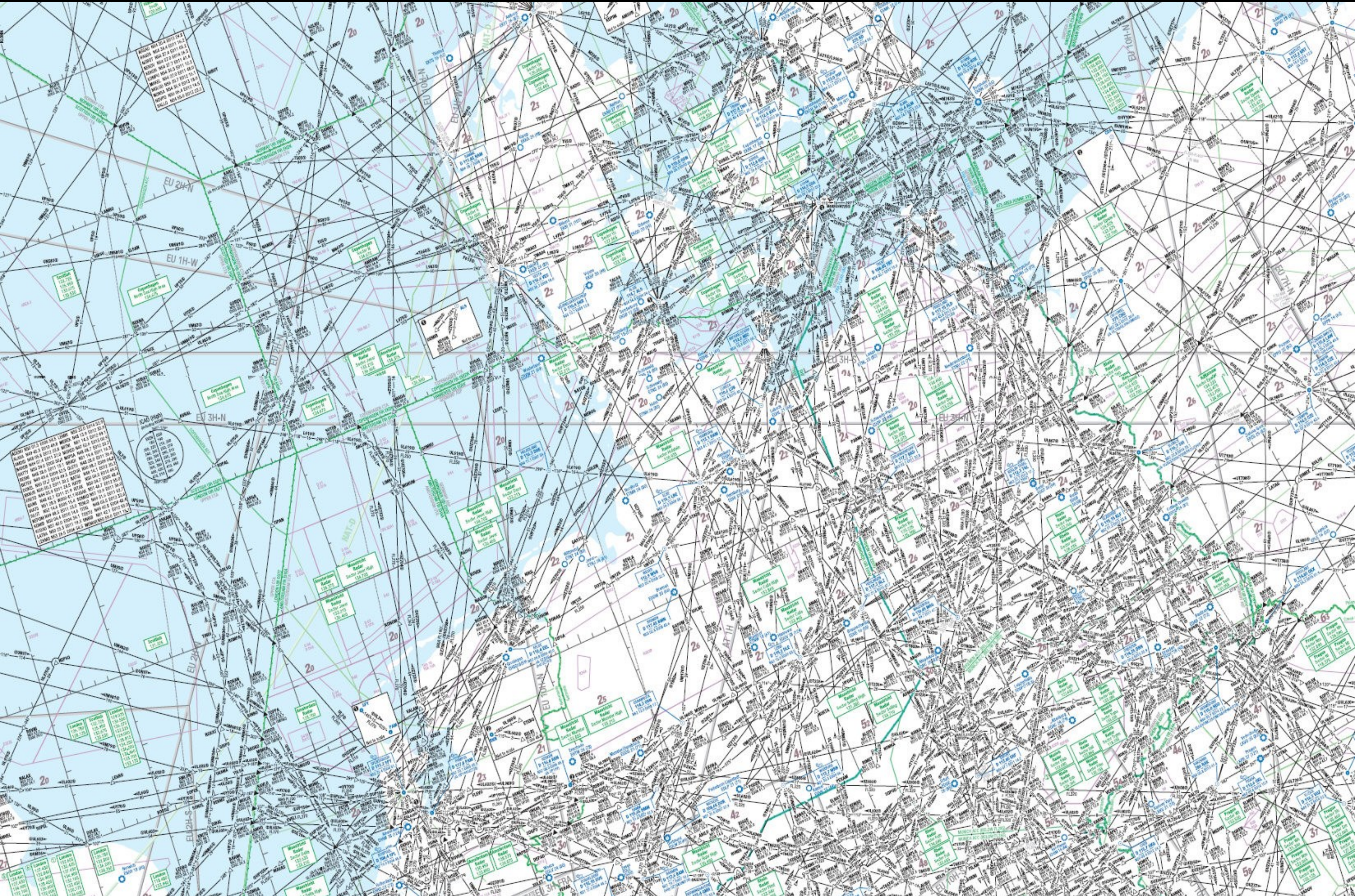
State of the Art in Shortest Paths

(Bast, Delling, Goldberg, Müller-Hannemann, Pajor, Sanders, Wagner, Werneck [2014])

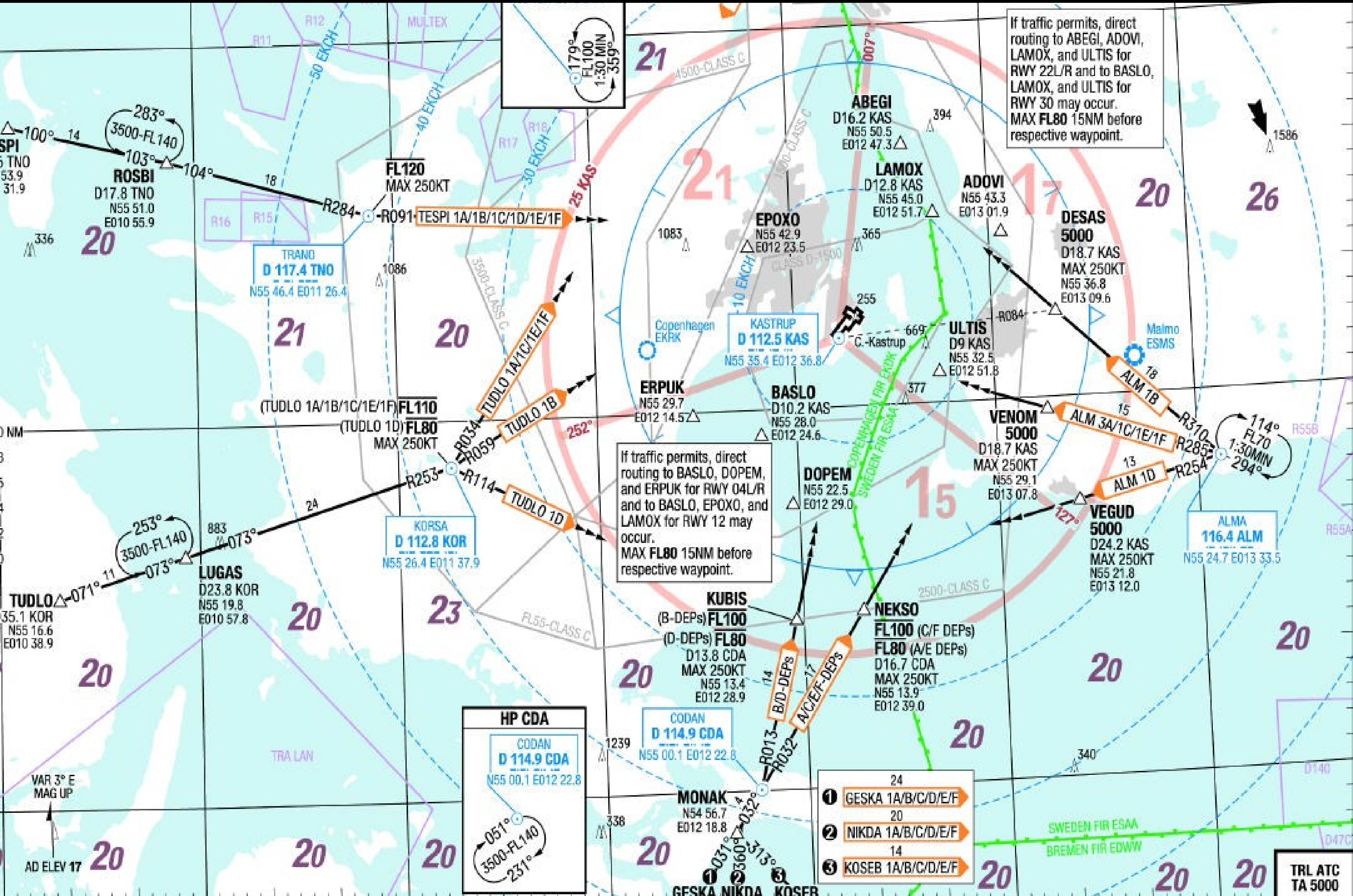
Table 1: Performance of various speedup techniques on Western Europe. Column *source* indicates the implementation tested for this survey.

		DATA STRUCTURES		QUERIES	
algorithm	source	space [GiB]	time [h:m]	scanned vertices	time [μs]
Dijkstra	[65]	0.4	–	9 300 000	2 550 000
Bidir. Dijkstra	[65]	0.4	–	4 800 000	1 350 000
CRP	[67]	0.9	1:00	2 766	1 650
Arc Flags	[65]	0.6	0:20	2 646	408
CH	[67]	0.4	0:05	280	110
CHASE	[65]	0.6	0:30	28	5.76
HLC	[70]	1.8	0:50	–	2.55
TNR	[13]	2.5	0:20	–	1.25
TNR+AF	[37]	5.4	1:45	–	0.99
HL	[70]	18.8	0:37	–	0.56
HL-∞	[5]	17.7	60:00	–	0.25
table lookup	[65]	1 208 358.7	145:30	–	0.06

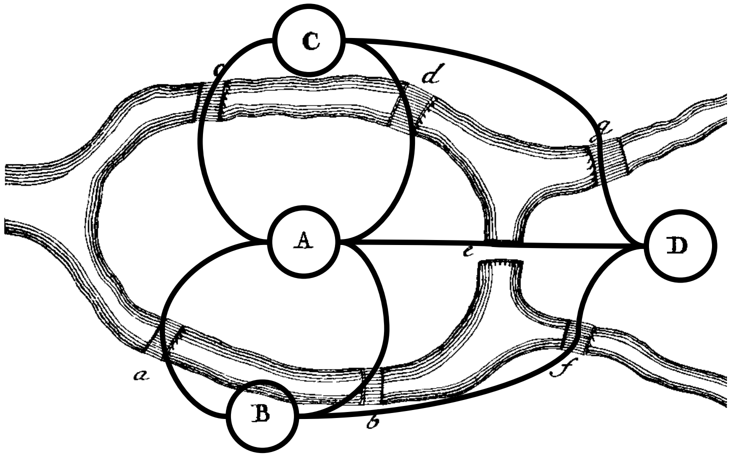
Airway network – Denmark and Germany



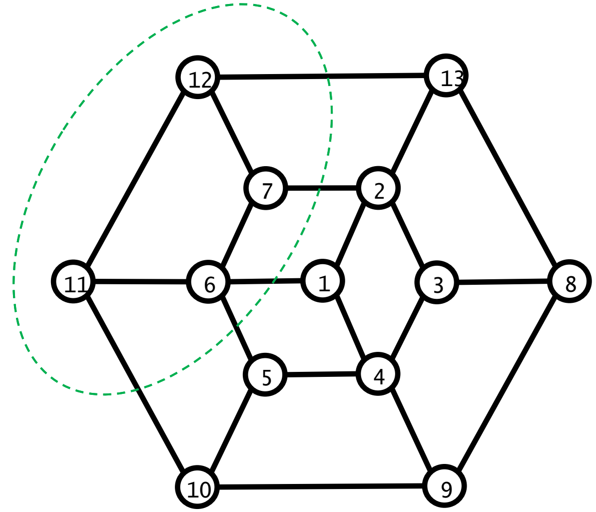
Airway network – Denmark



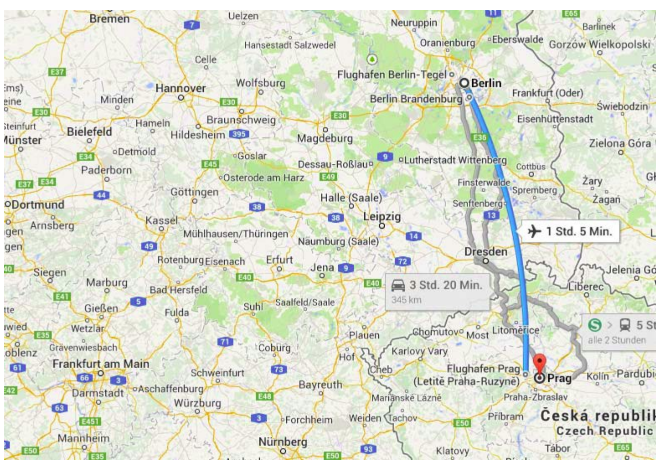
Constraints



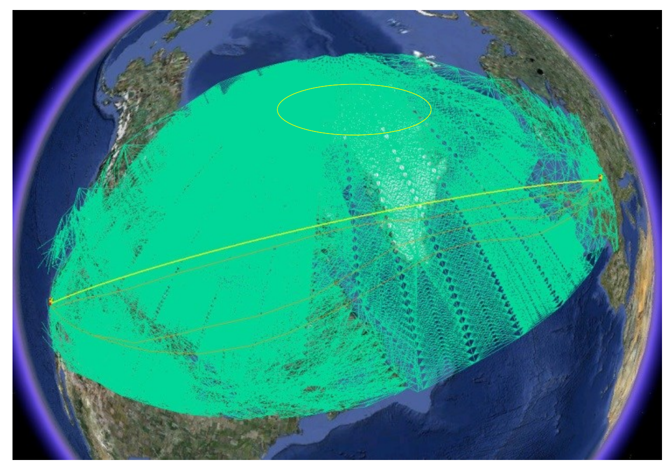
Euler Tour



Hamiltonian Cycle



Shortest Path



Aircraft Trajectory

1. Shortest Paths with Pair Constraints

Freie Universität Berlin

Updated on 12/12/13
Valid from 06/02/14

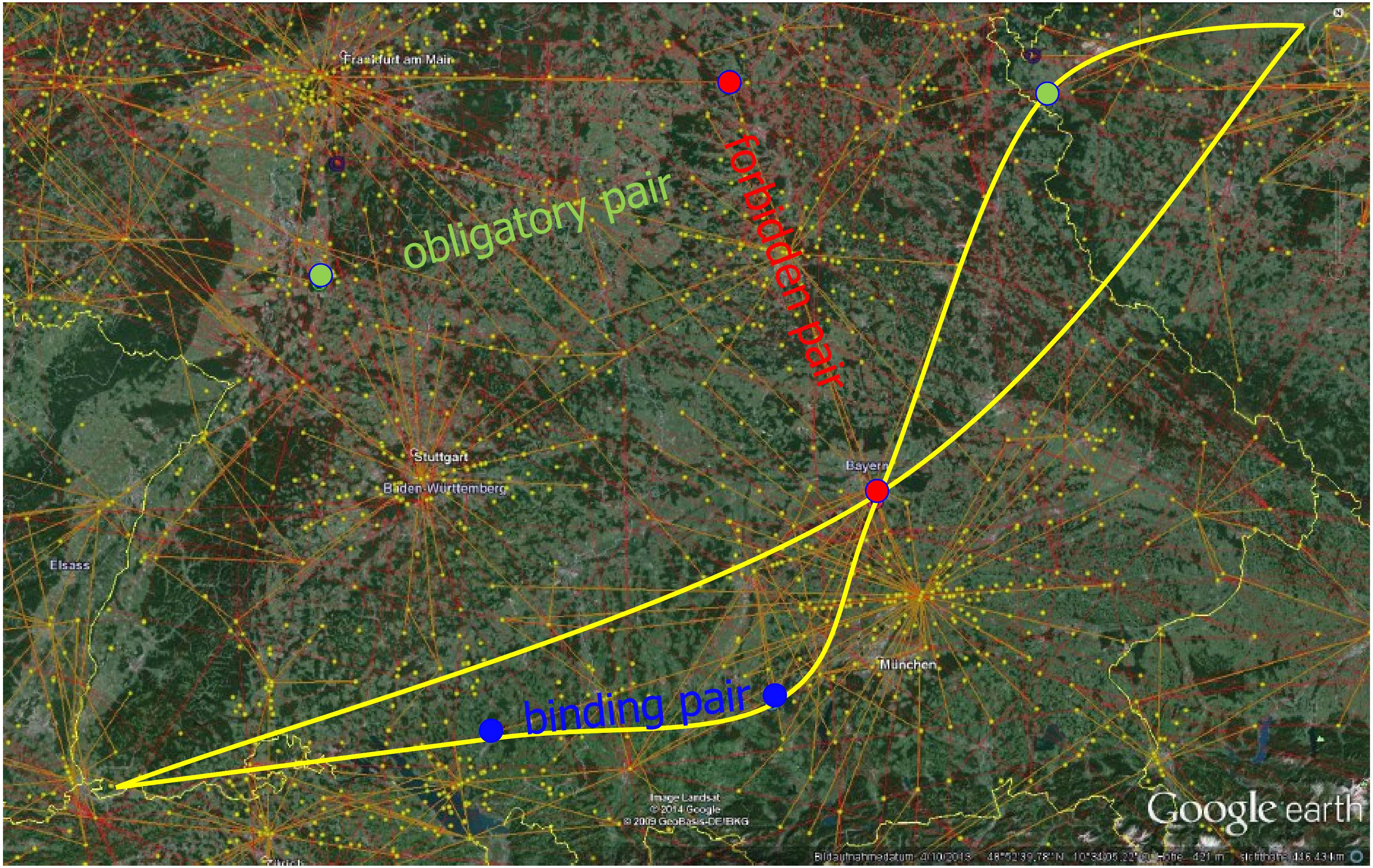
ROUTE RESTRICTIONS THROUGH GERMANY - ED

ANNEX ED
Page 1 of 92

AIRWAY	FROM - TO	RESTRICTION	ID No.	OPERATIONAL GOAL
	ABLOX	Not available for traffic DEP EDAB Except with ARR EDDB/DT, Havel Group	ED2567	SID requirement
		Not available for traffic DEP EDDM Except 1. ARR EDJA Below FL095 2. Type Jet	ED2894	SID requirement for EDDM departures
		Only available and compulsory for traffic DEP EDDM Via MILKA above FL245 1. Daily 22.30 (21.30) - 07.00 (06.00) 2. FRI 16.00 (15.00) - MON 07.00 (06.00) 3. During legal holidays	ED2895	SID requirement for EDDM departures Outside these times file SID MERSI Y110.
	BIBAG	Not available for traffic DEP EDDM Via L/UL605/Q104/Q118	ED3147	SID requirement
		With ARR Farnborough Group, London Group	EDYY1010	SID requirement Time refers to departure time EDDF.
	BODLA/ERGON/GOVEN	Compulsory for traffic DEP/Overflying EPWWFIR/UIR With ARR EDDB/DT	ED2876	To force traffic onto arrivals routes
	COL	Not available for traffic DEP EDDK Except 1. ARR Frankfurt Group, Frankfurt Y/Z Group 2. Training Flights	ED2576	SID requirement

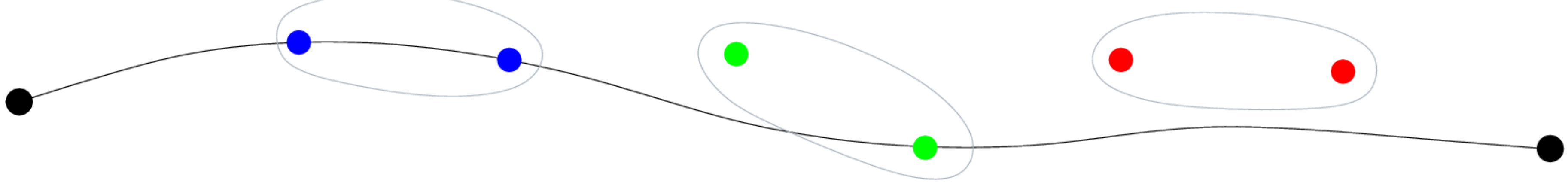
- If EDDF is the departure airport, then BIBTI must not be visited: EDDF $\in$ P $\Rightarrow$ BIBTI $\notin$ P (easy because departure is known)
- If MILKA is visited, then ALG must be visited: MILKA $\in$ P $\Rightarrow$ ALG $\in$ P (hard because visits are not known)

Forbidden, Obligatory, and Binding Pairs

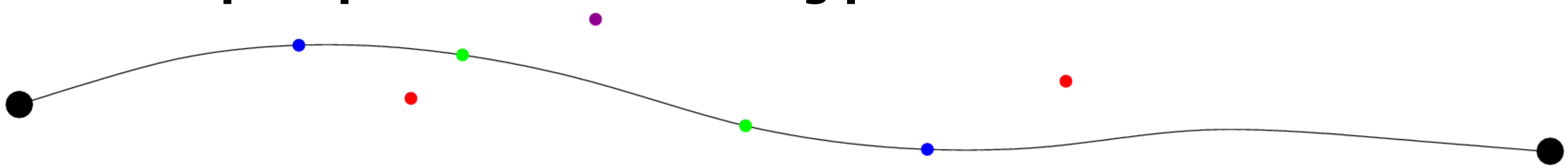


Forbidden, Obligatory, and Binding Pairs

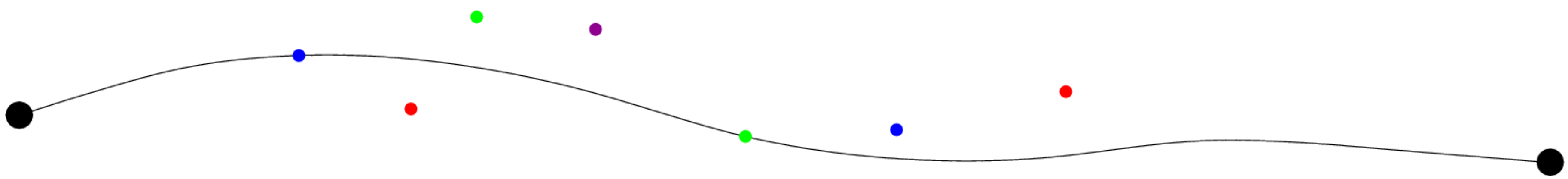
Given an ATS network, two nodes (waypoints) *s* and *t*, and a set of node-pairs *uv*, find a shortest *st*-path such that for every pair ...



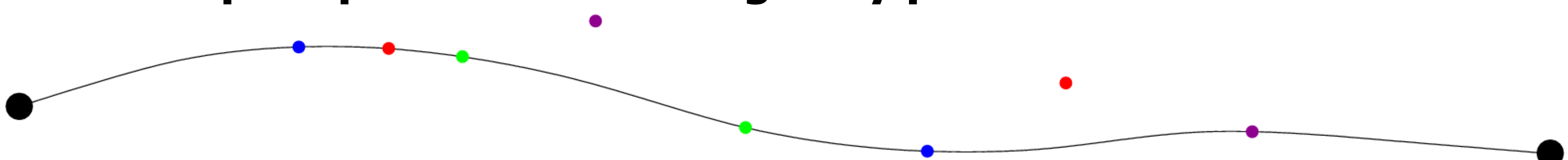
Shortest-path problem with binding pairs: u $\in$ P $\Rightarrow$ v $\in$ P



Shortest-path problem with forbidden pairs: u $\in$ P $\Rightarrow$ v $\notin$ P

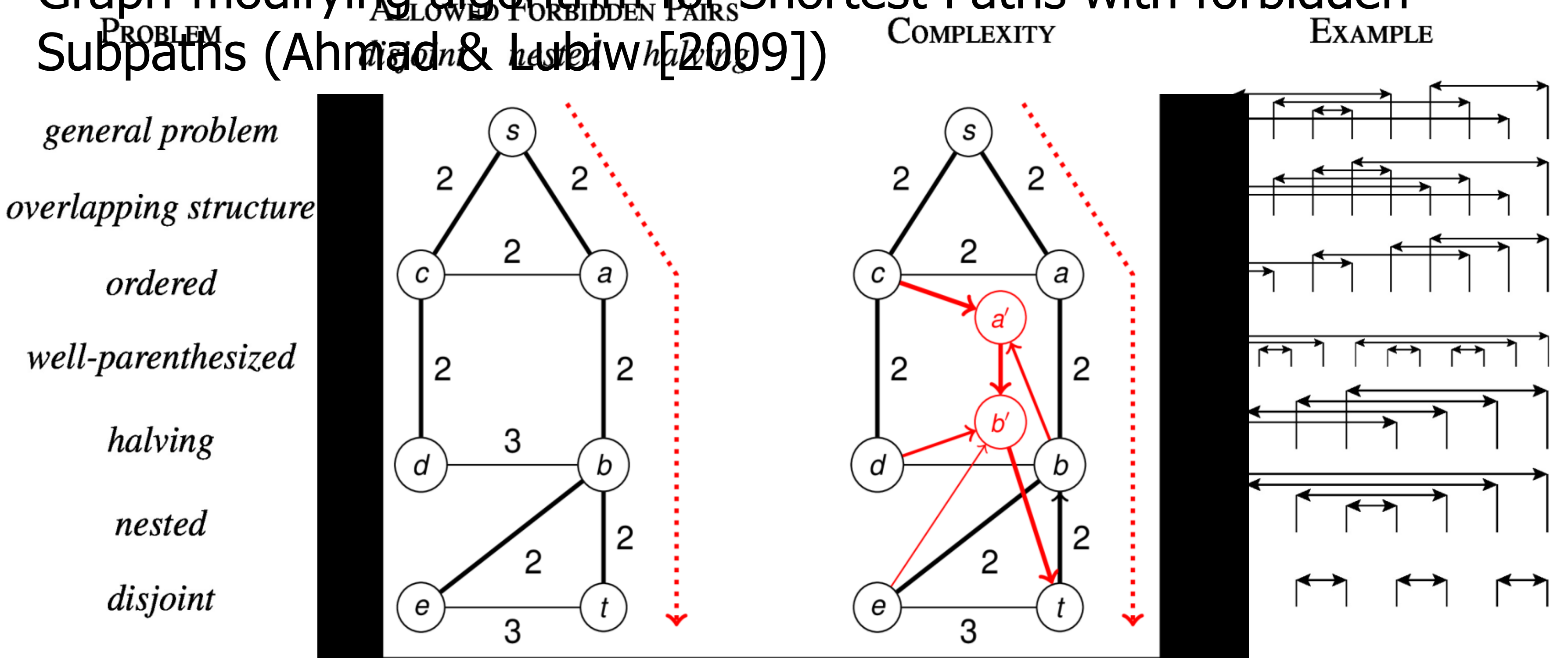


Shortest-path problem with obligatory pairs: u $\notin$ P $\Rightarrow$ v $\in$ P



Shortest Paths with Forbidden Pairs

- Applications in automatic software testing and bioinformatics (Gabow et al. [1976], Chen et al. [2001])
- NP- and APX-hard (Gabow et al. [1976], Hajiaghayi et al. [2010])
- Efficient contraction/dynamic programming algorithms for networks with special structures (Kolman and Pangrac [2009], Kovac [2011])
- Graph-modifying algorithm for Shortest Paths with forbidden Subpaths (Ahmad & Lubiw [2009])

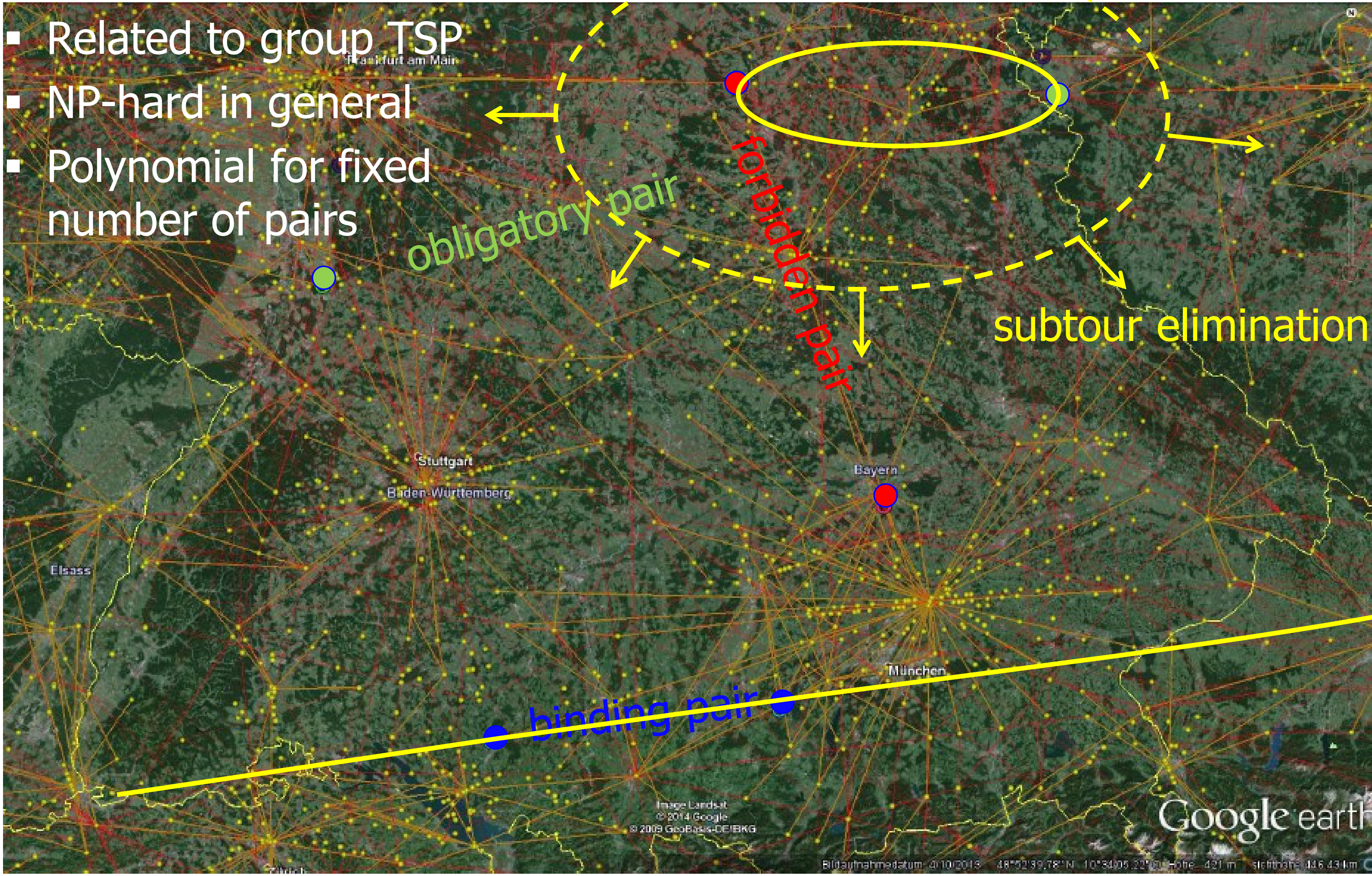


Shortest Paths with Binding Pairs

- Applications in automatic software testing
- NP-Hard even on acyclic digraphs (Ntafos, Hakimi [1979])
- No further literature

Shortest Paths with Obligatory Pairs

- Related to group TSP
- NP-hard in general
- Polynomial for fixed number of pairs



Shortest Paths with Pair Constraints

$$
\begin{array}{rcll}
\min & c^T x & & \\
x(\delta^-(v)) - x(\delta^+(v)) = & \delta_{st}(v) & & \forall v \in V \\
x(\delta^-(u)) + x(\delta^-(v)) \leq & 1 & & \forall uv \in F \\
x(\delta^-(u)) - x(\delta^-(v)) \leq & 0 & & \forall uv \in B \\
x(\delta^-(u)) + x(\delta^-(v)) \geq & 1 & & \forall uv \in O \\
 & & & \\
x_{uv} \in & \{0,1\} & & \forall uv \in A
\end{array}
$$

- No subtour elimination in acyclic digraphs

Proposition (Brückner [2015])

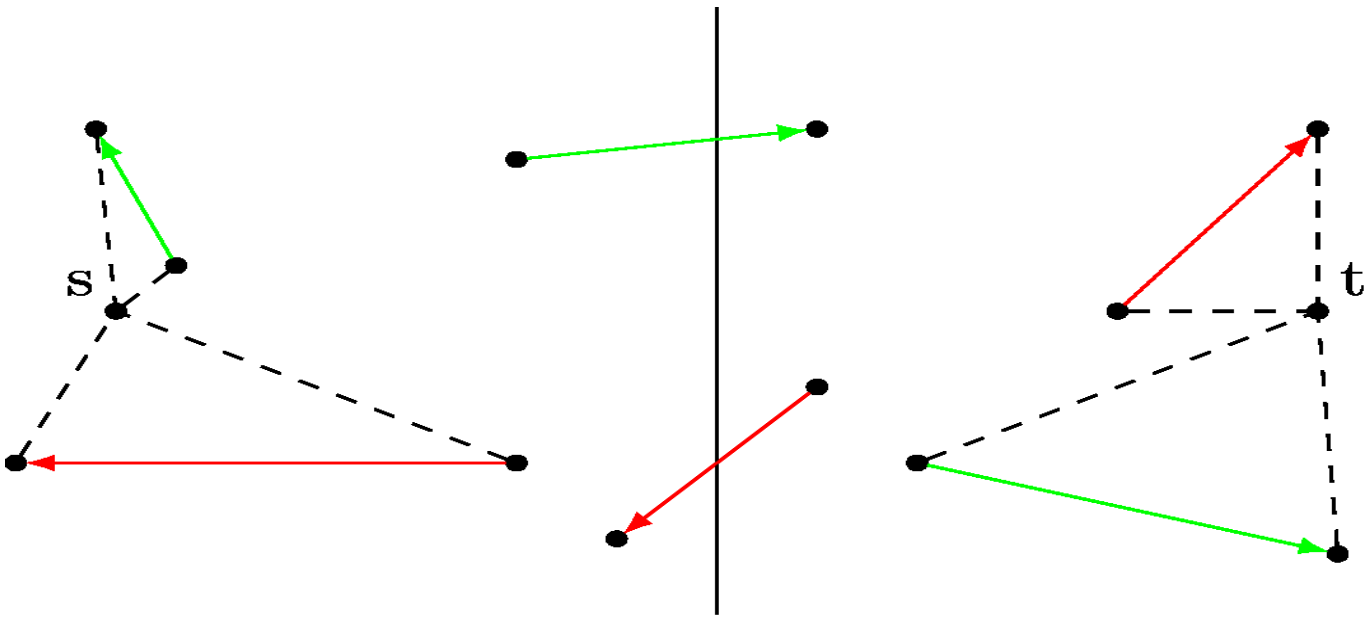
The SPPPC can be solved in polynomial time if the pairs are well-parenthesized.

Theorem (Blanco, B, Brückner, Hoang [2015])

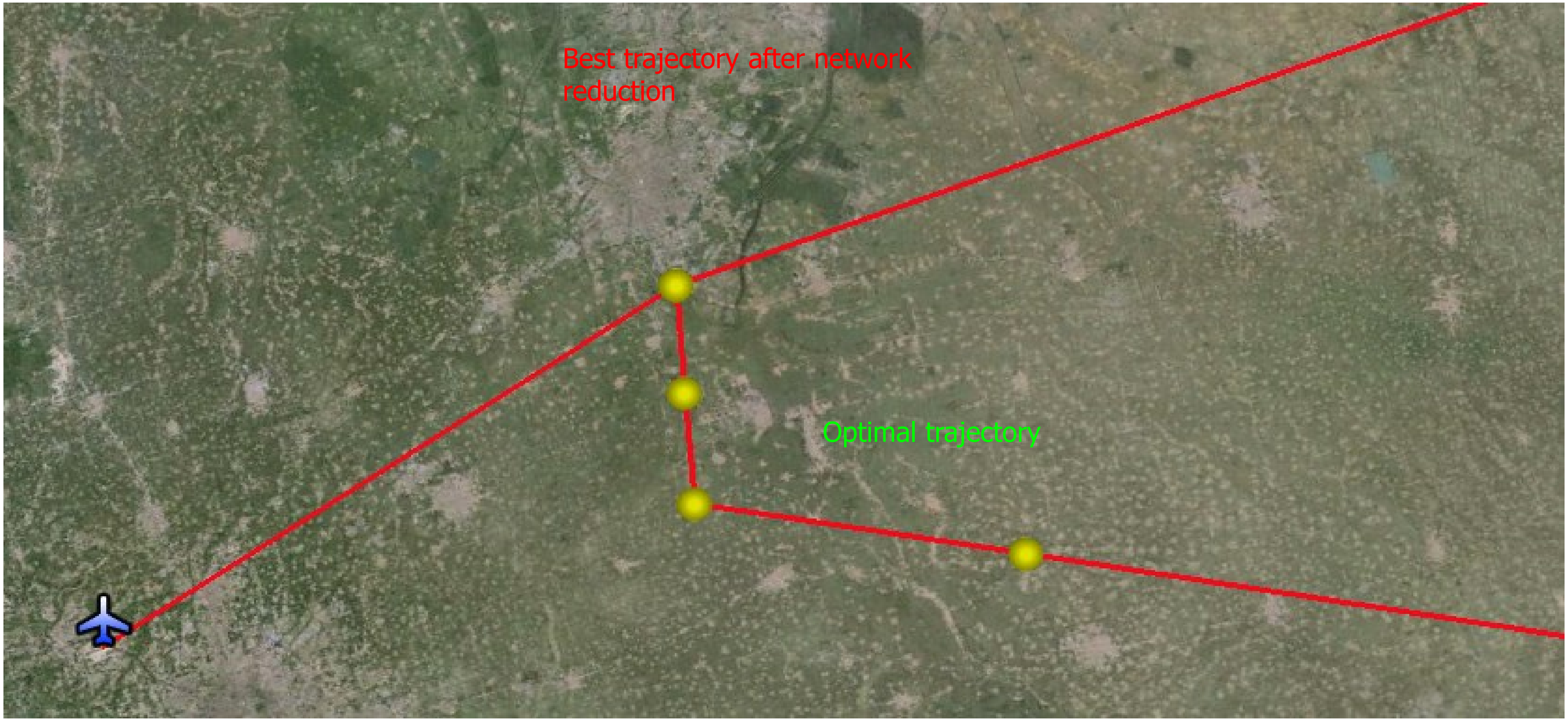
A complete linear description of the SPPPC polytope can be obtained if the pairs are disjoint.

Network Orientation

- Delete network segments pointing in the "wrong direction"
  - that point backward over the cut halfway between origin & destination
  - that point back to the origin (in the origin's hemisphere)
  - that point away from the destination (in the destination's hemisphere)
  - Results in acyclic network



Network Orientation Example Beijing-Hong Kong



- Errors near airports, mostly (but not always) small

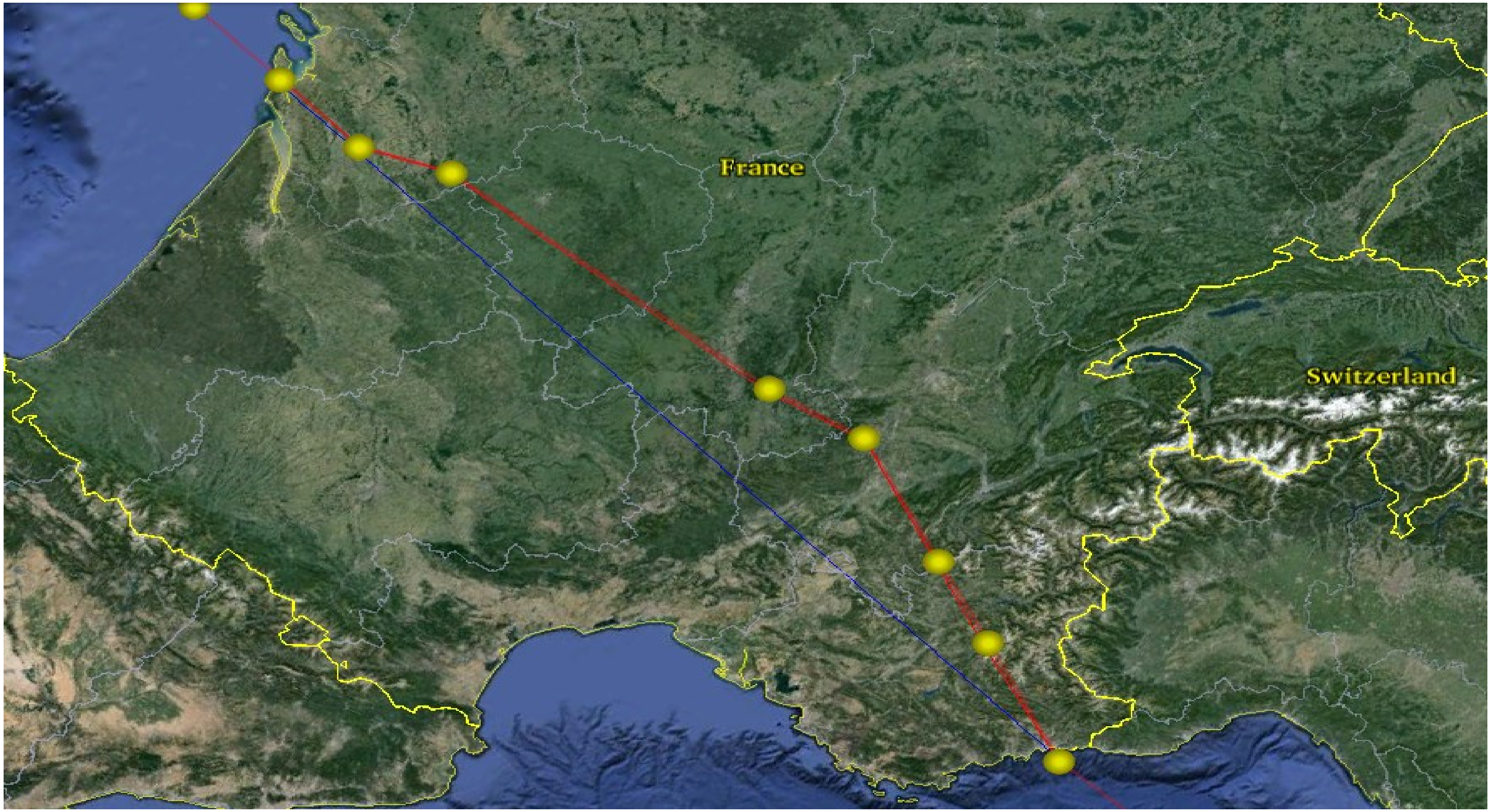
EUROCONTROL

Adjusted unit rates applicable to April 2014 flights

Please find hereunder the unit rates of route charges applicable to April 2014 flights, as well as the exchange rates used for their calculation, i.e. the average exchange rates for the month of March 2014 (monthly average of the "Closing Cross Rate" calculated by Reuters based on daily BID rate).

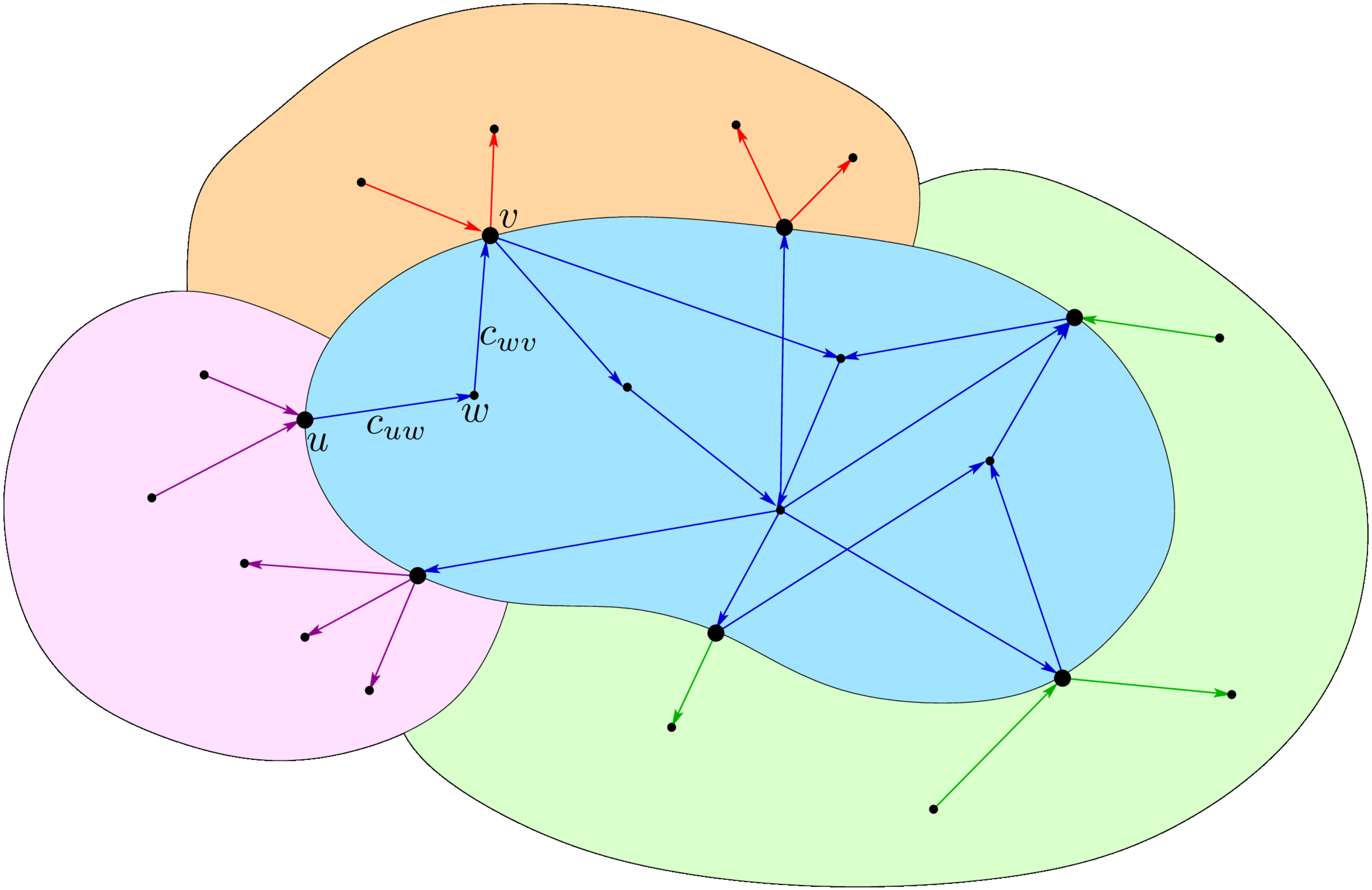
Zone	Taux unitaire Unit rate EUR	Taux de change Exchange rate 1 EUR =		
Portugal Santa Maria *	**10.60**	./.		
Belg.-Luxembourg *	**72.19**	./.		
Allemagne / Germany *	**77.47**	./.		
Finlande / Finland *	**52.21**	./.		
Royaume-Uni / United Kingdom	**84.85**	0.831957		GBP
Pays-Bas / Netherlands *	**66.62**	./.		
Irlande / Ireland *	**30.77**	./.		
Danemark / Denmark	**71.35**	7.46207		DKK
Norvège / Norway	**51.96**	8.29105		NOK
Pologne / Poland	**35.26**	4.19932		PLN
Suède / Sweden	**72.25**	8.86081		SEK
Lettonie / Latvia *	**28.59**	0.702804	***	LVL
Lituanie / Lithuania	**45.92**	3.45158		LTL
Espagne / Spain - Canarias *	**58.51**	./.		

ATC Charges

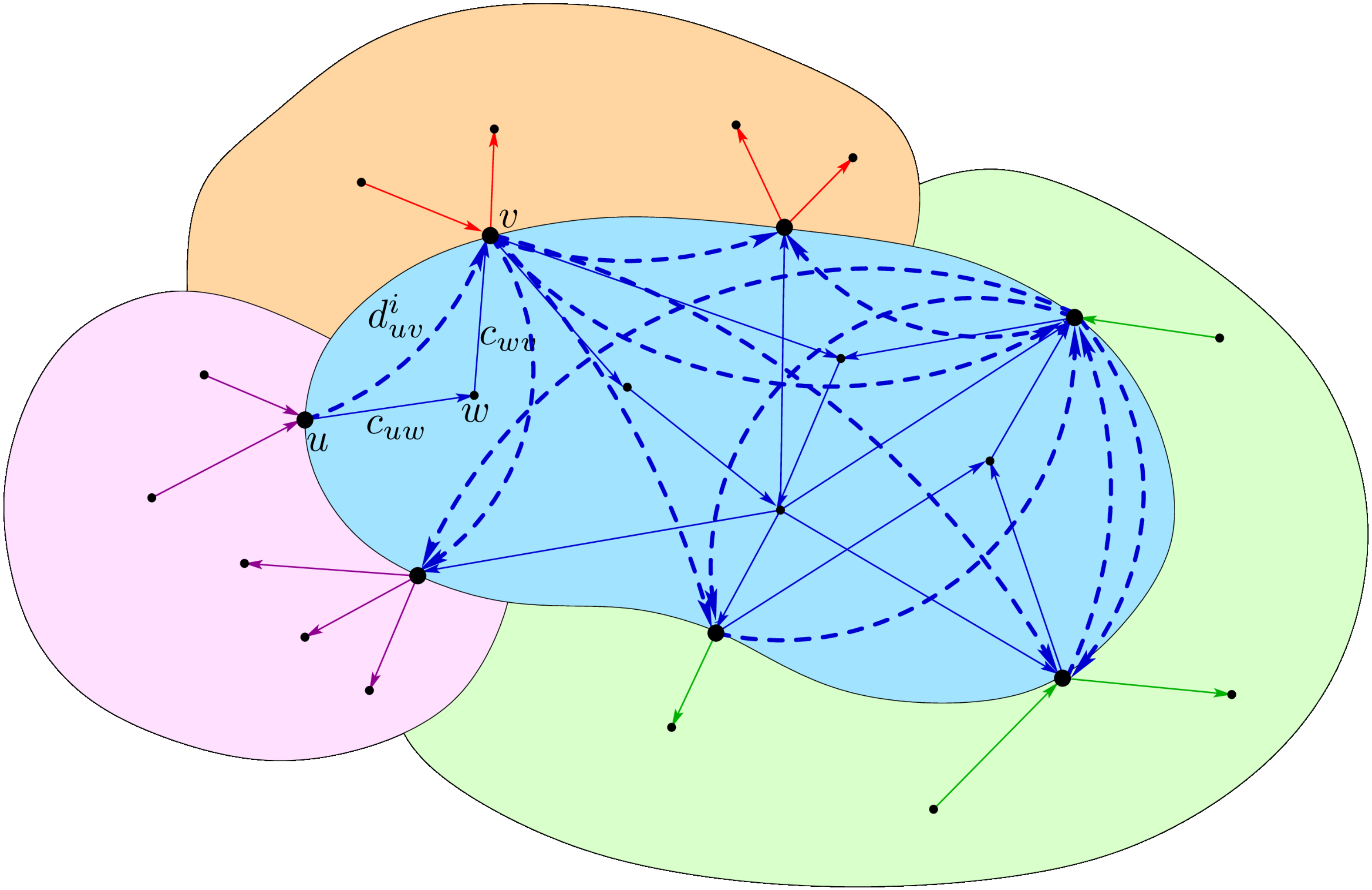


- Rate per flown km: easy to integrate in any search algorithm
- Rate per km in the great circle segment between entry and exit points: hard (Lido "pretends" it's the first model)

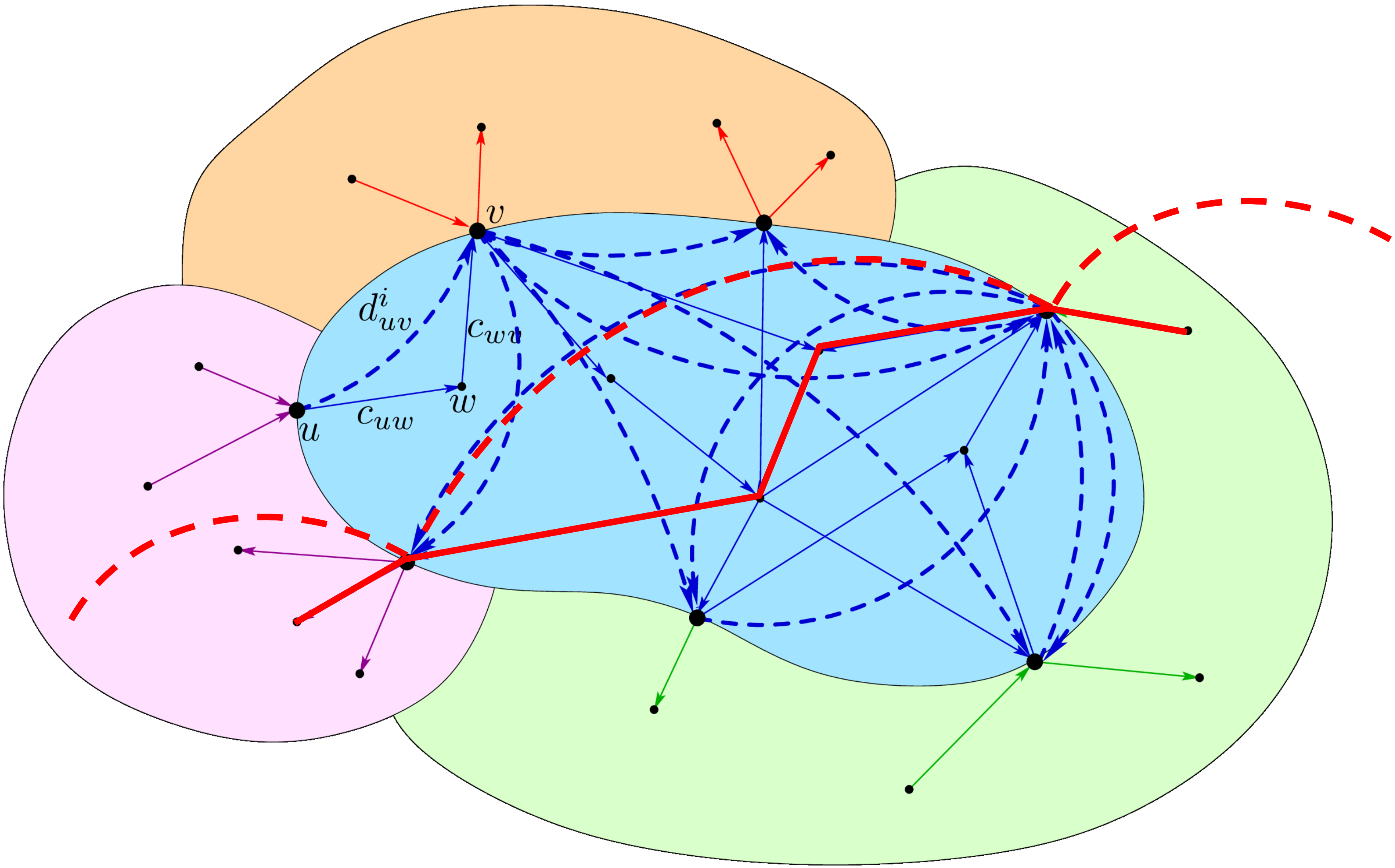
ATC Charges



ATC Charges



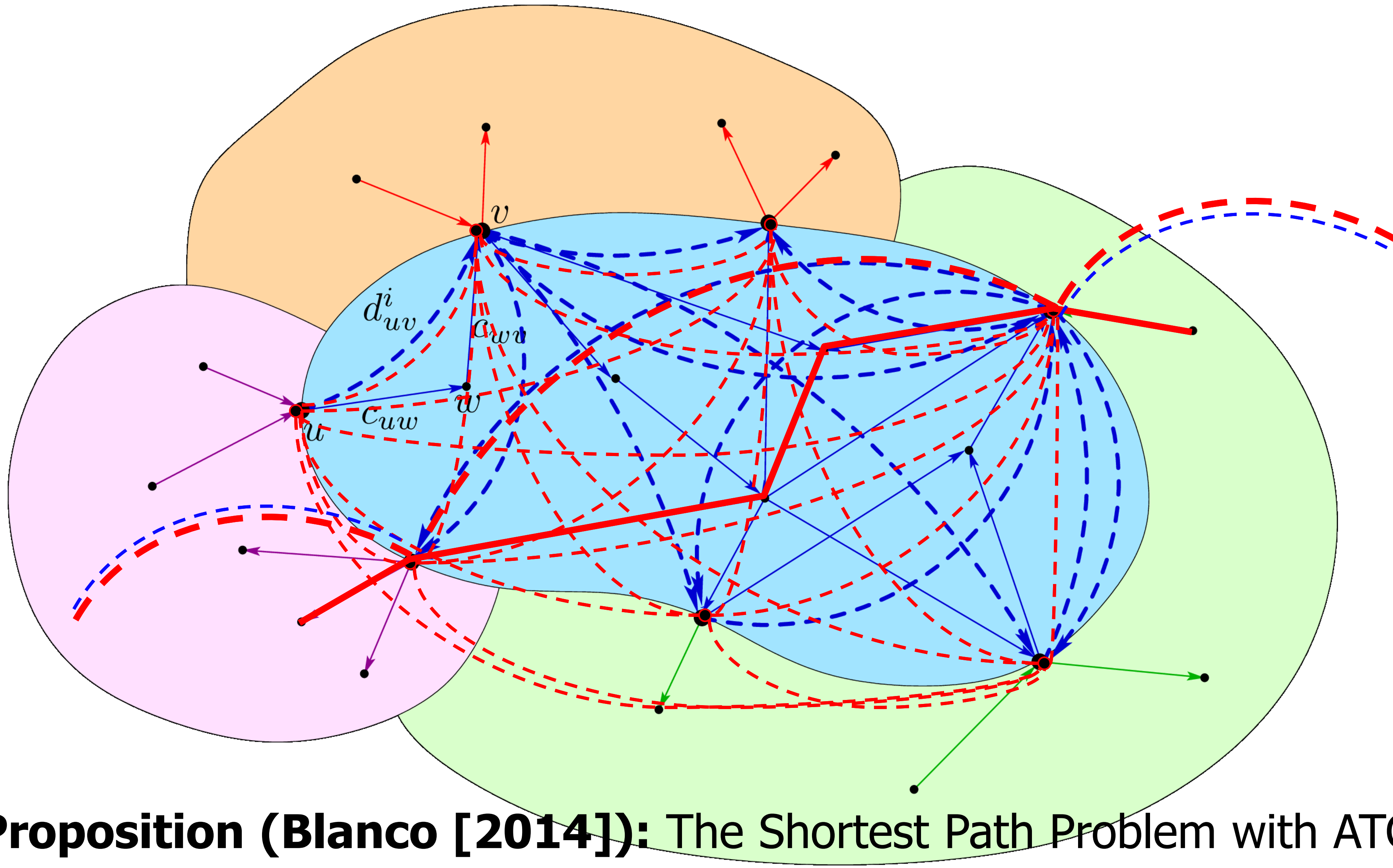
ATC Charges



ATC Charged Shortest Paths

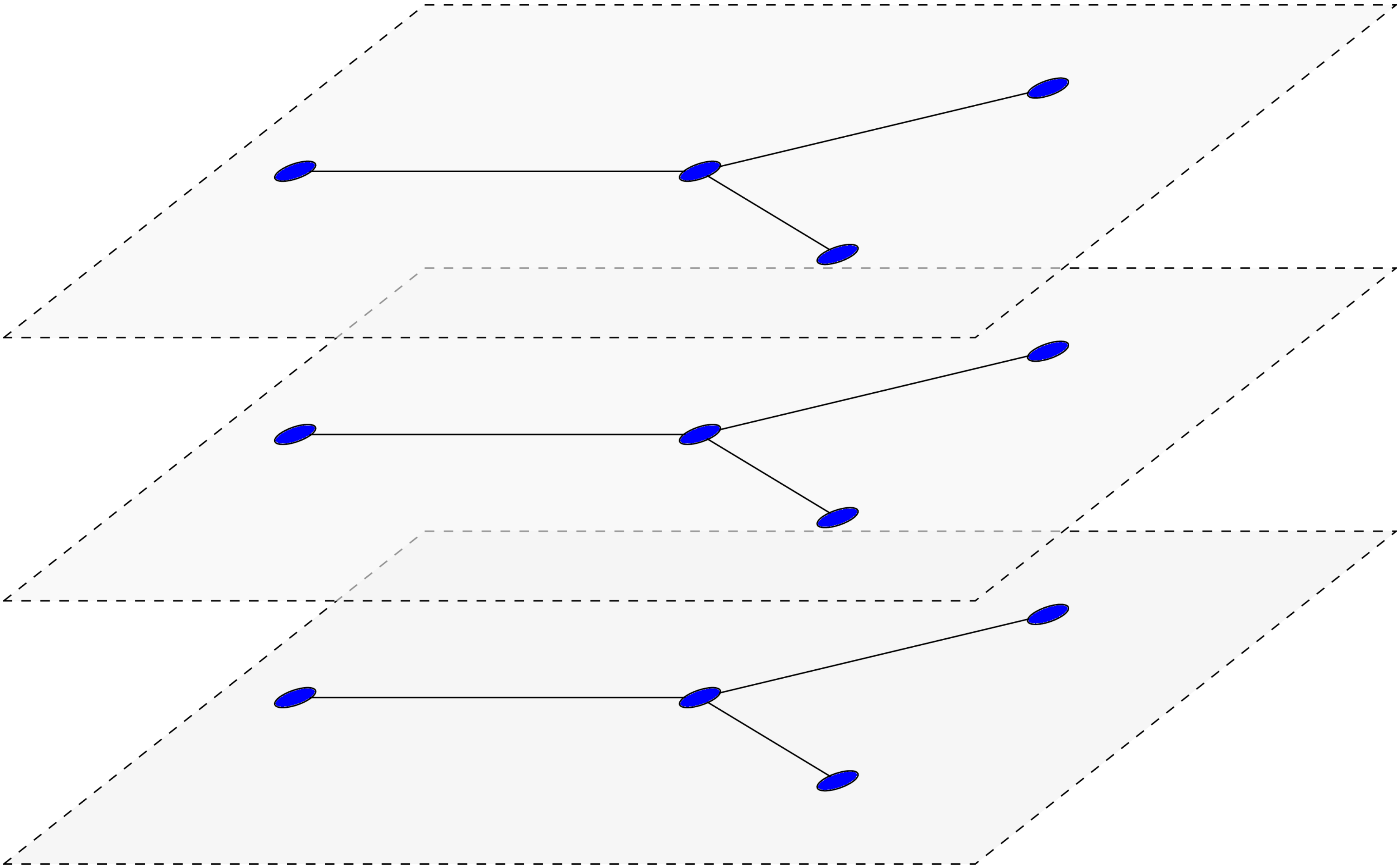
$$
\begin{aligned}
\min \quad & \sum_{(w,z)\in A} c_{wz} x_{wz} + \sum_i \sum_{(u,v)\in D_i} d^i_{uv} y_{uv} \\
\text{s.t.} \quad & \sum_{z:(w,z)\in A} x_{wz} - \sum_{u:(u,w)\in A} x_{uw} = \begin{cases} 1 & \text{if } w = s \\ -1 & \text{if } w = t \\ 0 & \text{else} \end{cases} && \forall w \in V \\
& \sum_{w:(u,w)\in A_i} x_{uw} = \sum_{v:(u,v)\in D_i} y_{uv} && \forall u \in \partial^-(V_i), \forall i \\
& \sum_{w:(w,v)\in A_i} x_{wv} = \sum_{u:(u,v)\in D_i} y_{uv} && \forall u \in \partial^+(V_i), \forall i \\
& \sum_i \sum_{v:(u,v)\in D_i} y_{uv} - \sum_i \sum_{w:(v,w)\in D_i} y_{vw} = \begin{cases} 1 & \text{if } v = s \\ -1 & \text{if } v = t \\ 0 & \text{else} \end{cases} && \forall v \in \bigcup_i \partial^{+/-}(V_i) \\
& x_{wz}, y^i_{uv} \in \{0,1\} && \forall (w,z) \in A, \\
& && \forall (u,v) \in D_i, \forall i
\end{aligned}
$$

ATC Charges

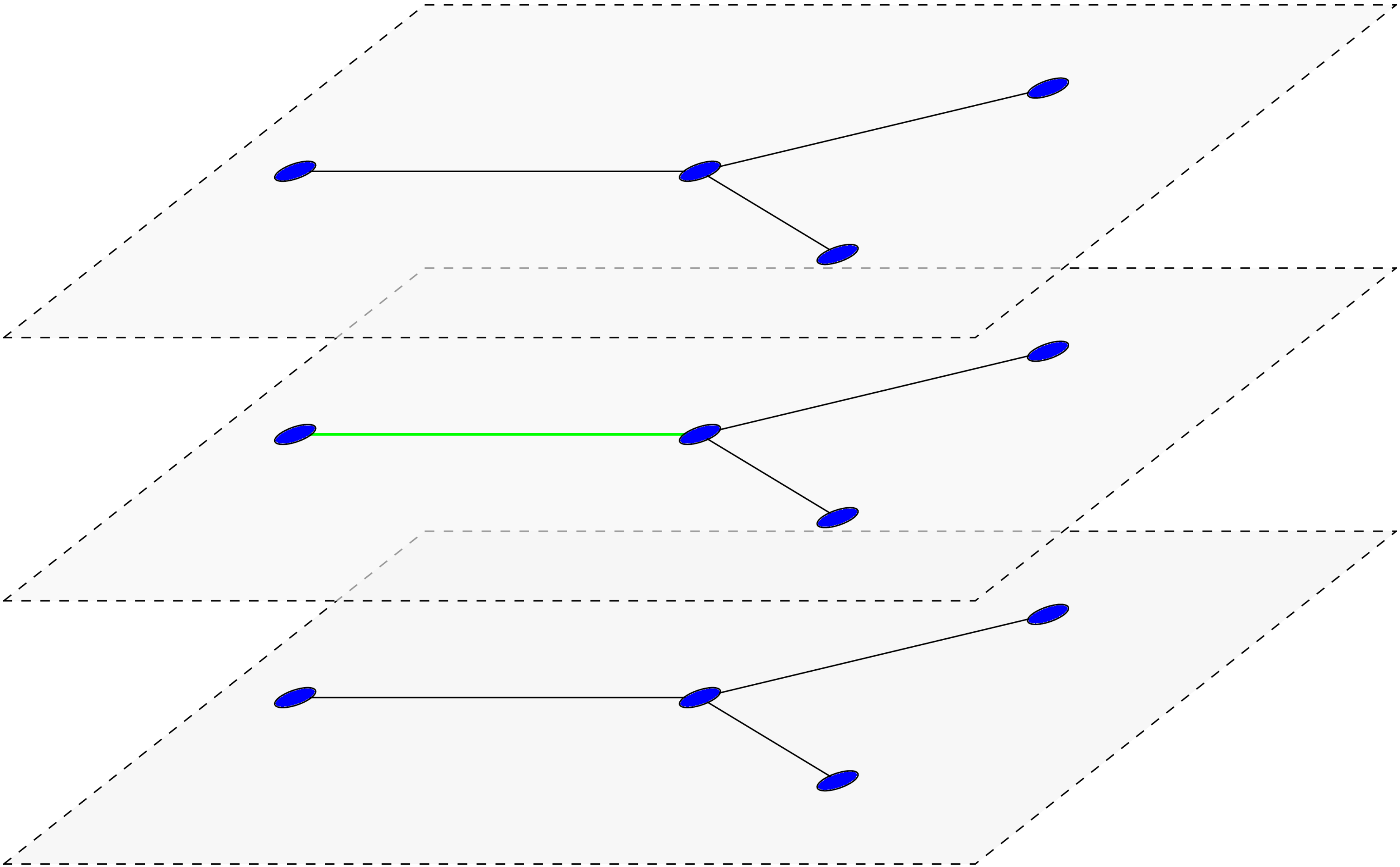


Proposition (Blanco [2014]): The Shortest Path Problem with ATC charges can be solved in polynomial time.

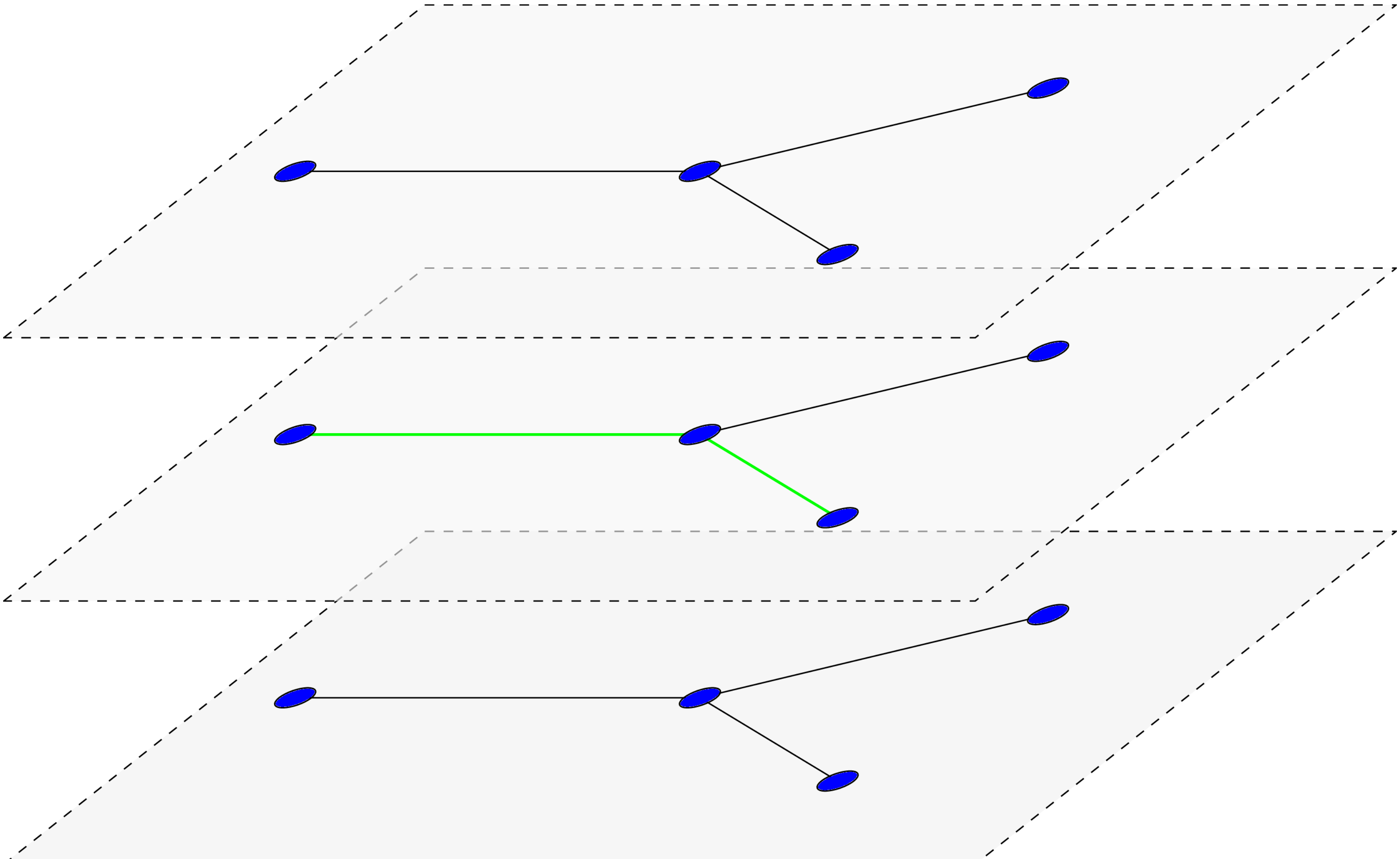
3. 3D-Shortest Paths



Horizontal segments are well defined.



Horizontal segments are well defined.



Horizontal segments are well defined.

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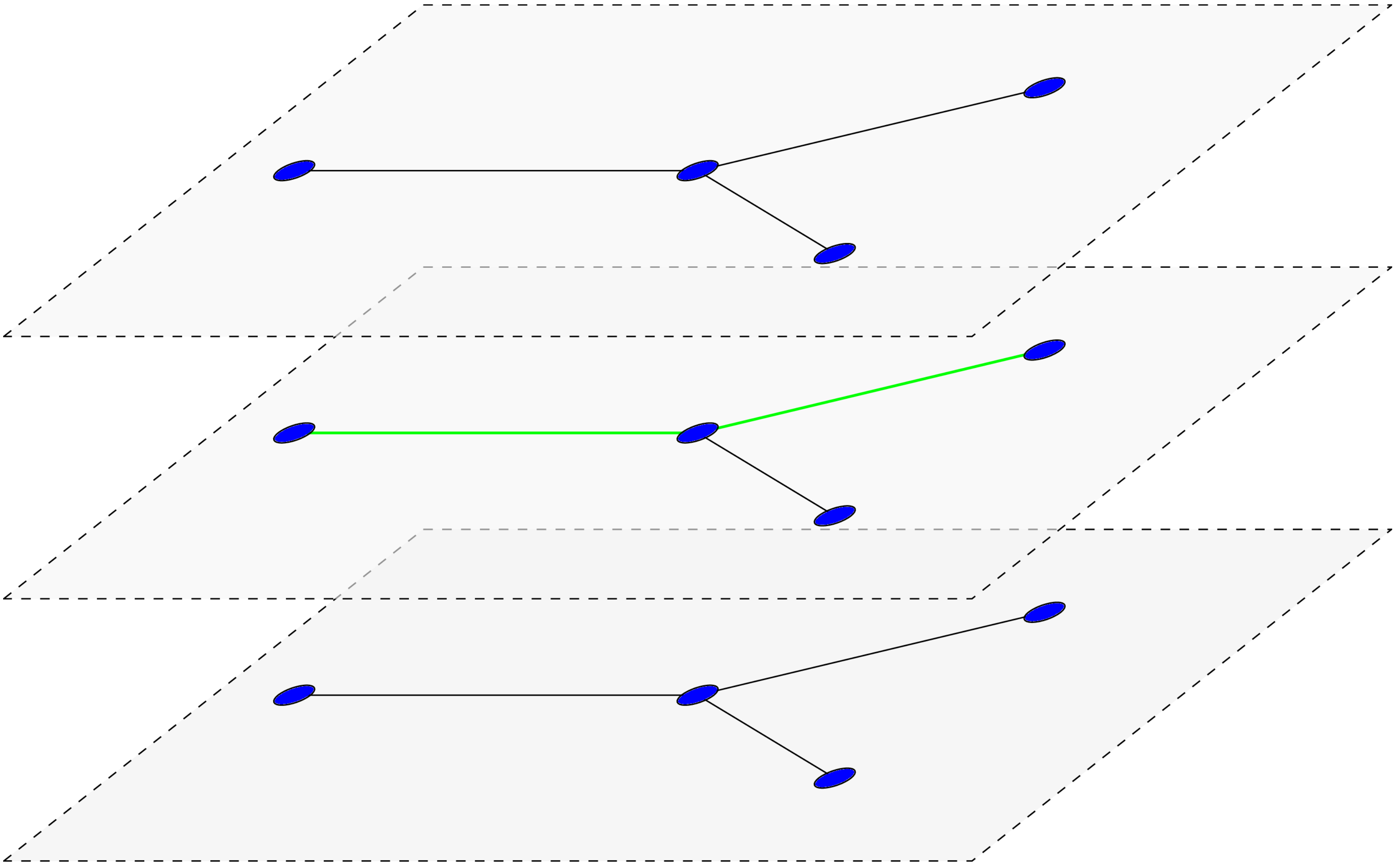
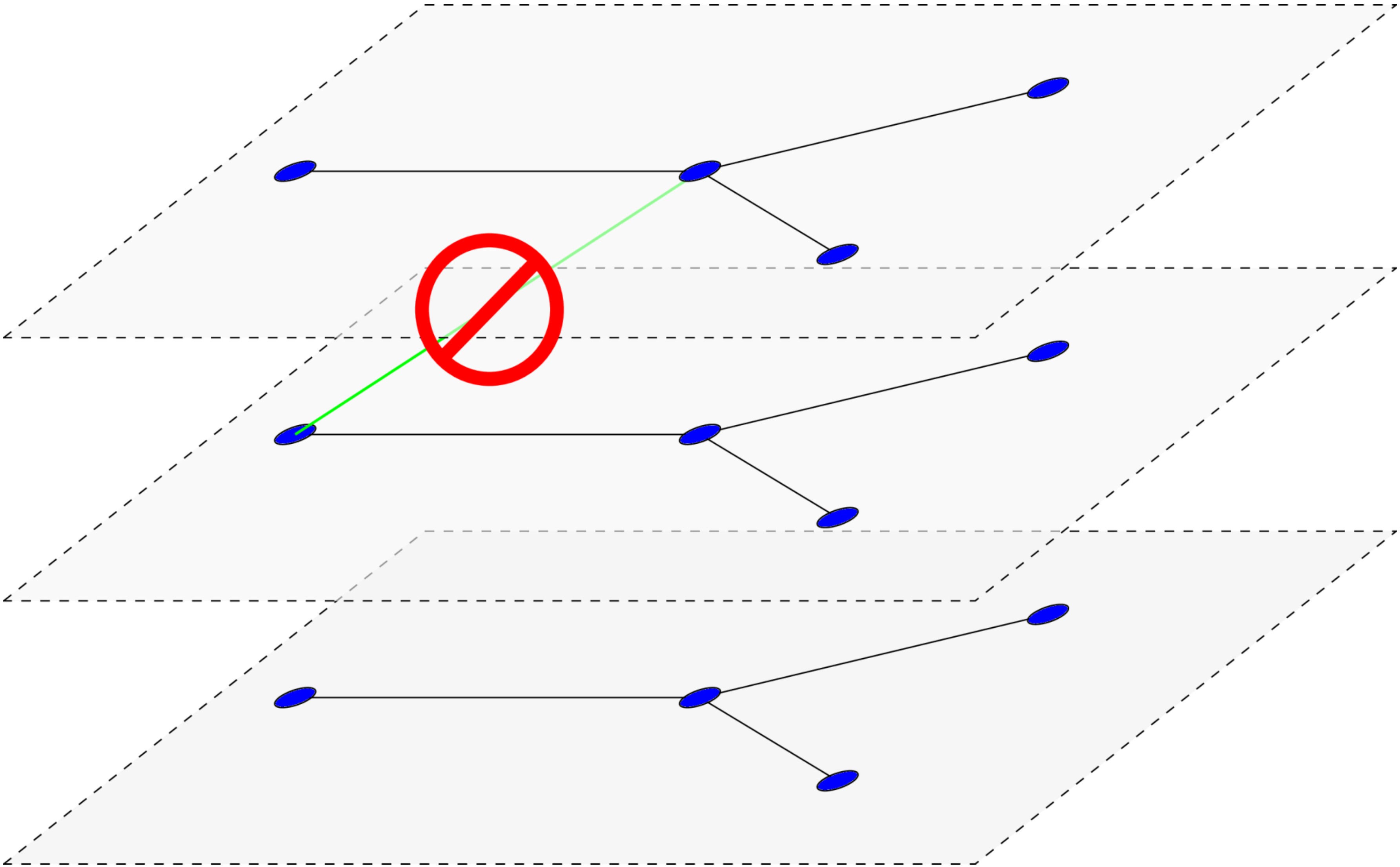
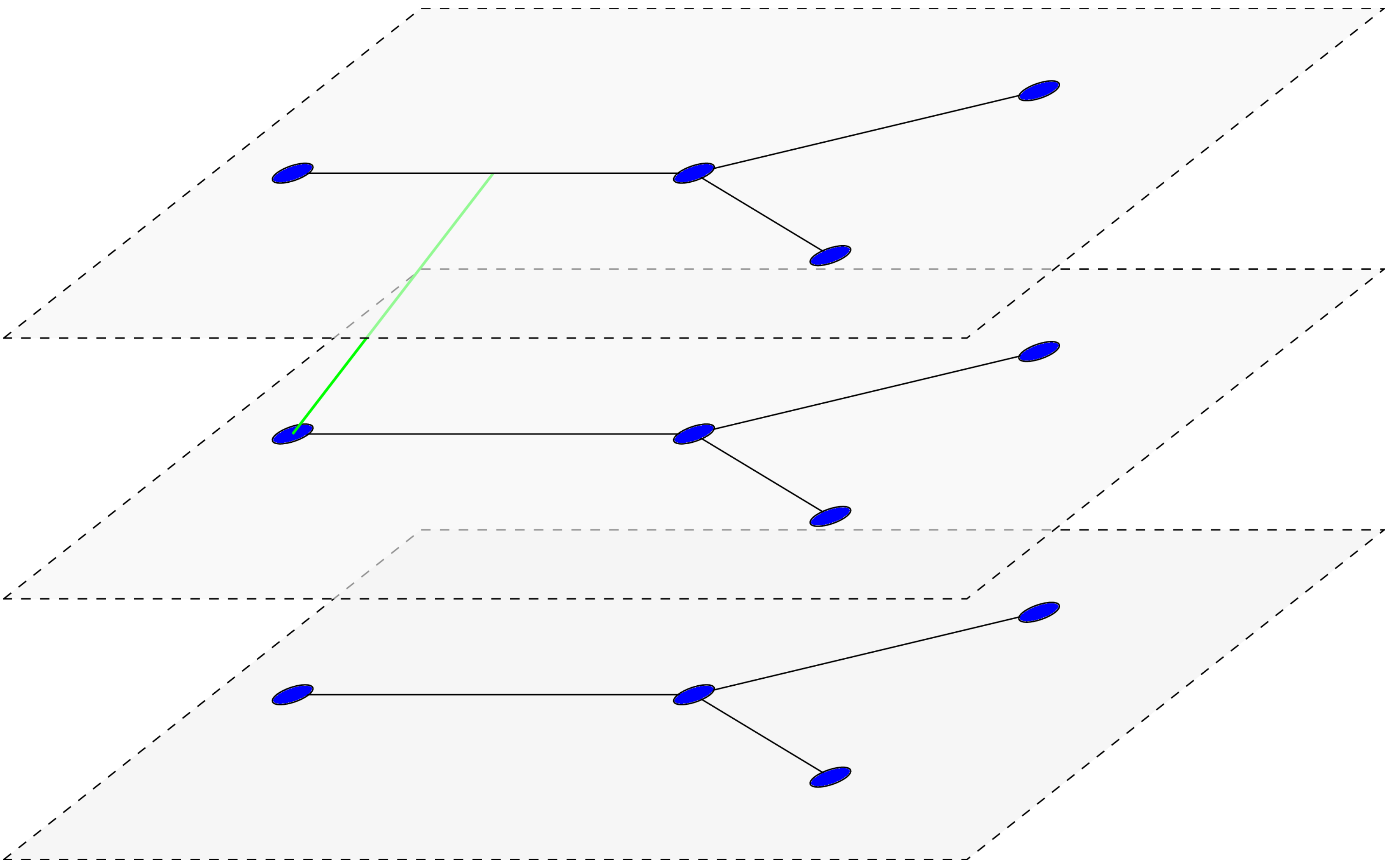


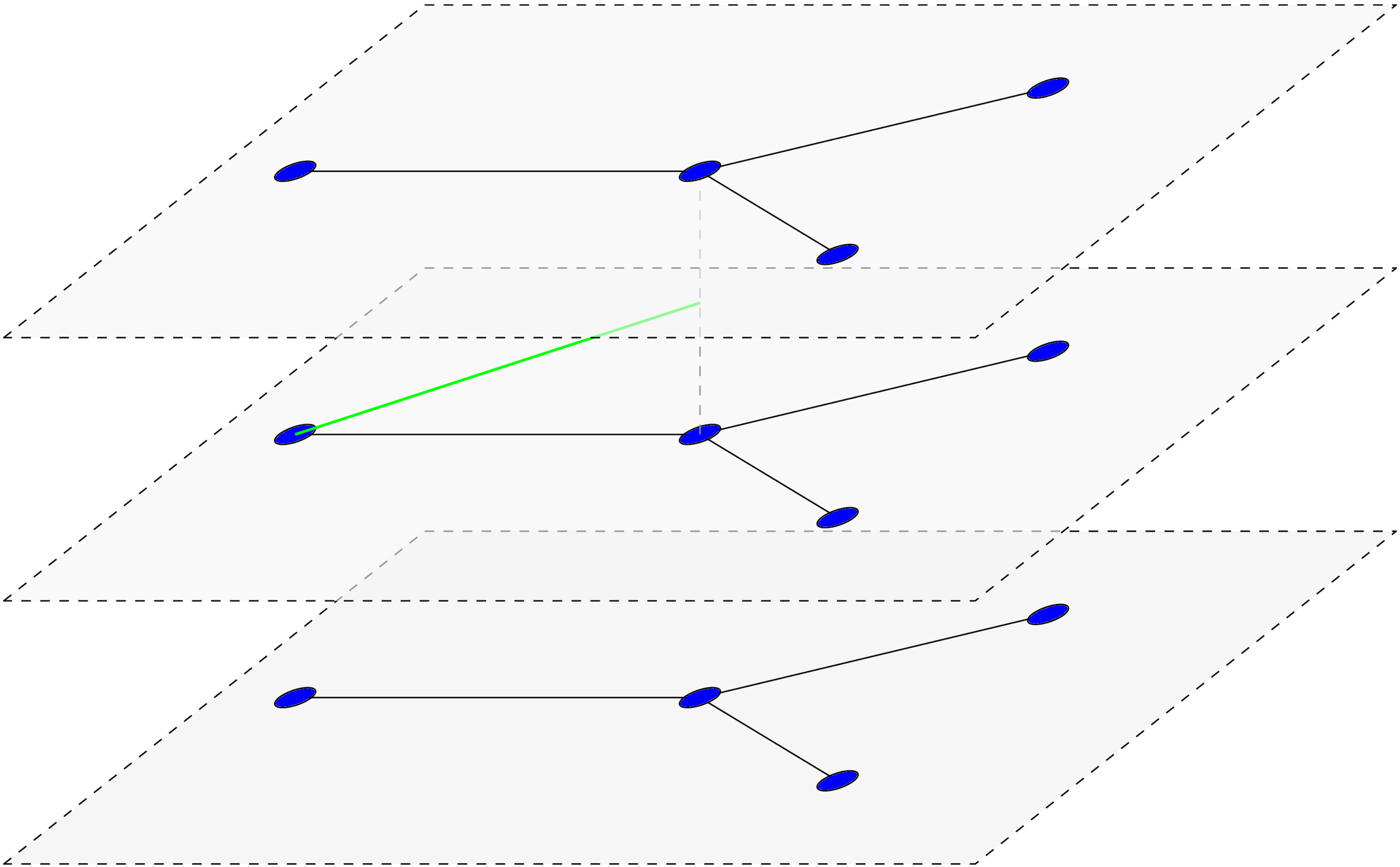
Climb/descent segments?



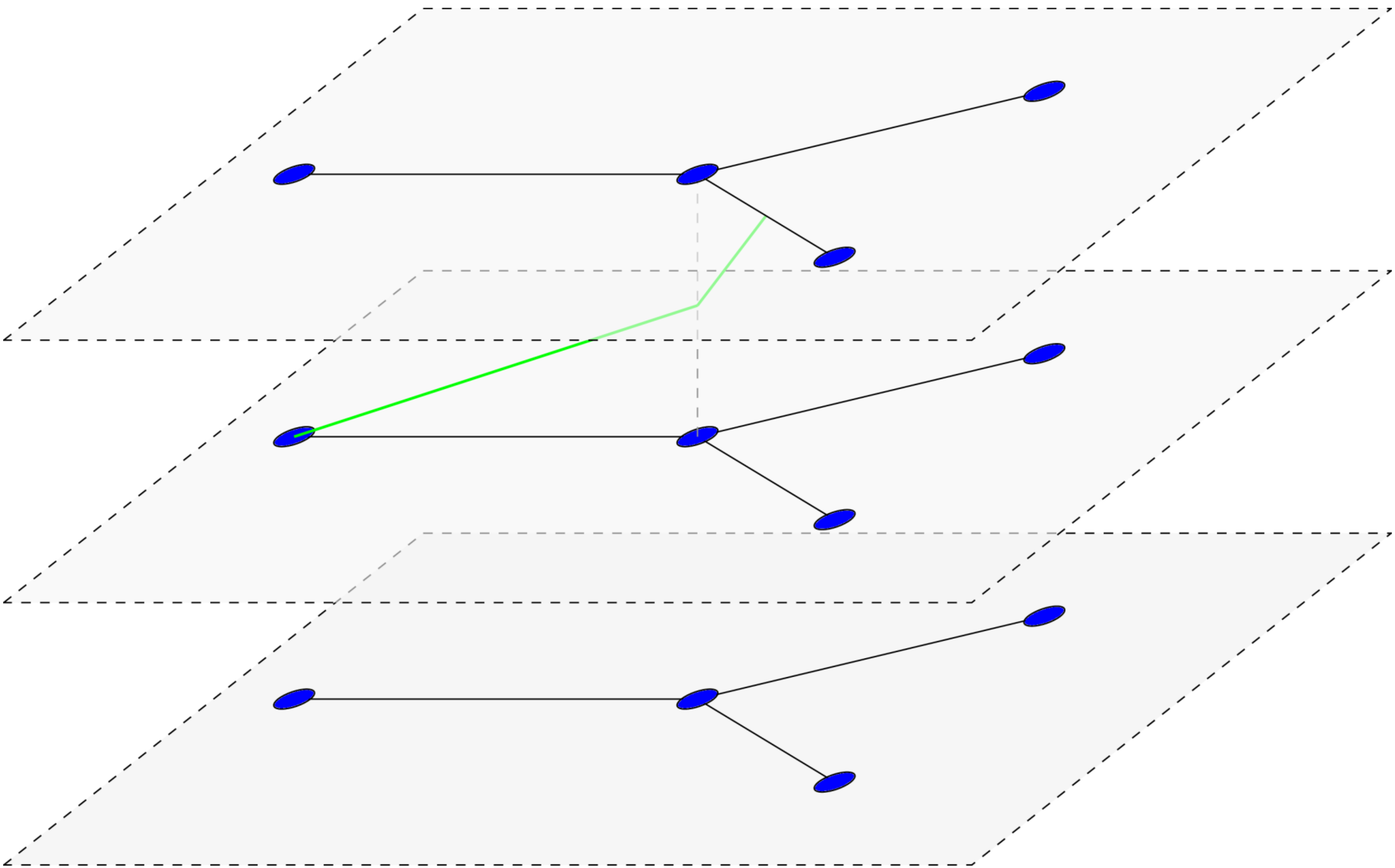
Vertical segments vary by weight, weather, speed, etc.



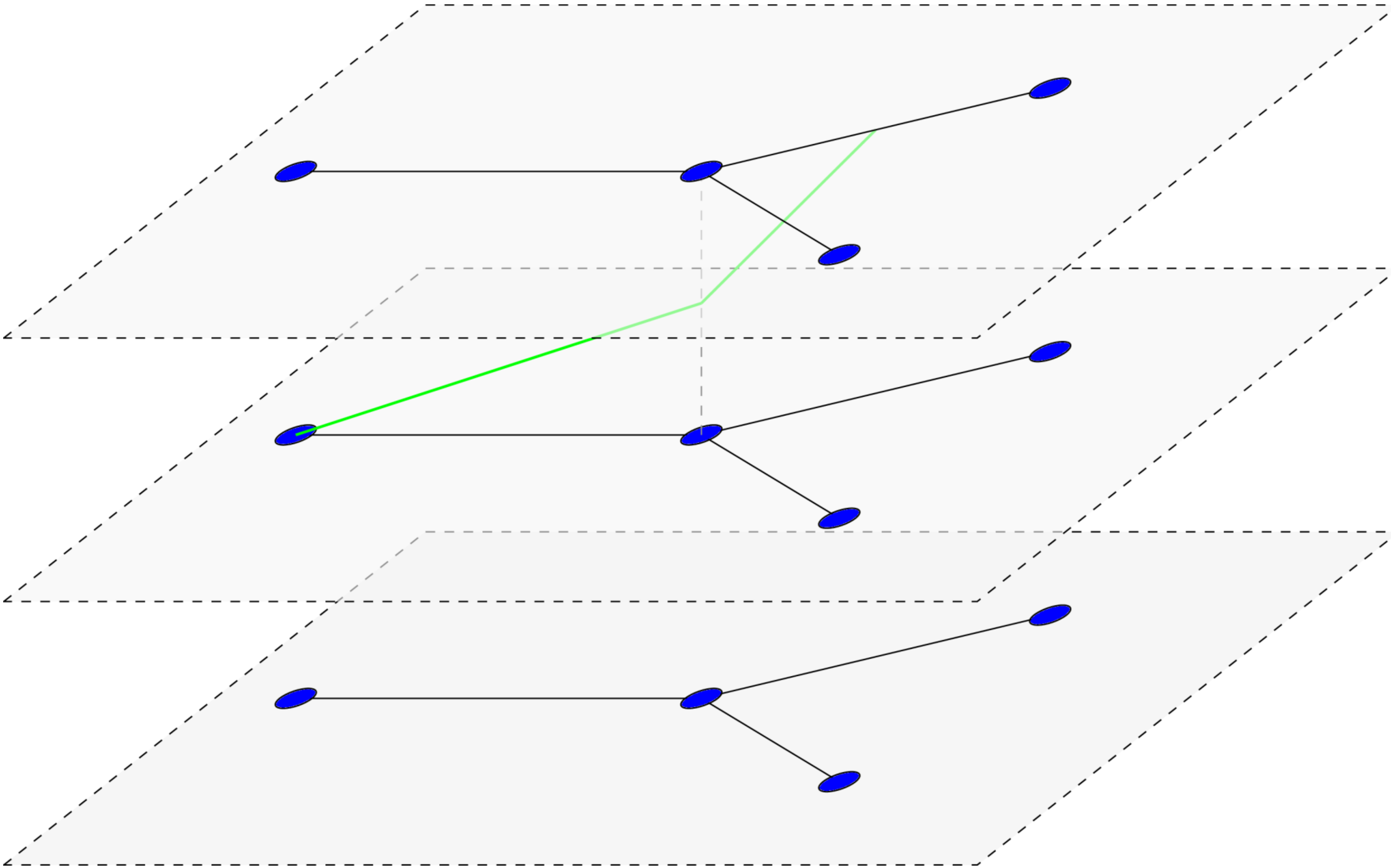
Vertical segments vary by weight, weather, speed, etc.



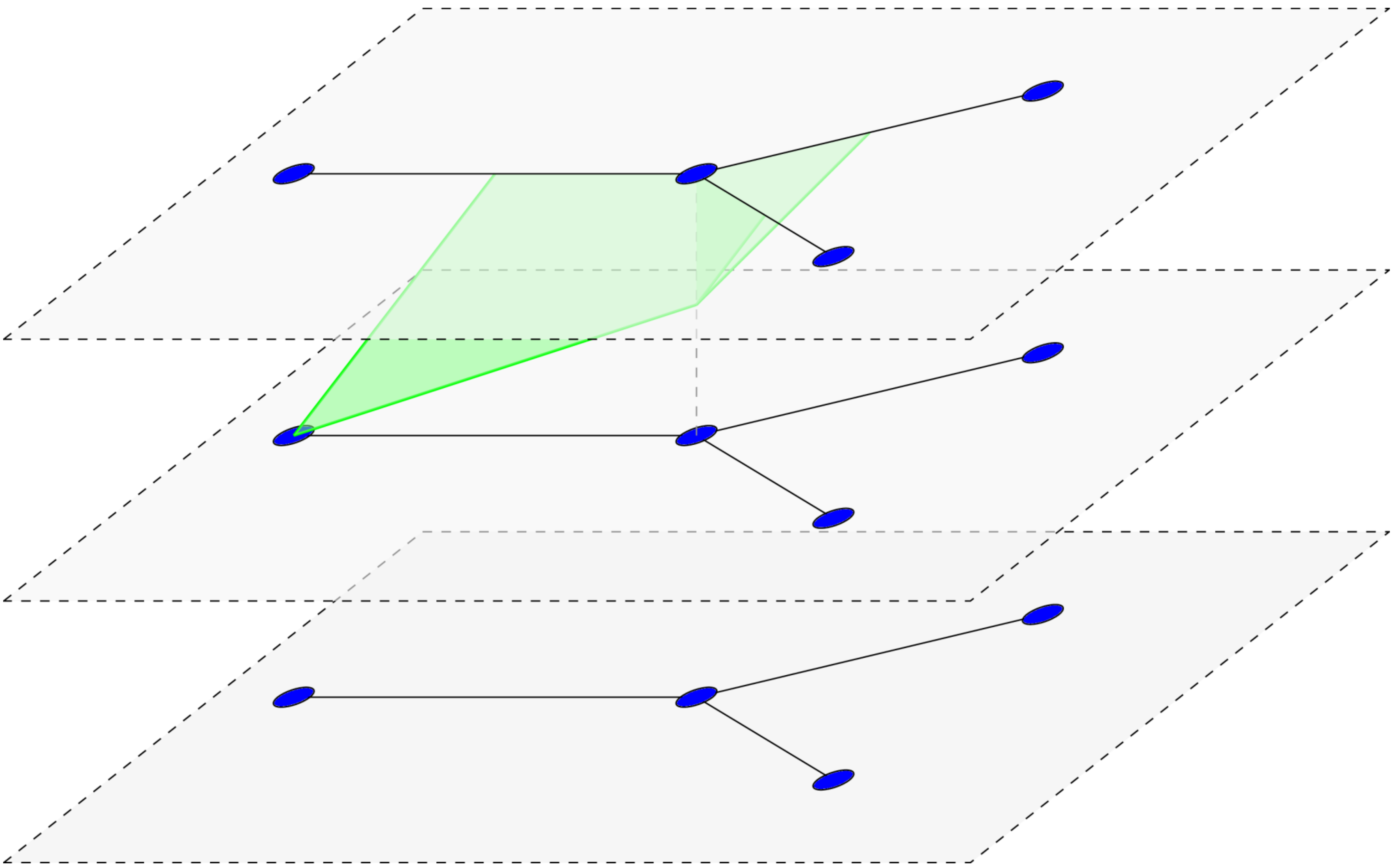
Vertical segments vary by weight, weather, speed, etc.



Vertical segments vary by weight, weather, speed, etc.



Vertical segments vary by weight, weather, speed, etc.



Cruise Performance

Constant speed, altitude, current weight, and specific range function

$$f(w_{curr}) = d$$

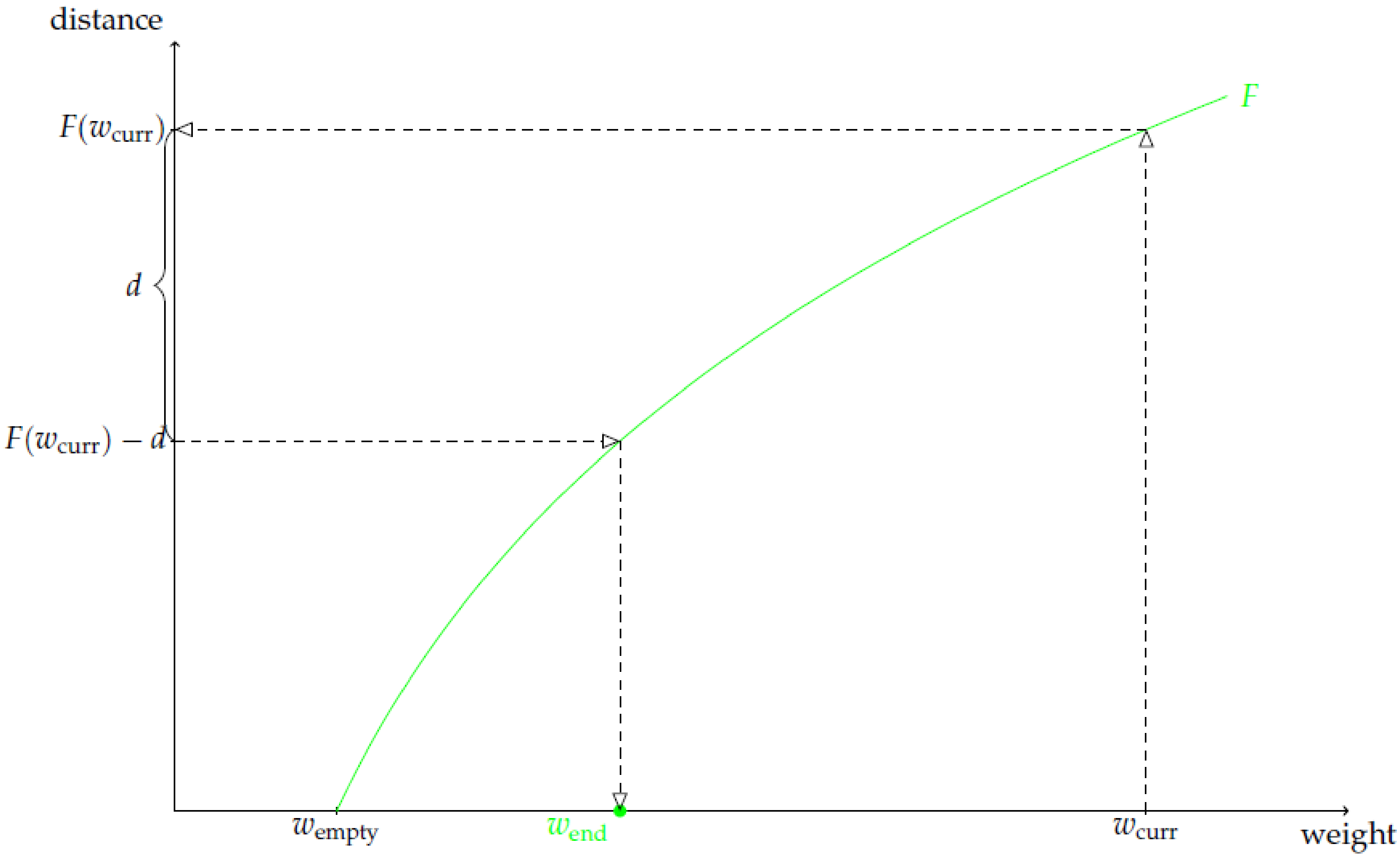
(how far can we fly with 1kg of fuel with weight w_{curr}). Then

$$d = \int_{w_{end}}^{w_{curr}} f(s)ds = F(w_{curr}) - F(w_{end}),$$

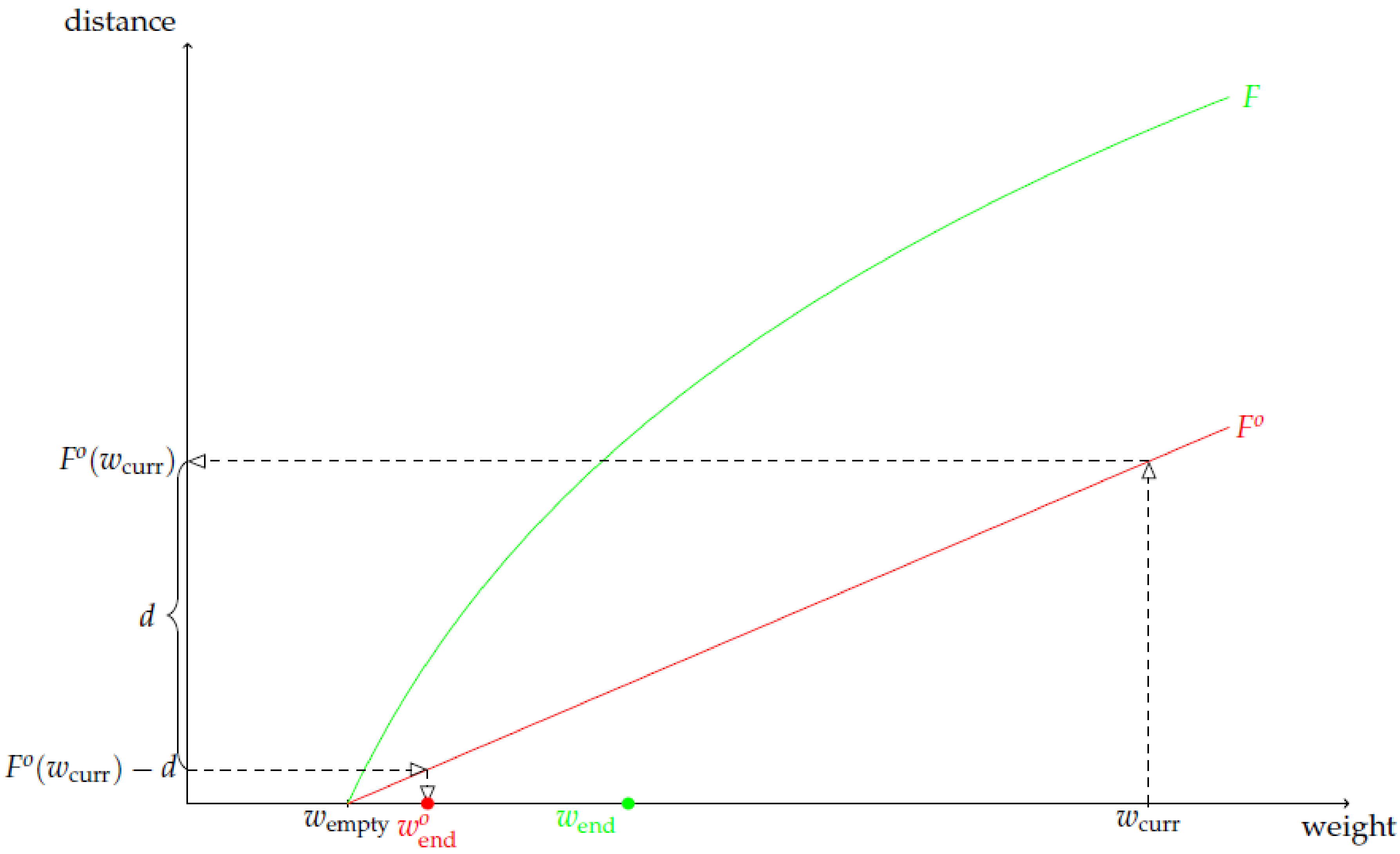
where F is the primitve integral of f, i.e.,

$$w_{end} = F^{-1}(F(w_{curr}) - d).$$

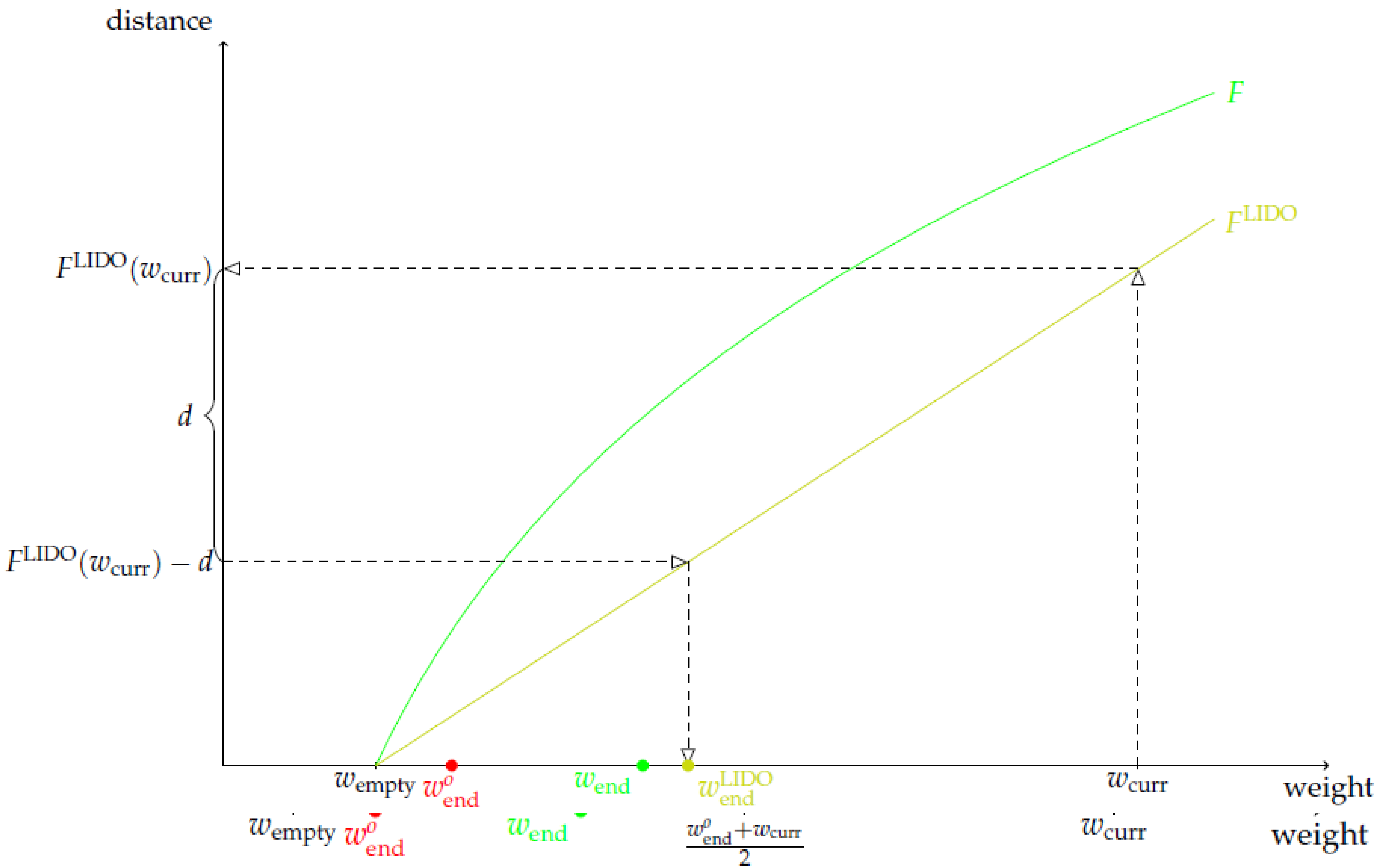
Cruise Performance



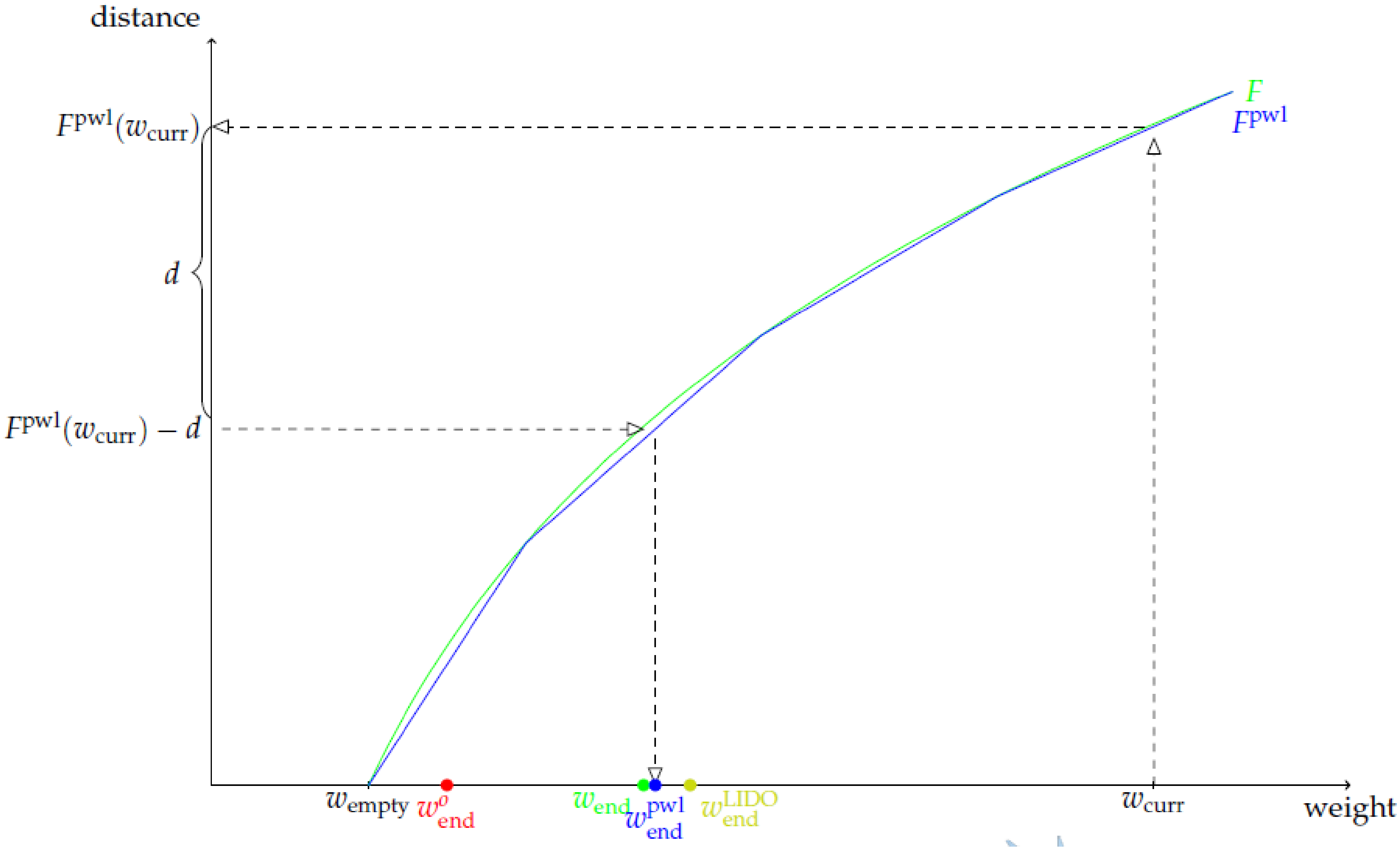
Overestimation



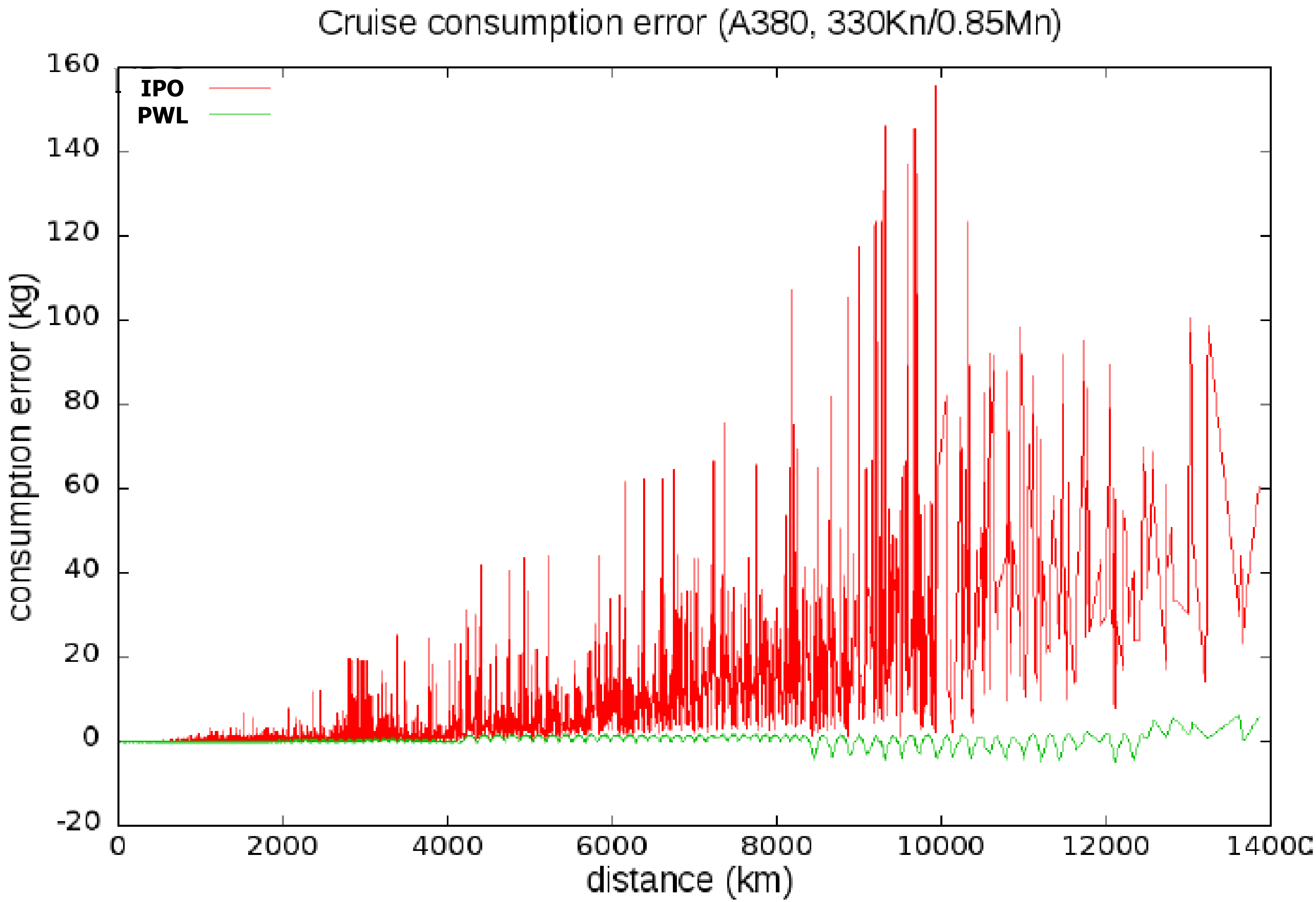
Interpolation



Piecewise Linear Approximation



Approximation Error



Approximation Error

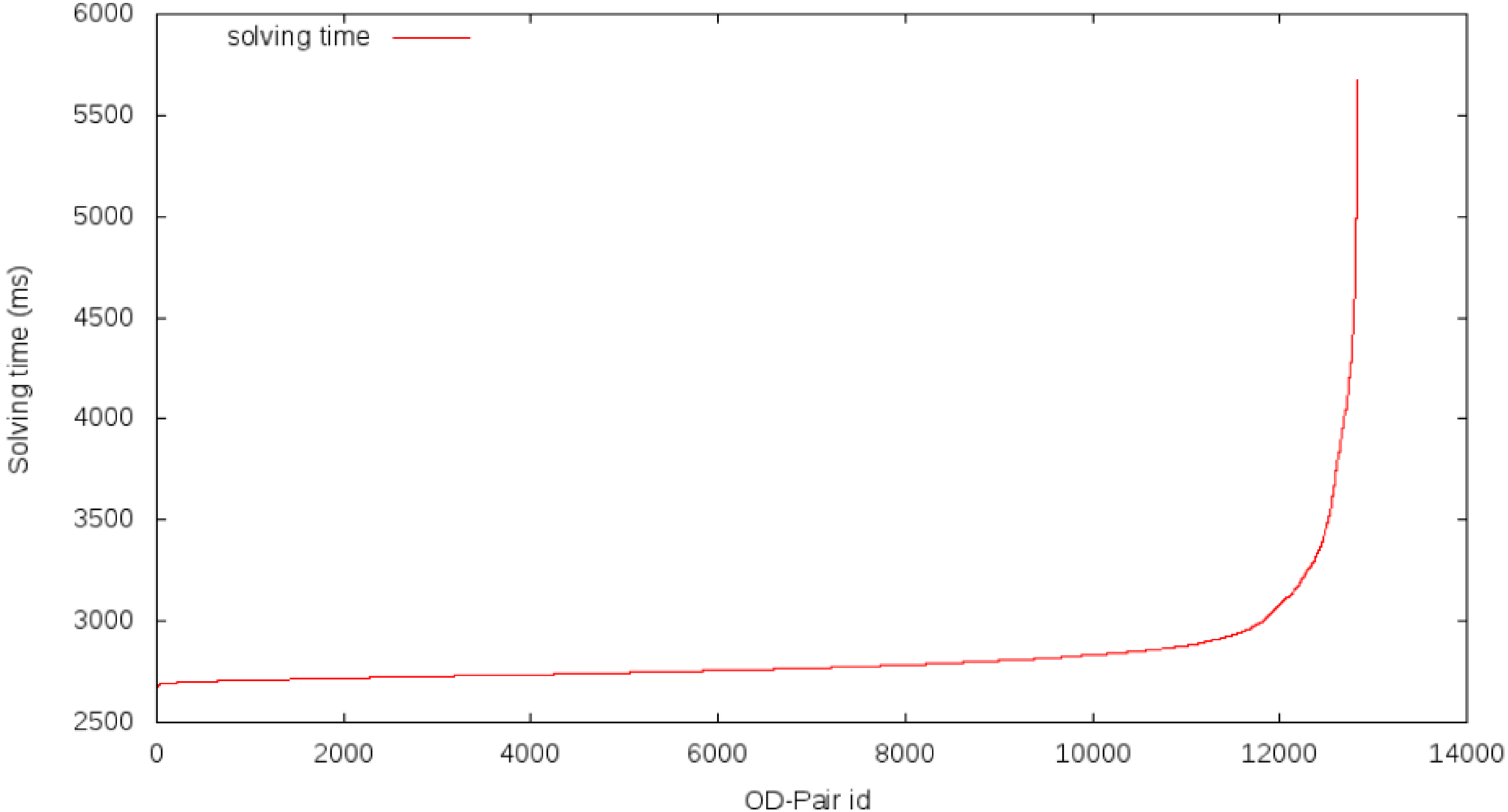
Proposition (Blanco, B, Hoang, Spiegel [2015])

Consider an aircraft cruising along several segments $e_0, \dots, e_k$ at a constant flight level at constant speed. Let w be the actual weight after the cruise phase, and w and $w^{\uparrow\downarrow}$ the values obtained by a piecewise liner underestimation of the primitive integral F of the specific range function f and a piecewise linear overestimation of its inverse F^{-1} using the same breakpoints with approximation errors $K^{\downarrow}$ and $K^{\uparrow}$, respectively. Then

$$|w - w^{\uparrow\downarrow}| \leq \max\left(K^{\downarrow} \cdot ||f^{-1}||_{\infty}, K^{\downarrow -1}\right),$$

independently of the number of segments.

Computational Results



A380, speed 0.83MN/300KIAS, constant altitude FL300, departure time 06.03.2014, 19:30:25

Thank you for your attention

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