Track Allocation and Path Configurations

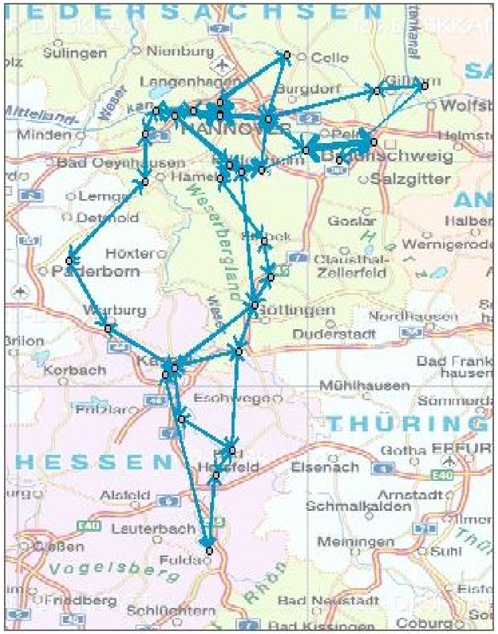
Ralf Borndörfer

2015 Workshop on
Combinatorial Optimization with Applications in Transportation and Logistics

Beijing, 29.07.2015

Track Allocation: Block Occupation

Track Allocation (Train Timetabling) Problem



- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006), Liebchen (2006)
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndoerfer, Schlechte (2005, 2007, 2009, 2010, 2011, 2012),** Caimi G., Fuchsberger M., Laumanns M., Schüpbach K. (2007)
- **Fischer, Helmberg, Janßen, Krostitz (2008, 2011)**
- Lusby, Larsen, Ehrgott, Ryan (2009)
- Caimi (2009), Klabes (2010)
- ...

Track Allocation (= Routes in Space-Time)

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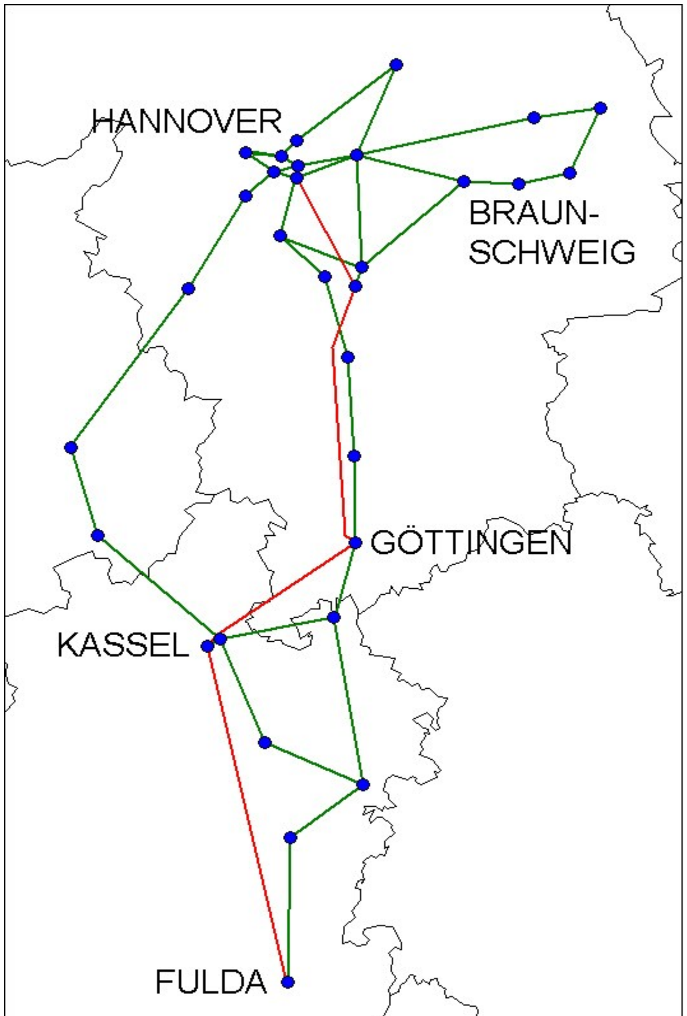
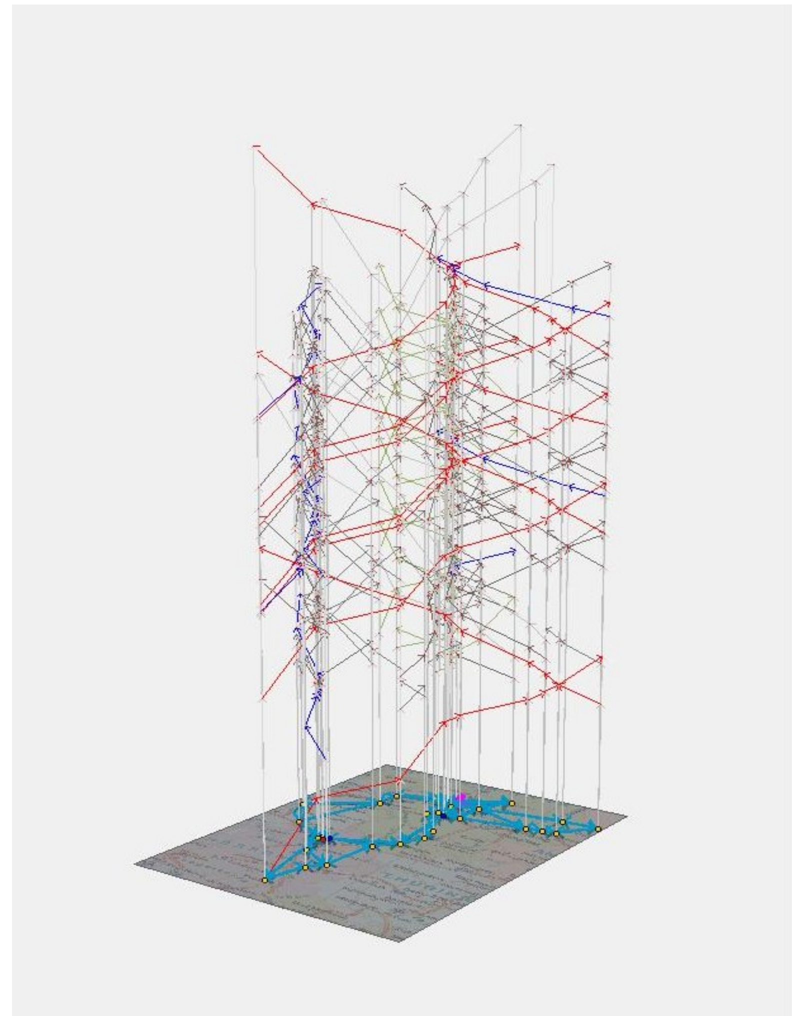
Railway Network + Trains



Train Routes + Timetable



Integrated Train Routing and Scheduling

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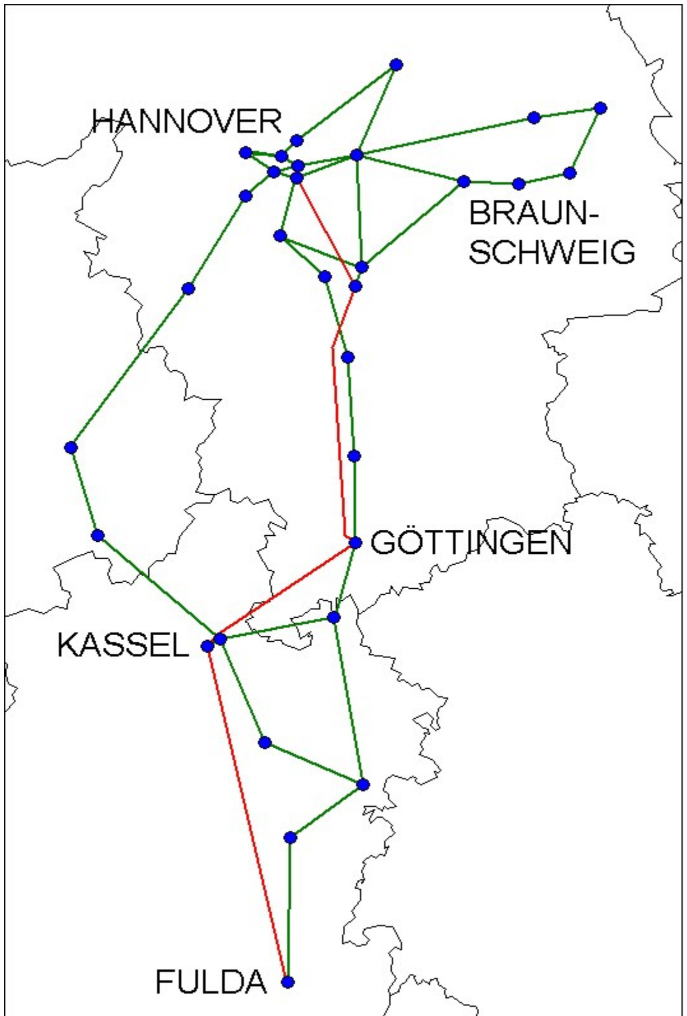
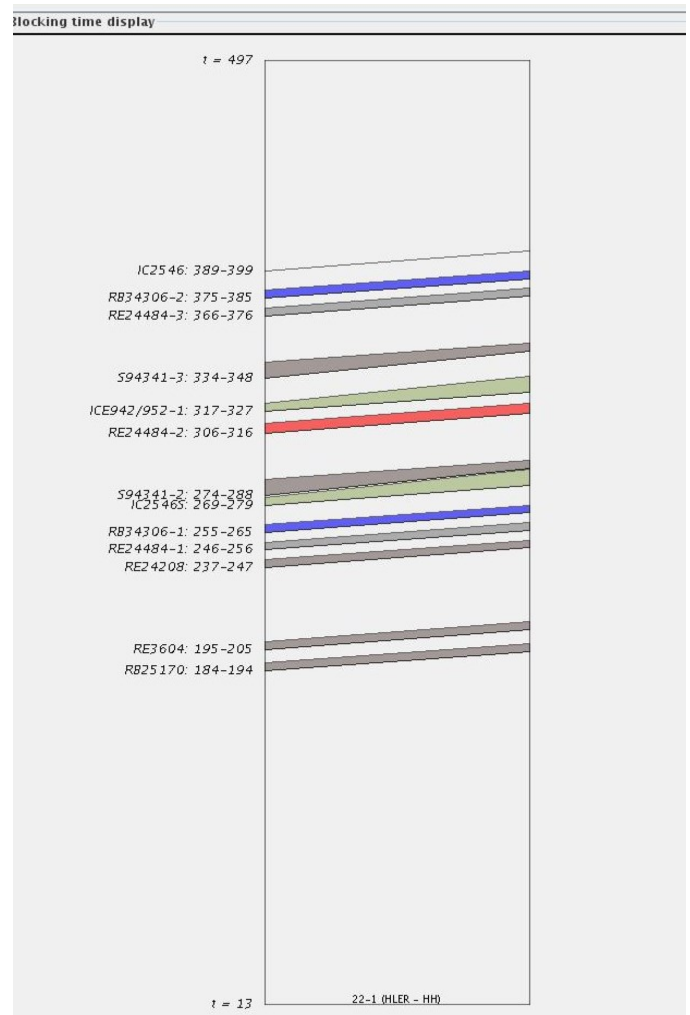
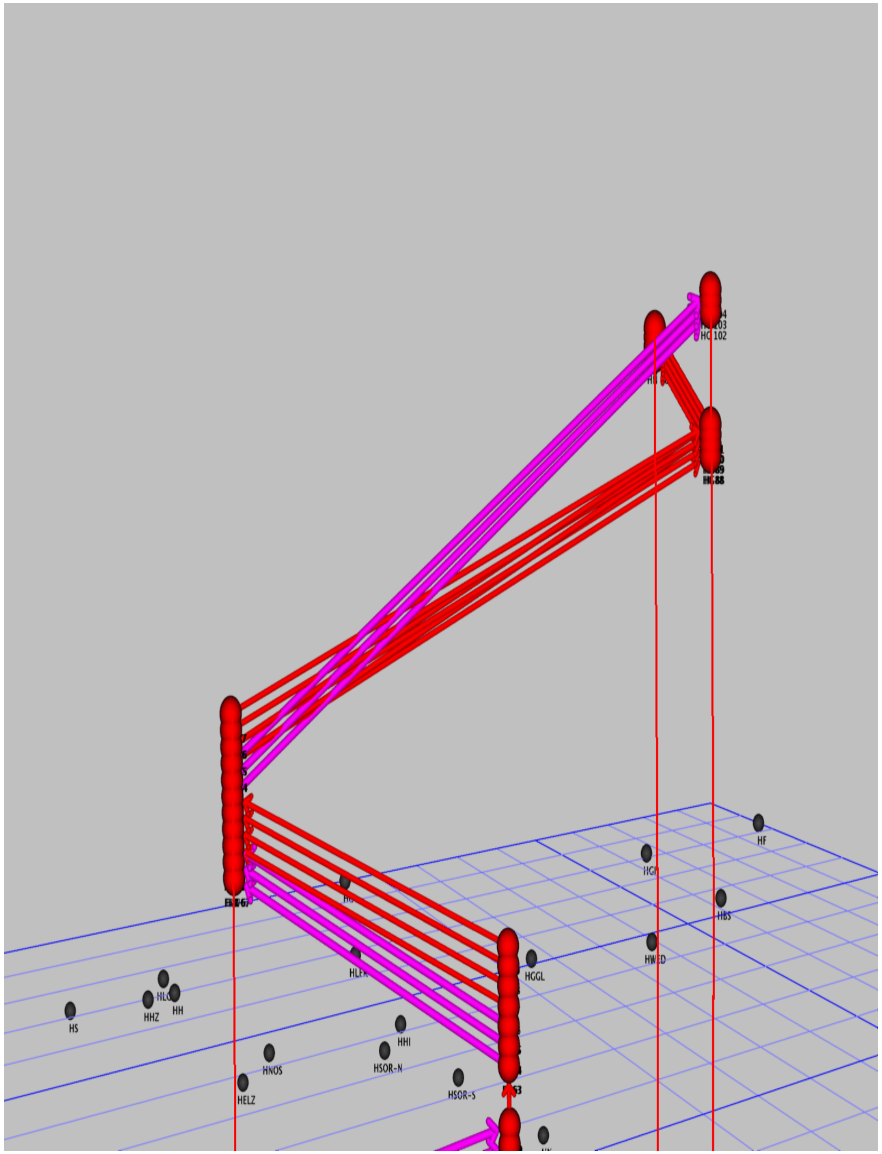
Routing: Flow

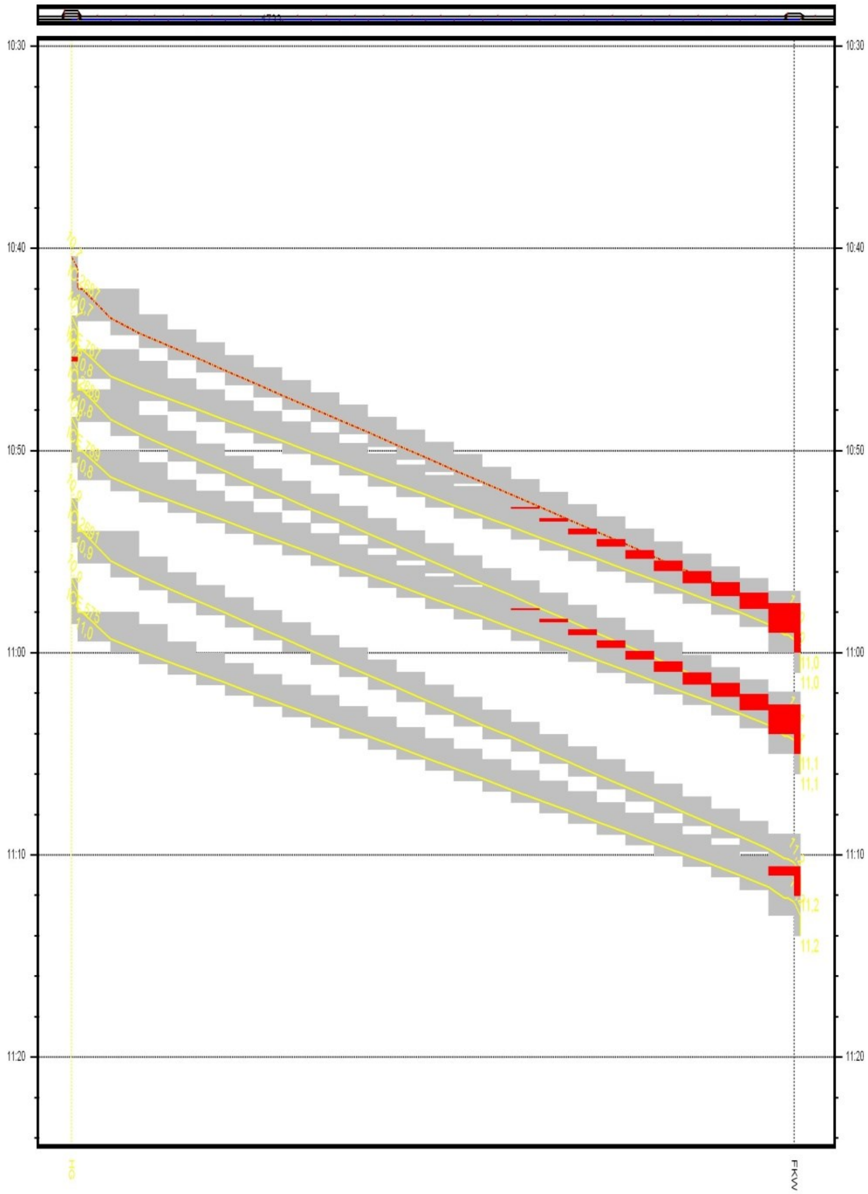


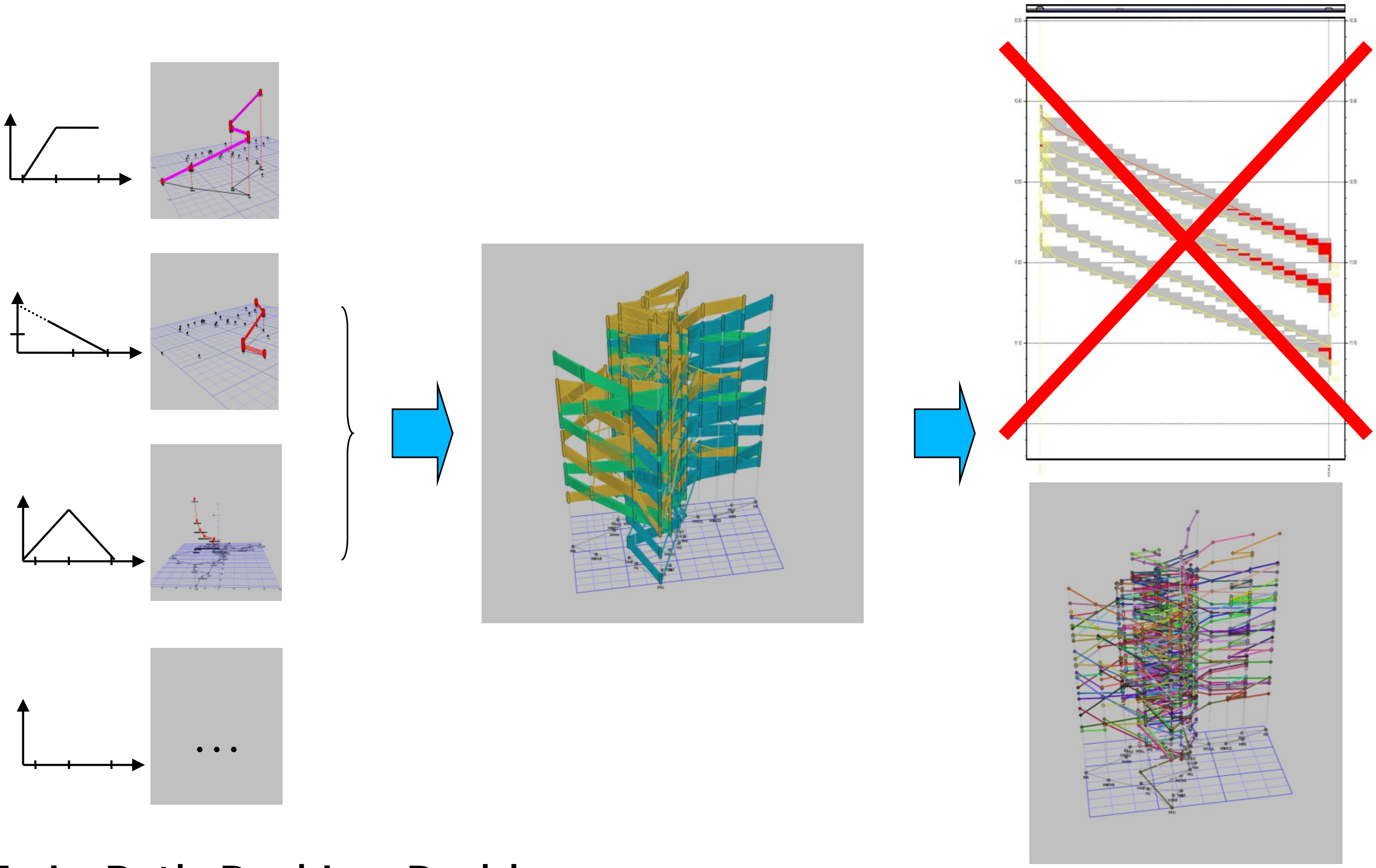
Scheduling: Headway constraints



Block Occupation Conflict

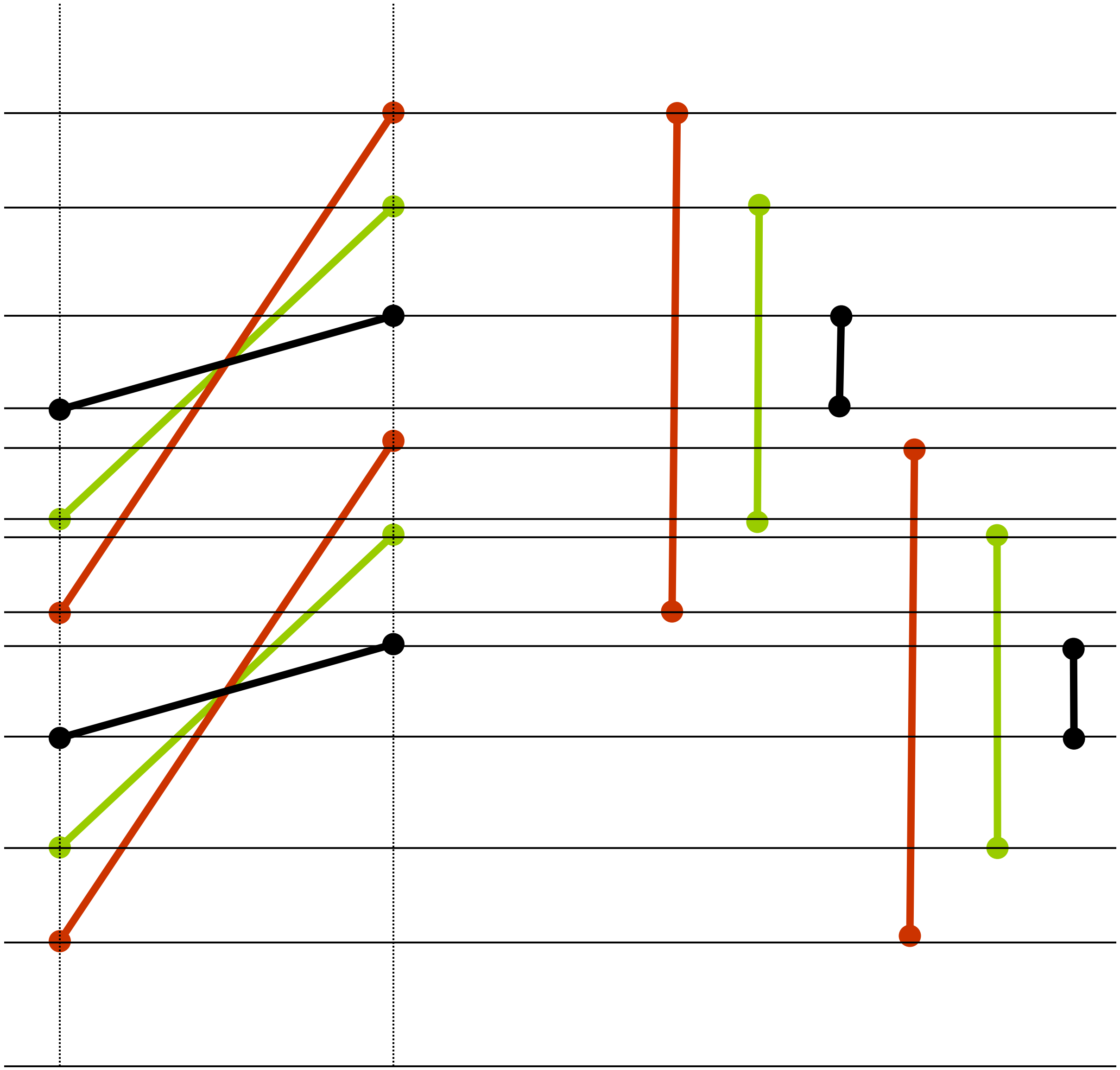




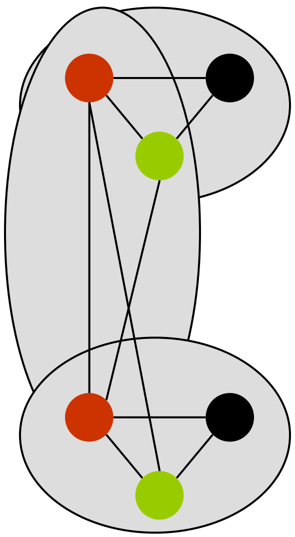


- Train Path Packing Problem

Multicommodity Flow + Arc Packing Model



- Block occupation conflict graph (here: interval graph, perfect)
- Conflict cliques
- Local for each block
- Packing constraints = local constraints



Arc Packing Model

(APP)	$\max \sum_{i\in I}\sum_{a\in A} c_a^i x_a^i$				
(i)	$\sum_{a\in\delta_i^+(v)} x_a^i - \sum_{a\in\delta_i^-(v)} x_a^i$	$=$	$\beta_i(v)$	$\forall v\in V, i\in I$	Trains
(ii)	$\sum_{(a,i)\in k} x_a^i$	$\leq$	1	$\forall k\in K$	Conflicts
(iii)	x_a^i	$\in$	$\{0,1\}$	$\forall a\in A, i\in I$	Integ.

Theorem (Lukac [2004]):

Let H be a quadrangle-linear headway matrix for train types Y, i.e.,

$$H(i,j)+H(j,k)\geq H(i,k)+H(j,j)\quad \forall i,j,k.$$

Then all maximal cliques in G_H are of the form

$$\bigcup_{i\in Y}\{i\}\times[t_i, t_i+H(i,i)-1],$$

if after setting $t_{i_0}=0$ for some $i_0\in Y$ the following holds for any different i,j:

$$H(i,i)-H(j,i)\leq t_j-t_i\leq H(i,j)-H(j,j).$$

Arc Packing Model

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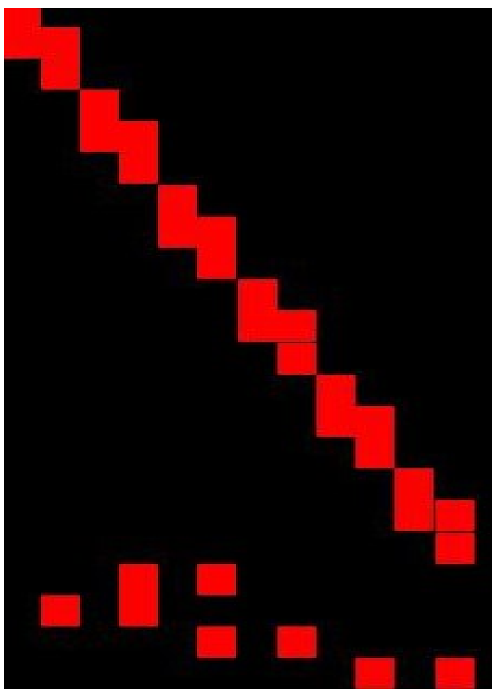
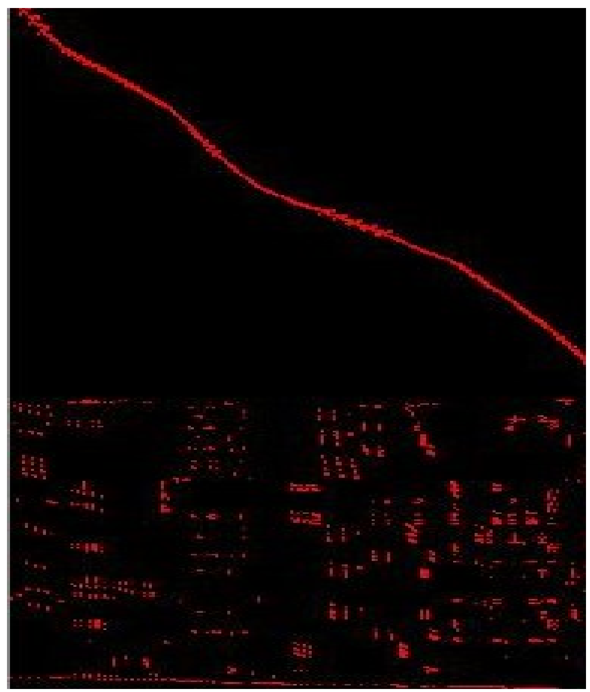
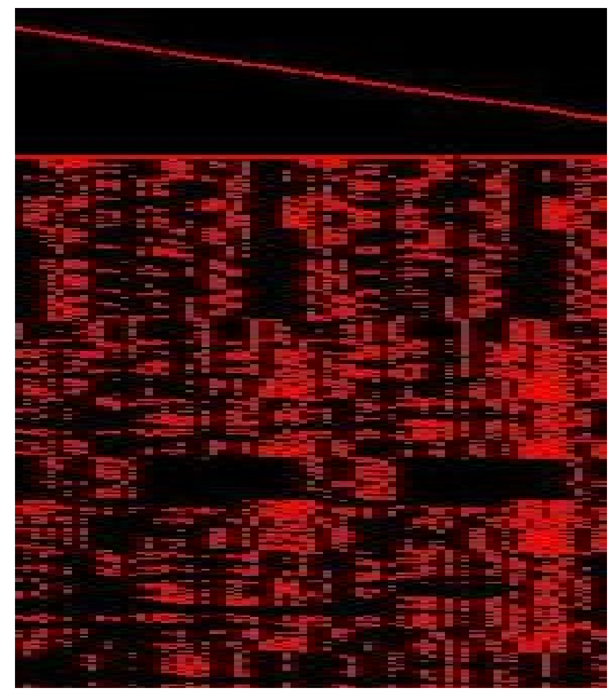
$$
\begin{array}{llcccl}
\text{(APP)} & \max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i & & & & \\
\text{(i)} & \sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i & = & \beta_i(v) & \forall v \in V, i \in I & \text{Trains} \\
\text{(ii)} & \sum_{(a,i) \in k} x_a^i & \leq & 1 & \forall k \in K & \text{Conflicts} \\
\text{(iii)} & x_a^i & \in & \{0,1\} & \forall a \in A, i \in I & \text{Integ.}
\end{array}
$$

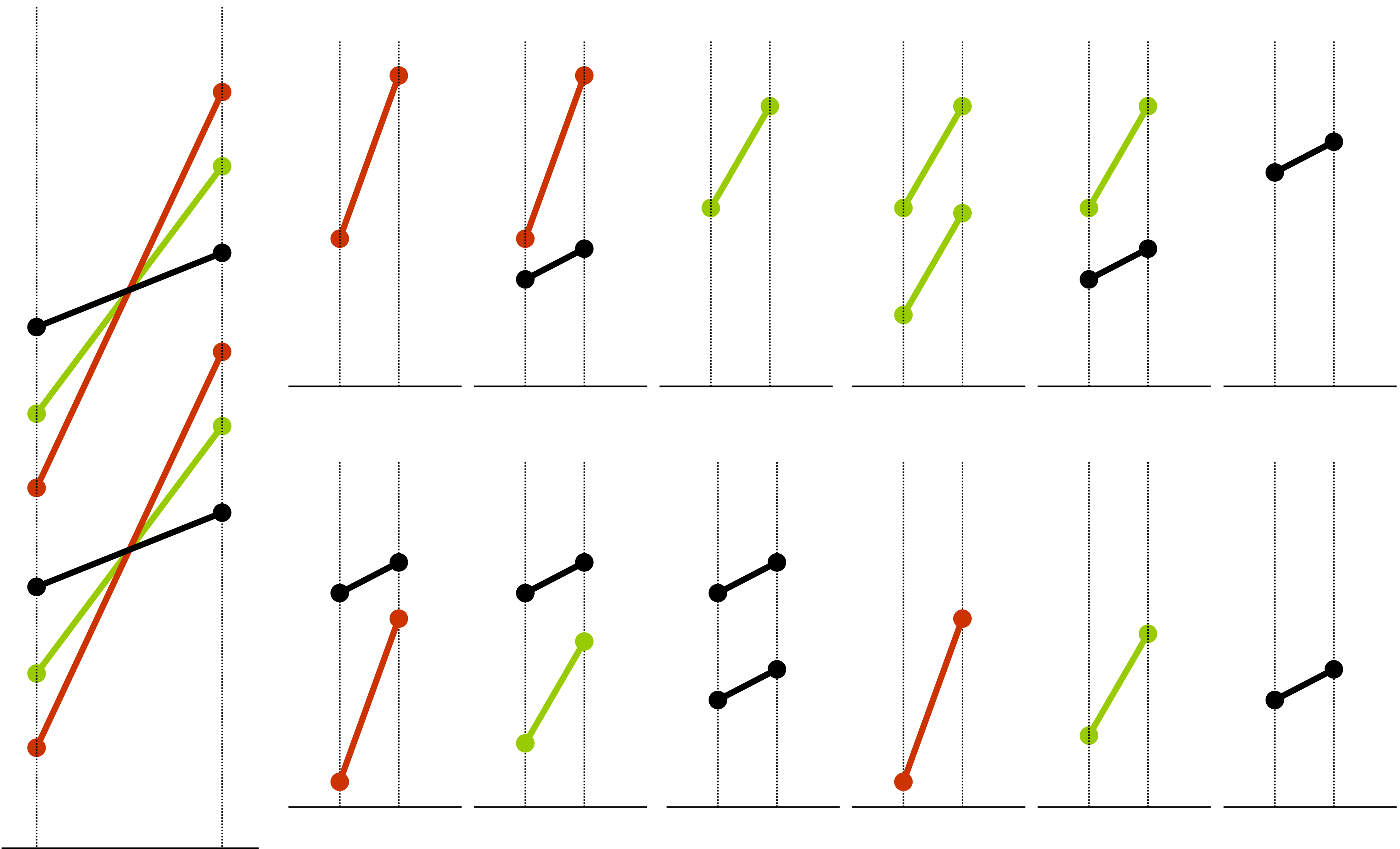






Block Occupation Configurations

(B., Schlechte [2007])



Arc Packing and Arc Configuration Model

$$
\begin{array}{llllll}
\text{(APP)} & \max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i & & & & \\
\text{(i)} & \sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i & = & \beta_i(v) & \forall v \in V, i \in I & \text{Trains} \\
\text{(ii)} & \sum_{(a,i) \in k} x_a^i & \leq & 1 & \forall k \in K & \text{Conflicts} \\
\text{(iii)} & x_a^i & \in & \{0,1\} & \forall a \in A, i \in I & \text{Integ.}
\end{array}
$$

$$
\begin{array}{llllll}
\text{(ACP)} & \max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i & & & & \\
\text{(i)} & \sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i & \leq & \beta_i(v) & \forall v \in V, i \in I & \text{Trains} \\
\text{(ii)} & \sum_{q \in Q_j} y_q & = & 1 & \forall j \in J & \text{Configs} \\
\text{(iii)} & \sum_{a \in q \in Q} y_q & = & x_a^i & \forall a \in A, i \in I & \text{Coupling} \\
\text{(iv)} & x_a^i & \in & \{0,1\} & \forall a \in A, i \in I & \text{Integ.} \\
\text{(v)} & y_q & \in & \{0,1\} & \forall q \in Q & \text{Integ.}
\end{array}
$$

Arc Packing and Configuration Model

(APP)	$\max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i$				
(i)	$\sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i$	$=$	$\beta_i(v)$	$\forall v \in V, i \in I$	Trains
(ii)	$\sum_{(a,i) \in k} x_a^i$	$\leq$	1	$\forall k \in K$	Conflicts
(iii)	x_a^i	$\in$	$\{0,1\}$	$\forall a \in A, i \in I$	Integ.

(ACP)	$\max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i$				
(i)	$\sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i$	$\leq$	$\beta_i(v)$	$\forall v \in V, i \in I$	Trains
(ii)	$\sum_{q \in Q_j} y_q$	$=$	1	$\forall j \in J$	Configs
(iii)	$\sum_{a \in q \in Q} y_q$	$\geq$	$\sum_{i \in I} x_a^i$	$\forall a \in A$	Coupling
(iv)	x_a^i	$\in$	$\{0,1\}$	$\forall a \in A, i \in I$	Integ.
(v)	y_q	$\in$	$\{0,1\}$	$\forall q \in Q$	Integ.

Arc Packing and Path Configuration Model

(APP)	$\max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i$				
(i)	$\sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i$	$=$	$\beta_i(v)$	$\forall v \in V, i \in I$	Trains
(ii)	$\sum_{(a,i) \in k} x_a^i$	$\leq$	1	$\forall k \in K$	Conflicts
(iii)	x_a^i	$\in$	$\{0,1\}$	$\forall a \in A, i \in I$	Integ.

(PCP)	$\max \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c_a^i x_p$				
(i)	$\sum_{p \in P_i} x_p$	$\leq$	1	$\forall i \in I$	Trains
(ii)	$\sum_{q \in Q_j} y_q$	$\leq$	1	$\forall j \in J$	Configs
(iii)	$\sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q$	$\leq$	0	$\forall a \in A$	Coupling
(iv)	x_p	$\in$	$\{0,1\}$	$\forall p \in P$	Nonneg.
(v)	y_q	$\in$	$\{0,1\}$	$\forall q \in Q$	Nonneg.

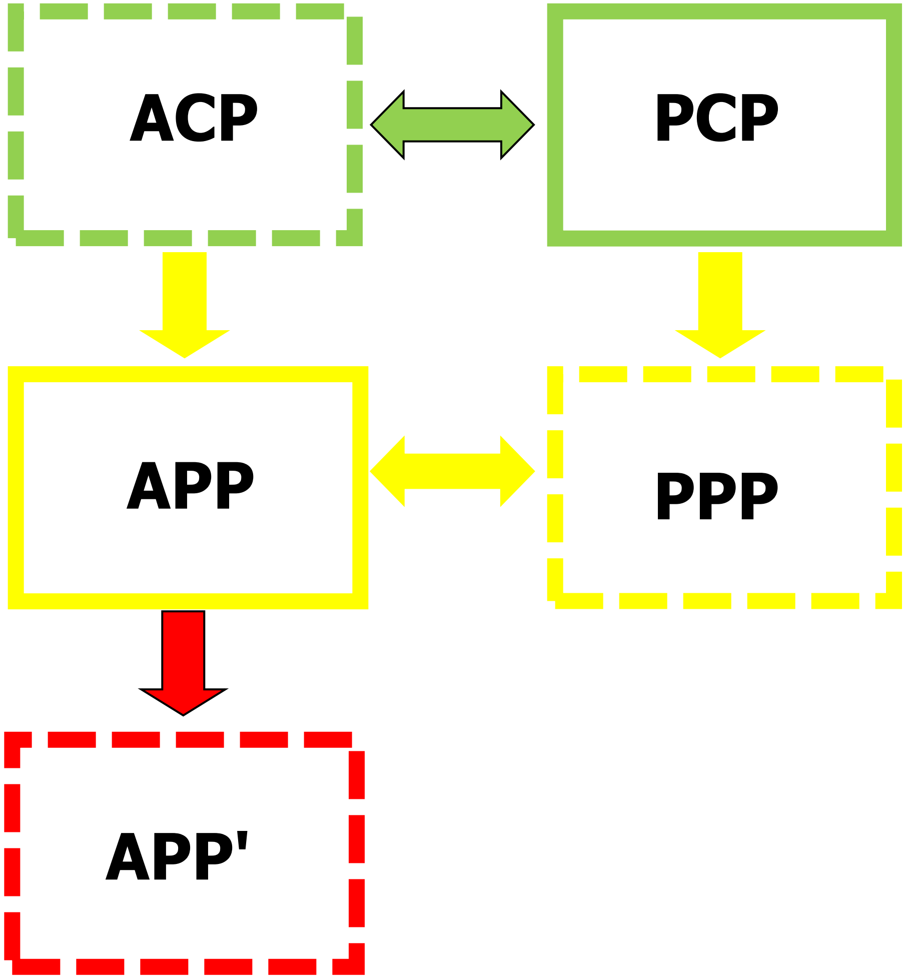
Model Comparison

Theorem (B., Schlechte [2007]):

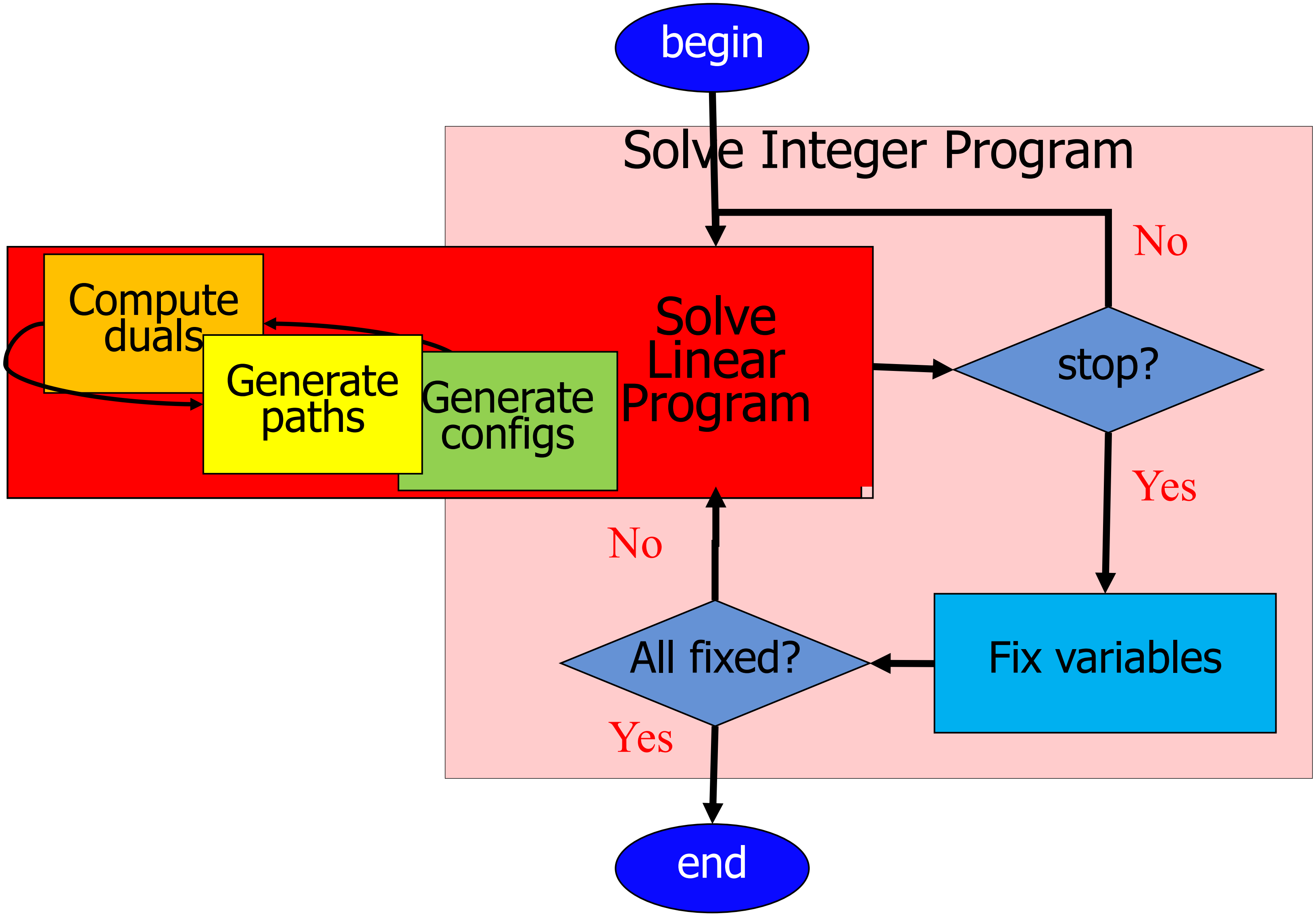
$$v_{IP}(PCP) = v_{IP}(ACP)$$
$$= v_{IP}(APP) = v_{IP}(PPP)$$
$$= v_{IP}(APP')$$

and

$$v_{LP}(PCP) = v_{LP}(ACP)$$
$$\leq v_{LP}(APP) = v_{LP}(PPP)$$
$$\leq v_{LP}(APP').$$

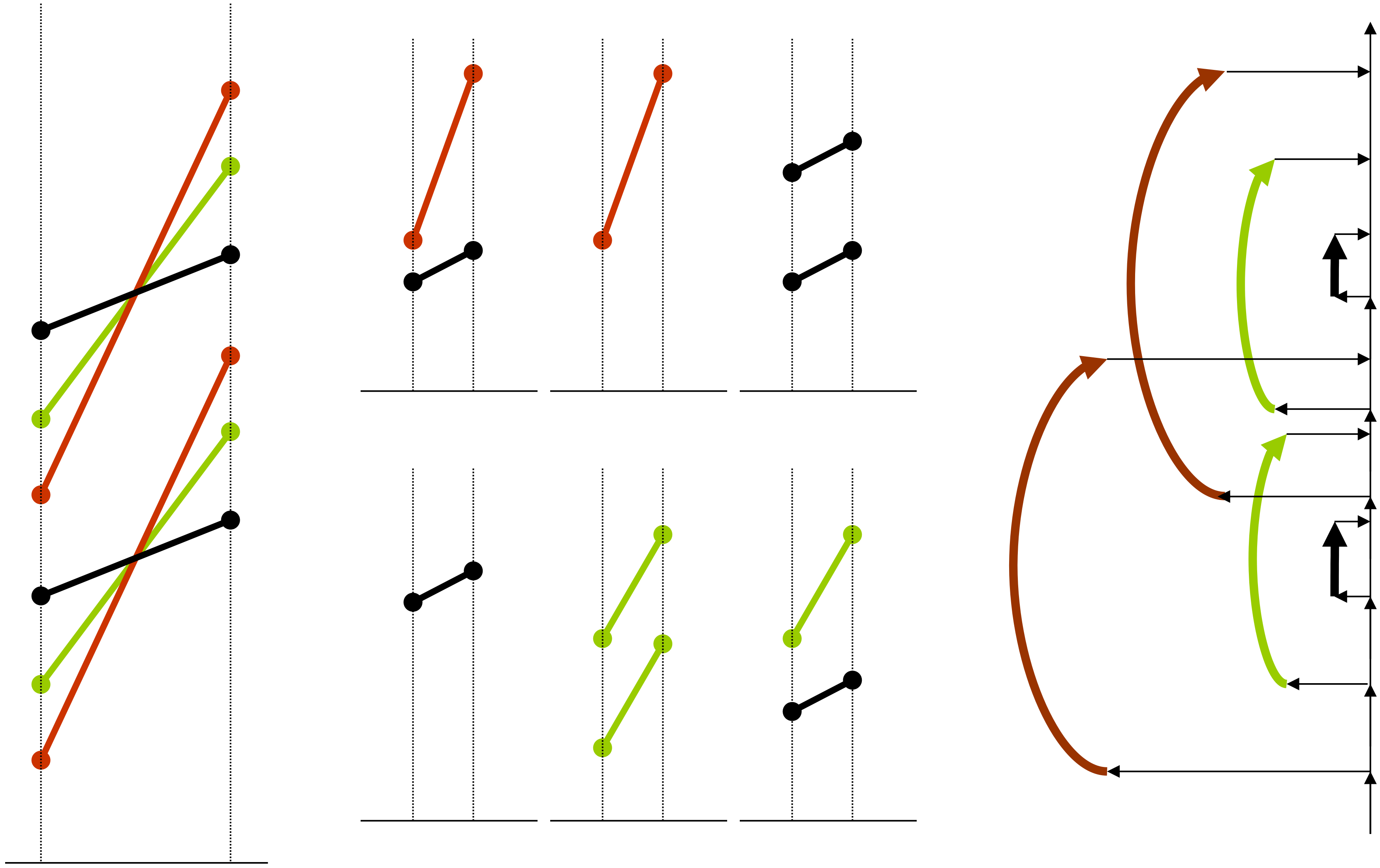


Column Generation Algorithm

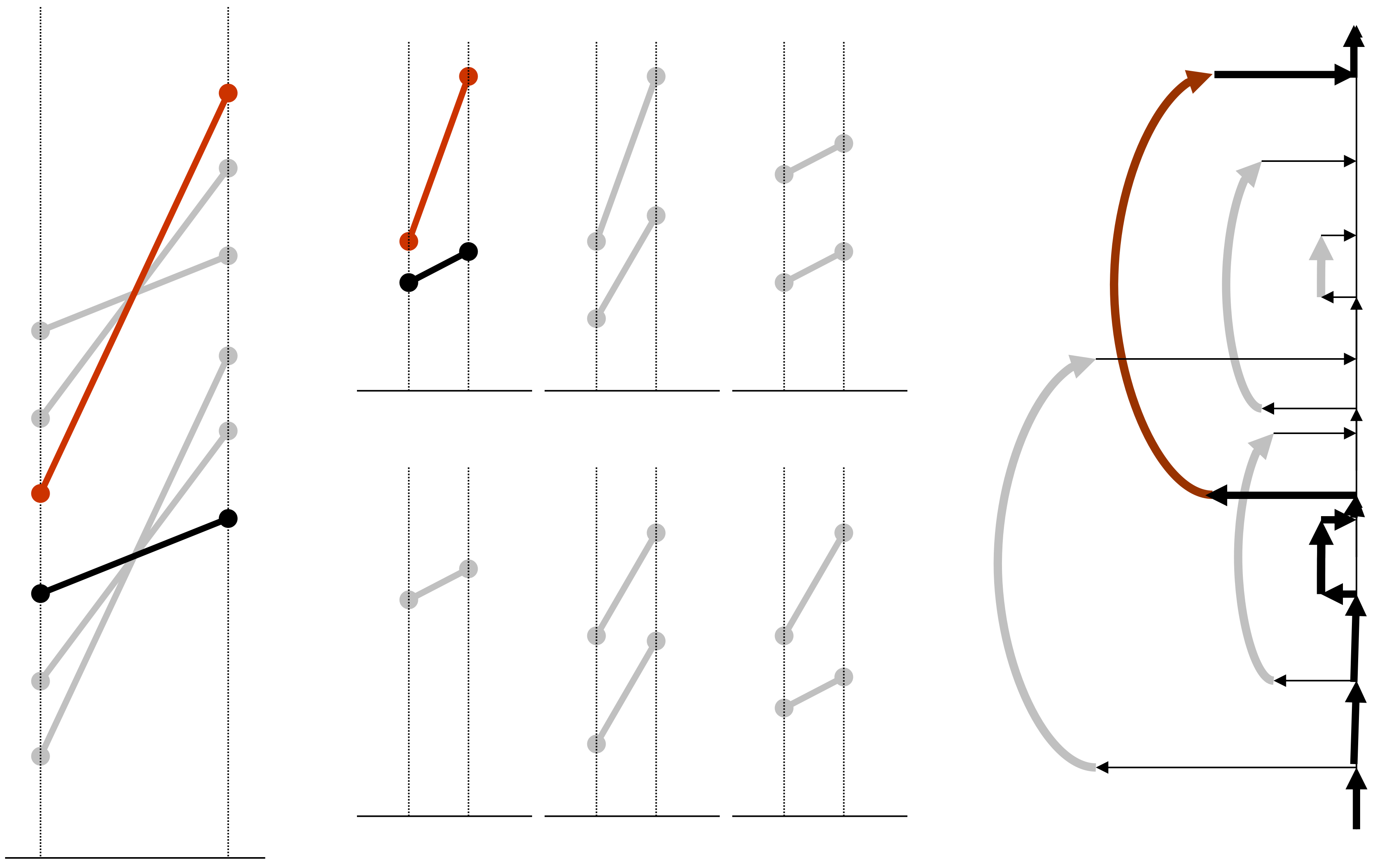


Configuration Pricing = Shortest Path Problem

(B., Schlechte [2007])



Configuration Pricing = Shortest Path Problem



Configuration LP

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$$\text{(PLP)} \max \sum_{i\in I} \sum_{p\in P_i} \sum_{a\in p} c_a^i x_p$$

$$\begin{array}{llllll}
\text{(i)} & \sum_{p\in P_i} x_p & \le 1 & \forall i \in I & \text{Trains} & (\gamma_i) \\
\text{(ii)} & \sum_{q\in Q_j} y_q & \le 1 & \forall j \in J & \text{Configs} & (\pi_j) \\
\text{(iii)} & \sum_{a\in p\in P} x_p - \sum_{a\in q\in Q} y_q & \le 0 & \forall a \in A & \text{Coupling} & (\lambda_a) \\
\text{(iv)} & x_p & \ge 0 & \forall p \in P & \text{Nonneg.} & \\
\text{(v)} & y_q & \ge 0 & \forall q \in Q & \text{Nonneg.} &
\end{array}$$

$$\text{(DUA)} \min \sum_{i\in I} \gamma_i + \sum_{j\in J} \pi_j$$

$$\begin{array}{lllll}
\text{(i)} & \gamma_i + \sum_{a\in p} \lambda_a & \ge \sum_{a\in p} c_a^i & \forall p \in P_i, i \in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a\in q} \lambda_a & \ge 0 & \forall q \in Q_j, j \in J & \text{Configs} \\
\text{(iii)} & \gamma, \pi, \lambda & \ge 0 & &
\end{array}$$

Configuration Pricing = Shortest Path Problem

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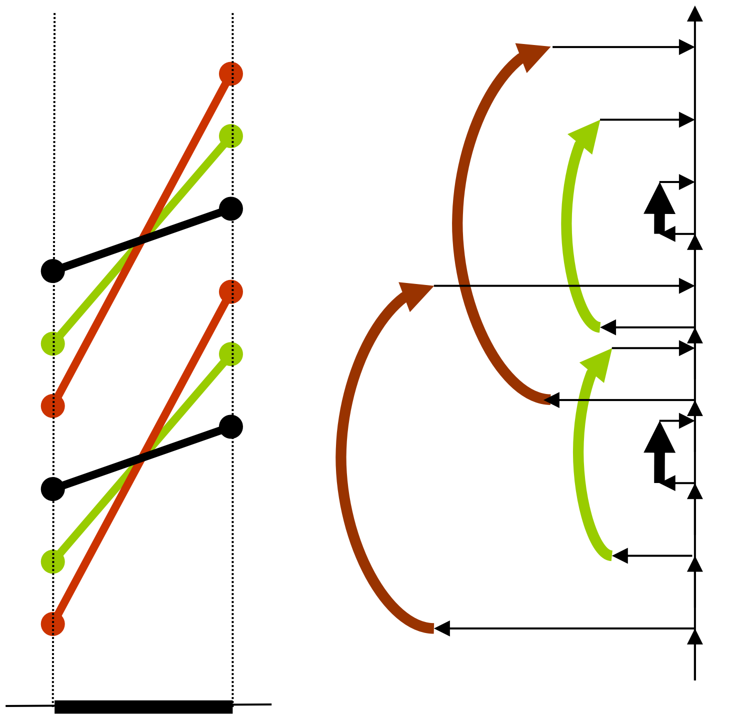
$$
\begin{array}{llcll}
\text{(DUA)} & \min \sum_{i \in I} \gamma_i + \sum_{j \in J} \pi_j & & & \\
\text{(i)} & \gamma_i + \sum_{a \in p} \lambda_a & \geq \sum_{a \in p} c_a^i & \forall p \in P_i, i \in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a \in q} \lambda_a & \geq 0 & \forall q \in Q_j, j \in J & \text{Configs} \\
\text{(iii)} & \gamma, \pi, \lambda & \geq 0 & &
\end{array}
$$

Prop. (B., Schlechte [2007]):

Config pricing is an acyclic shortest path problem with arc weights

$$\bar{c}_a = -\lambda_a,$$

that can be solved in polynomial time.



Path Pricing = Shortest Path Problem

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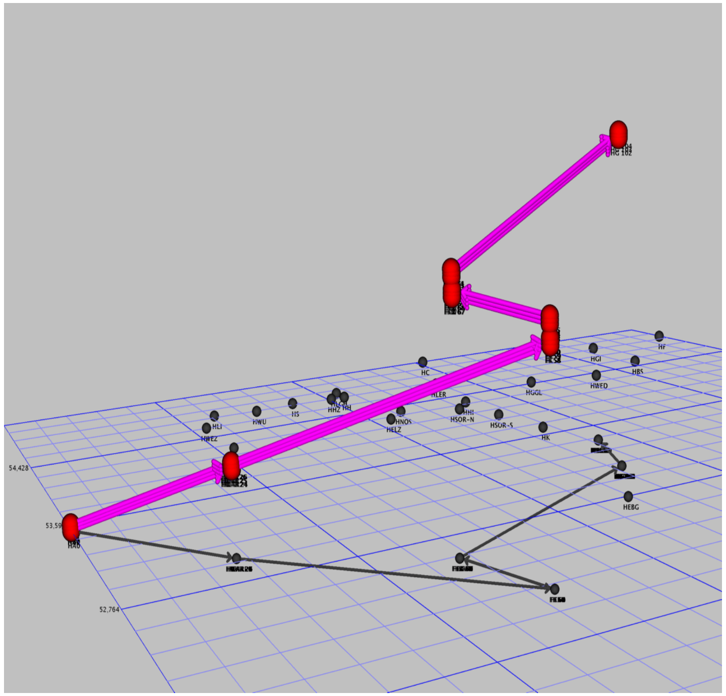
$$
\begin{array}{llcll}
\text{(DUA)} & \min \sum_{i\in I}\gamma_i + \sum_{j\in J}\pi_j & & & \\
\text{(i)} & \gamma_i + \sum_{a\in p}\lambda_a & \geq \sum_{a\in p} c_a^i & \forall p\in P_i, i\in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a\in q}\lambda_a & \geq 0 & \forall q\in Q_j, j\in J & \text{Configs} \\
\text{(iii)} & \gamma,\pi,\lambda & \geq 0 & &
\end{array}
$$

Prop. (B., Schlechte [2007]):

Route pricing is an acyclic shortest path problem with arc weights

$$\bar{c}_a = -c_a + \lambda_a,$$

that can be solved in polynomial time.



Model Comparison

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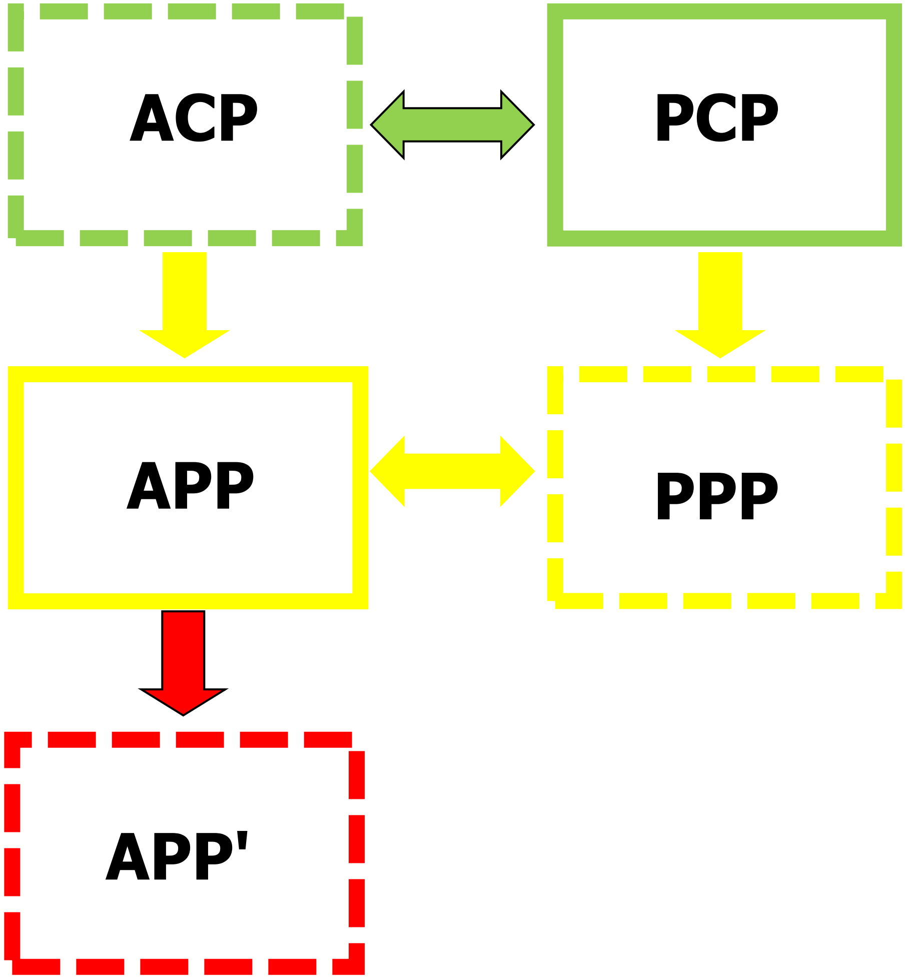
Theorem (B., Schlechte [2007]):

$$v_{IP}(PCP) = v_{IP}(ACP)$$
$$= v_{IP}(APP) = v_{IP}(PPP)$$
$$= v_{IP}(APP')$$

and

$$v_{LP}(PCP) = v_{LP}(ACP)$$
$$\leq v_{LP}(APP) = v_{LP}(PPP)$$
$$\leq v_{LP}(APP').$$

$v_{LP}(PCP)$ and $v_{LP}(ACP)$ can be computed in polynomial time.



A Dual Bound

$$
\begin{array}{llll}
\text{(DUA)} & \min \sum_{i \in I} \gamma_i + \sum_{j \in J} \pi_j & & \\
\text{(i)} & \gamma_i + \sum_{a \in p} \lambda_a \geq \sum_{a \in p} c_a^i & \forall p \in P_i, i \in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a \in q} \lambda_a \geq 0 & \forall q \in Q_j, j \in J & \text{Configs} \\
\text{(iii)} & \gamma, \pi, \lambda \geq 0 & &
\end{array}
$$

$$
\eta_i = \max_{p \in P_i} \sum_{a \in p} (c_a^i - \lambda_a) - \gamma_i \Rightarrow \eta_i + \gamma_i \geq \sum_{a \in p} (c_a^i - \lambda_a) \ \forall i \in I, p \in P_i
$$

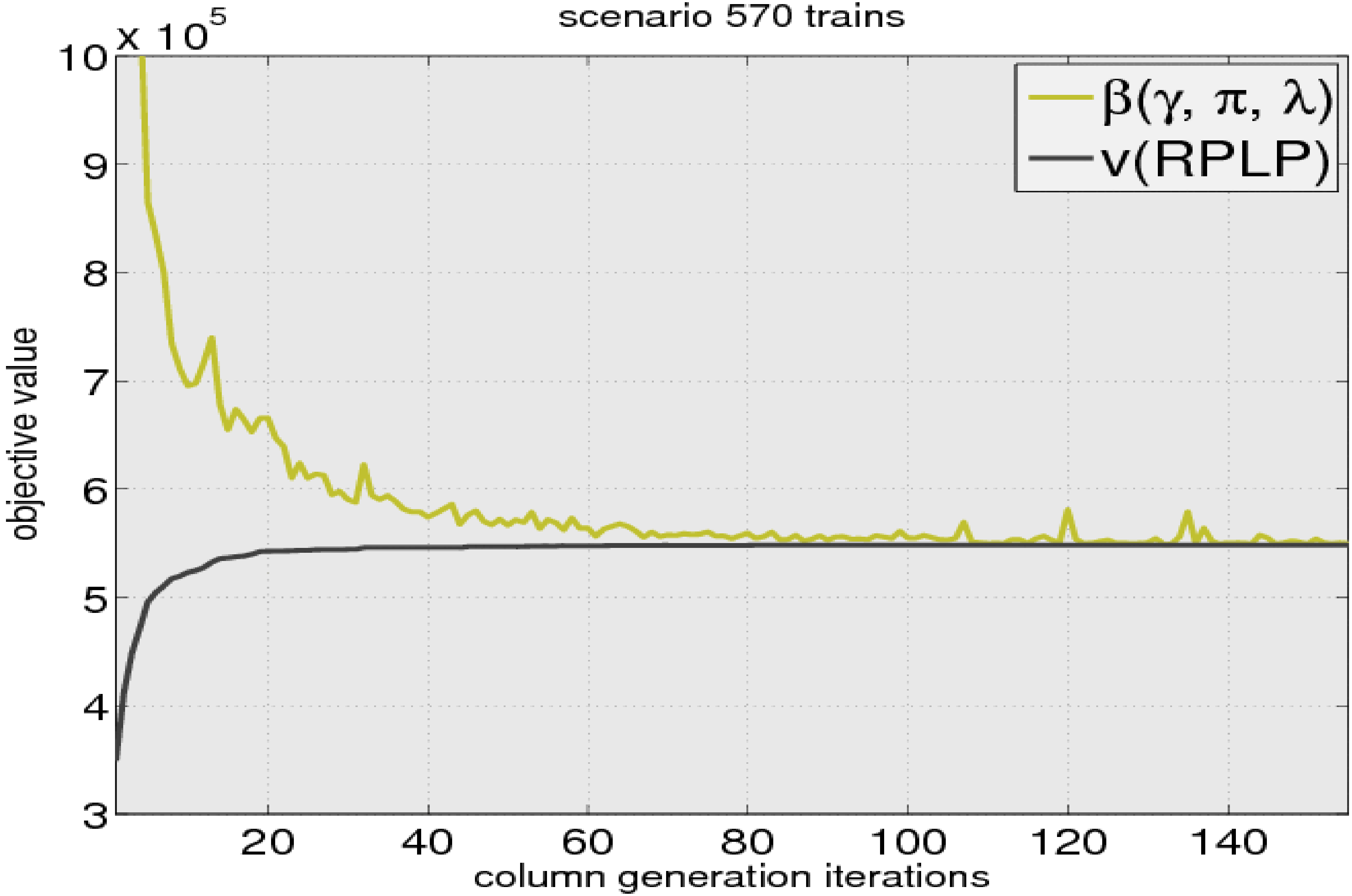
$$
\theta_j = \max_{q \in Q_j} \sum_{a \in q} \lambda_a - \pi_j \Rightarrow \theta_j + \pi_j \geq \sum_{a \in p} \lambda_a \ \forall j \in J, q \in Q_j
$$

Prop. (B., Schlechte [2007]): $([\eta+\gamma]^+, [\theta+\pi]^+, \lambda)$ is dual feasible for PLP and

$$\beta(\gamma, \pi, \lambda) = \Sigma\, [\eta+\gamma]^+_i + \Sigma\, [\theta+\pi]^+_j$$

is an upper bound on v(PLP).

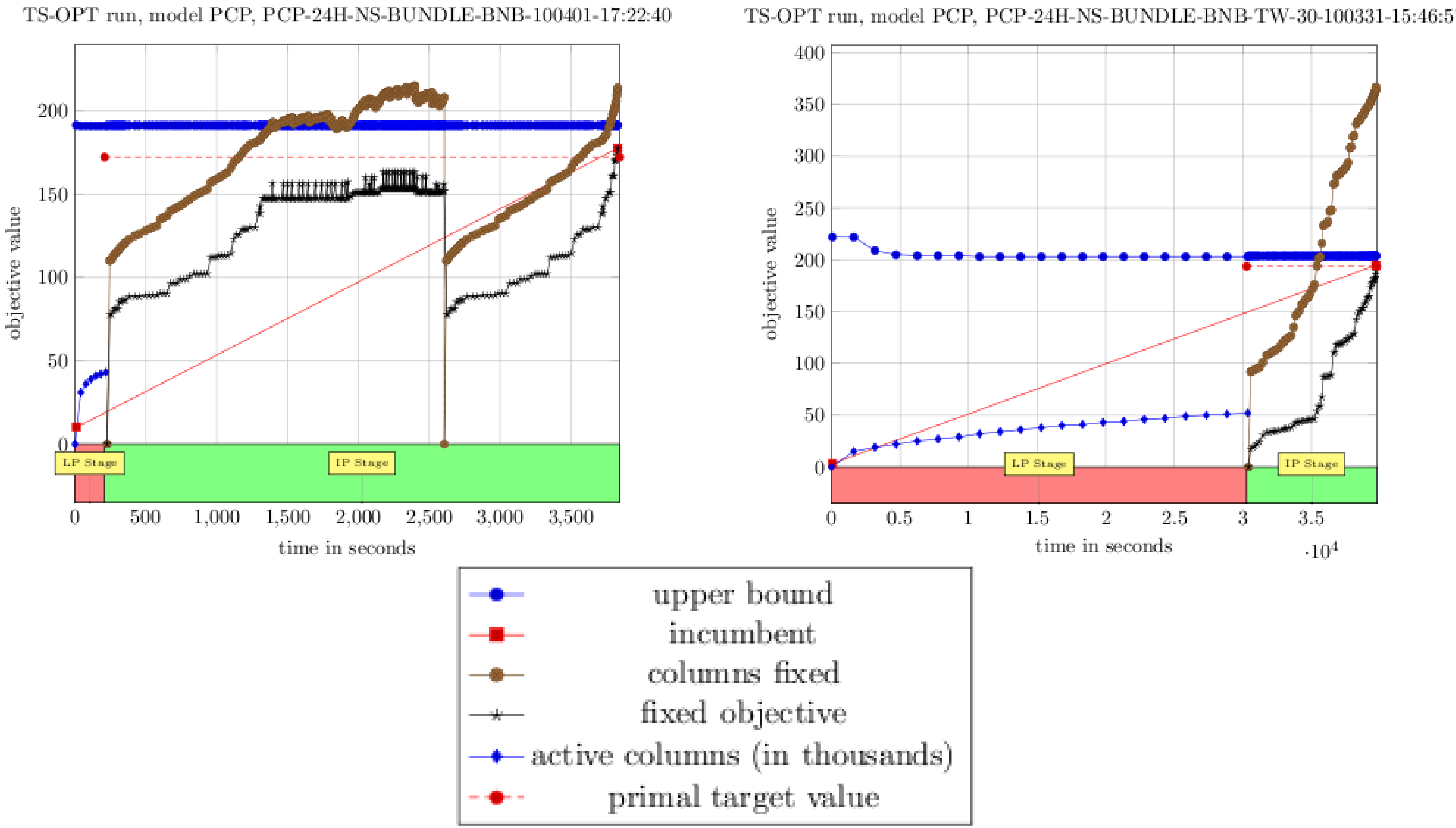
Solving the LP-Relaxation



Rapid Branching & Track Allocation

(B., Schlechte, Weider, Reuther [2013])

- ▷ HaKaFu, req32, 1140 requests, 30 mins time windows



Packing vs. Configuration Model

(Schlechte [2011])

τ	$\#rows$	$\#cols$	$\#trains$	ub^*	$v(\mathrm{LP})$	v^*	gap in %	t_Σ in s	$\#bbn$
0	288	316	29	37.10	**37.10**	**37.10**	–	8.87	1
2	962	1005	67	99.92	**99.92**	**99.92**	–	8.20	1
4	3134	3589	121	219.05	222.92	219.05	–	8.90	68
6	5552	6372	143	238.67	246.25	238.67	–	9.49	570
8	9584	10515	161	260.77	279.99	260.77	–	11.14	569
10	15481	15726	185	309.54	322.47	309.54	–	12.93	518
12	23135	21730	198	336.63	348.29	334.93	0.51	3609.85	1521298
14	33004	28569	220	375.97	387.26	373.94	0.54	3612.16	1209431
16	47245	36673	239	401.50	408.92	399.81	0.42	3612.97	773386
18	66181	45882	254	439.78	458.45	438.08	0.39	3613.58	462670
20	93779	56951	257	456.57	458.45	451.76	1.06	3613.94	303575

- Scenario REQ_36 from `ttplib.zib.de` (Packing ↑, Config ↓)

τ	$\#rows$	$\#cols$	$\#trains$	ub^*	$v(\mathrm{LP})$	v^*	gap in %	t_Σ in s	$\#bbn$
0	835	920	29	37.10	**37.10**	**37.10**	–	8.44	1
2	2418	2895	67	99.92	**99.92**	**99.92**	–	8.41	1
4	6920	9345	121	219.05	**219.05**	**219.05**	–	8.63	1
6	11129	16329	143	238.67	242.72	238.67	–	9.39	1
8	17393	27470	161	260.77	269.25	260.77	–	13.29	280
10	24825	41517	185	309.54	314.04	309.54	–	46.41	577
12	33156	57149	198	334.93	342.66	334.93	–	110.98	528
14	42282	74862	220	373.94	381.45	373.94	–	259.62	780
16	53142	96729	239	399.81	405.33	399.81	–	1467.77	1485
18	65378	124115	254	438.08	450.48	438.08	–	2399.55	512
20	79697	156674	257	454.77	458.30	451.76	0.67	3618.53	421

Track Allocation and Train Timetabling

Article	*Stations*	*Tracks*	*Trains*	*Modell/Approach*
Szpigel [1973]	6	5	10	Packing/Enumeration
Brännlund et al. [1998]	17	16	26	Packing/ Lagrange, BAB
Caprara et al. [2002]	74 (17)	73 (16)	54 (221)	Packing/ Lagrange, BAB
B. & Schlechte [2007]	37	120	570	Config/PAB
Caprara et al. [2007]	102 (16)	103 (17)	16 (221)	Packing/PAB
Fischer et al. [2008]	656 (104)	1210 (193)	117 (251)	Packing/Bundle, IP Rounding
Lusby et al. [2008]	???	524	66 (31)	Packing/BAP
B. & Schlechte [2010]	37	120	>1.000	Config/Rapid Branching

- BAB: Branch-and-Bound
- PAB: Price-and-Branch
- BAP: Branch-and-Price

Thank you for your attention

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