Line Planning and Steiner Path Connectivity

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Combinatorial Optimization with Applications in Transportation and Logistics

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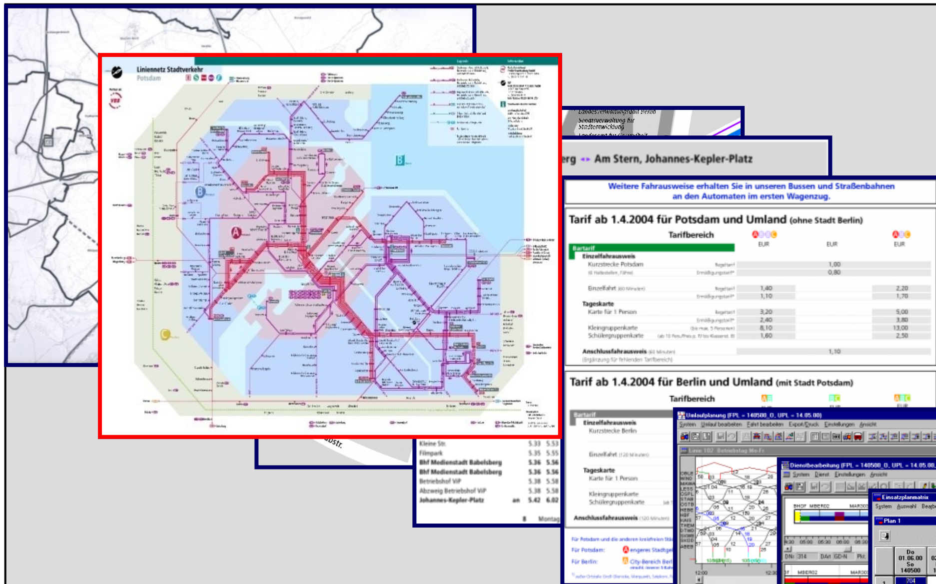
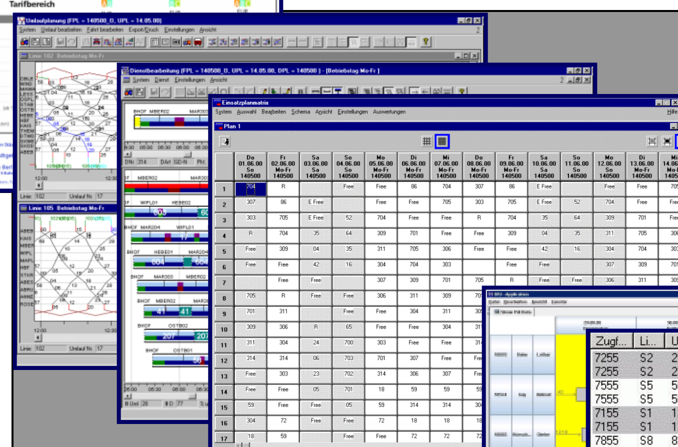
Overview

- Service Design
- Steiner Path Connectivity
- Line Planning
- Project Stadt+

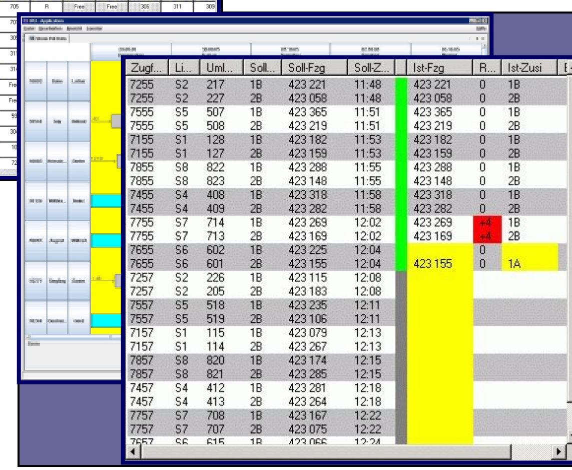
Planning Problems in Public Transit



Service Design

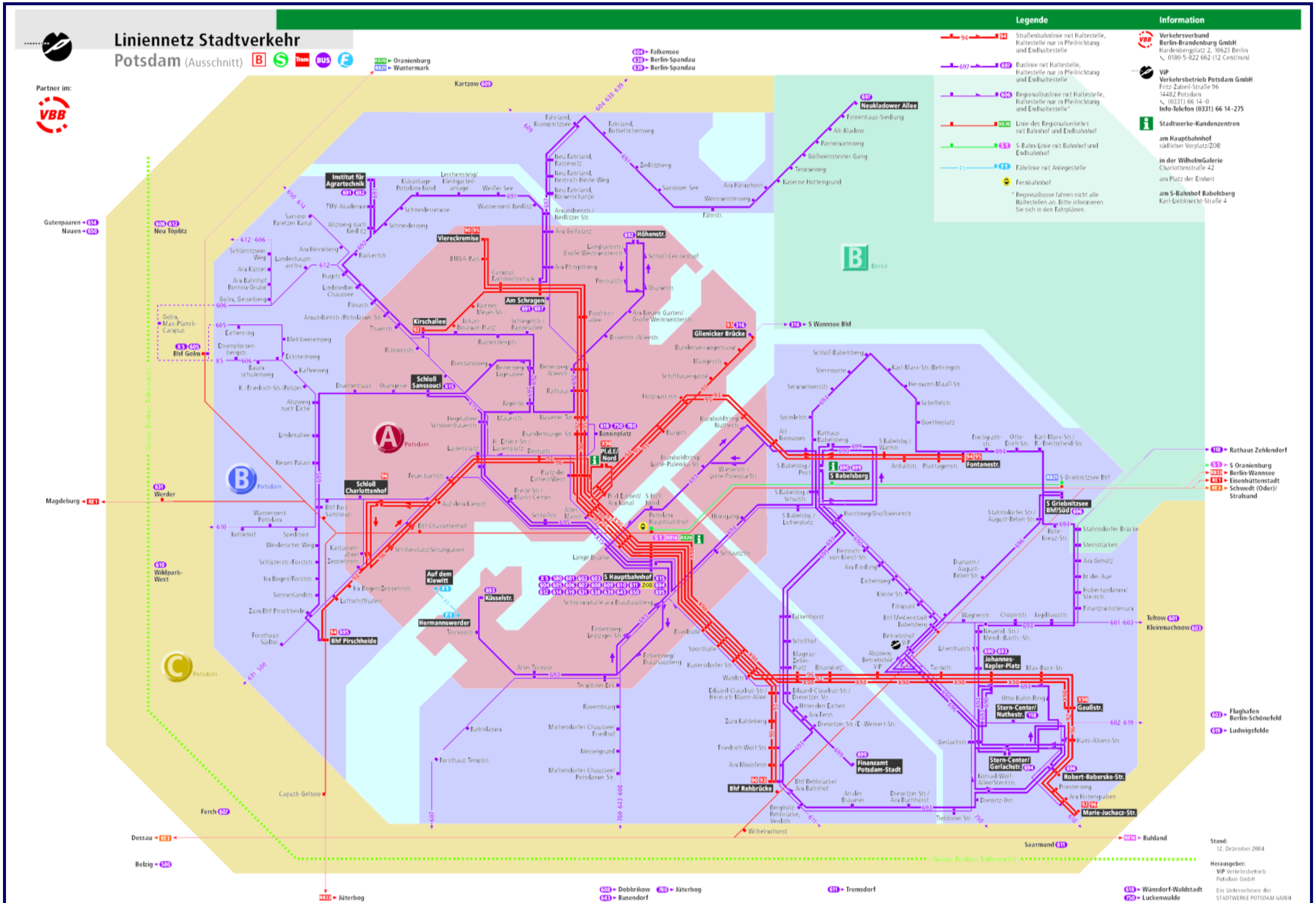


Operational Planning



Operations Control

Line Plan


Liniennetz Stadtverkehr
Potsdam (Ausschnitt)
Legende
Information
Partner im:
Kartzow
Neukladower Allee
Glienicker Brücke
Schloß Sanssouci
Schloß Charlottenhof
Kirschallee
Am Schragen
Viereckremise
Bassinplatz
S Hauptbahnhof
Auf dem Kiewitt
Hermannswerder
Küsselstr.
Bhf Pirschheide
Bhf Rehbrücke
Finanzamt Potsdam-Stadt
S Babelsberg
Fontanestr.
S Griebnitzsee Bhf/Süd
Johannes-Kepler-Platz
Gaußstr.
Robert-Baberske-Str.
Marie-Juchacz-Str.
Stern-Center/ Nuthestr.
Stern-Center/ Gerlachstr.
Höhenstr.
S Wannsee Bhf
Rathaus Zehlendorf
S Oranienburg
Berlin-Wannsee
Eisenhüttenstadt
Schwedt (Oder)/ Stralsund
Flughafen Berlin-Schönefeld
Ludwigsfelde
Teltow
Kleinmachnow
Ruhland
Saarmund
Werder
Magdeburg
Wildpark-West
Ferch
Dessau
Belzig
Jüterbog
Dobbrikow
Busendorf
Tremsdorf
Wünsdorf-Waldstadt
Luckenwalde
Gutenpaaren
Nauen
Neu Töplitz
Bhf Golm
Oranienburg
Wustermark
Falkensee
Berlin-Spandau

Line Planning and Steiner Path Connectivity

Line Planning Problem

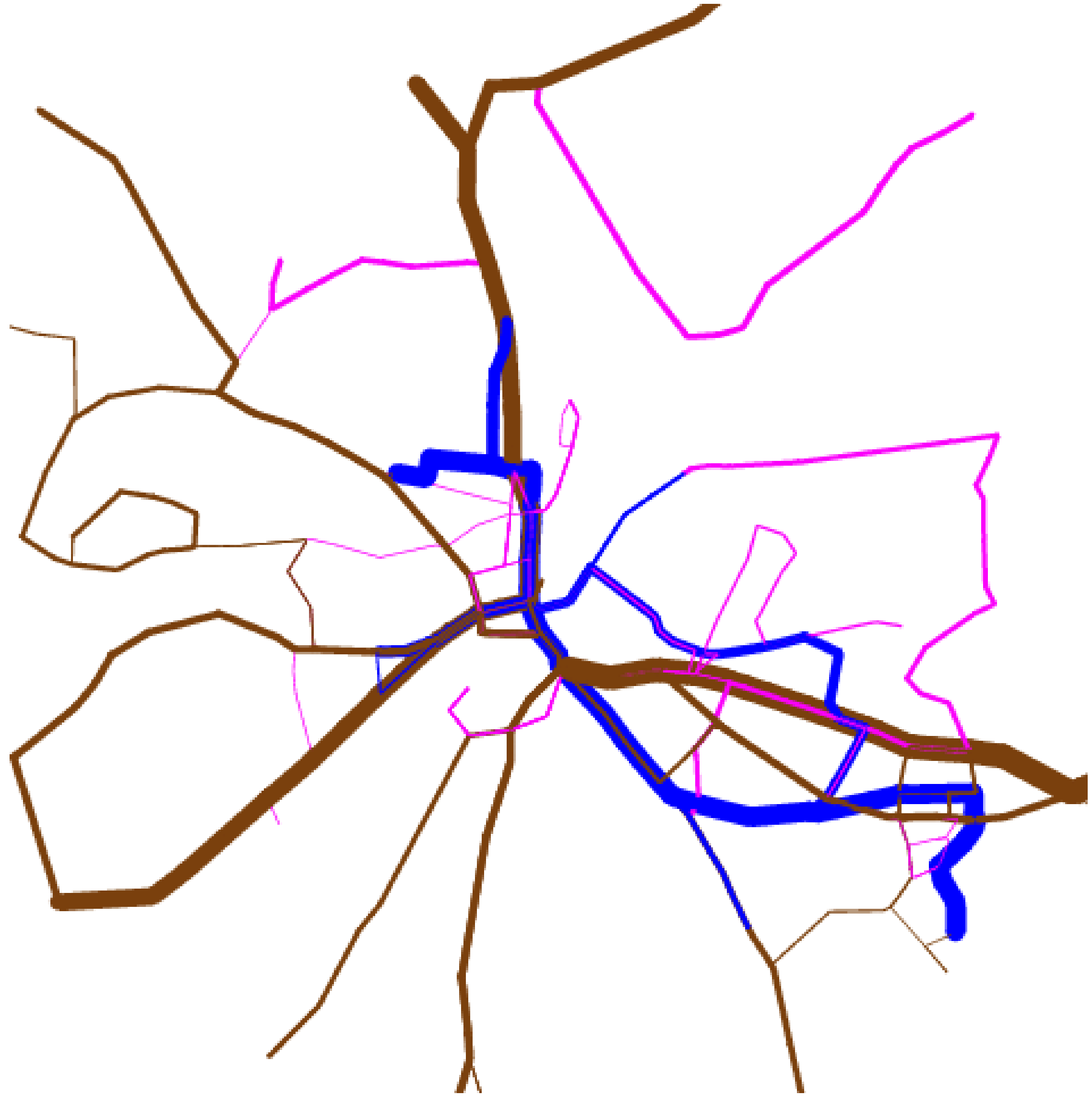
Find a cost minimal set of lines and associated frequencies, s.t. a given set of travel demands can be transported in minimal time.

Steiner Path Connectivity Problem

Find a cost minimal set of paths that provide enough capacity to route a fastest multi-commodity flow.

Features

- Bicriteria problem (cost vs. quality)
- Passenger behavior (transfers)



Graphics: JavaView, F4

Line Planning and Steiner Path Connectivity

Line Planning Problem

- Find lines and frequencies to satisfy a given demand

Objectives

- Minimize operation costs
- Minimize travel time

Input

- Public transport network
- OD-matrix of travel demands
- Operation costs and travel times

Output

- Set of lines and frequencies
- Passenger flow

Steiner Path Connectivity Problem

- Find paths to connect a set of terminal nodes

Objectives

- Minimize path costs

Input

- Undirected graph
- Set of terminal nodes
- Path costs

Output

- Connecting set of paths

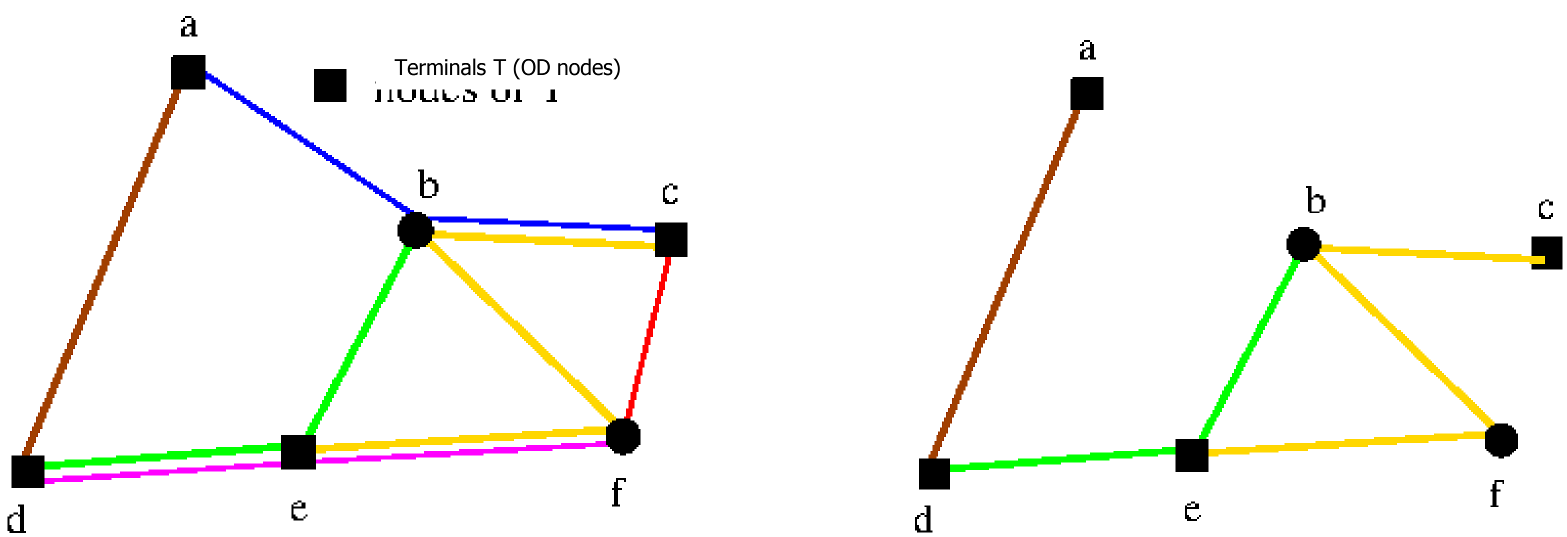
Steiner (Path) Connectivity Problem

(B., Karbstein., Pfetsch [2009])

$$\min \sum_{\ell \in \mathcal{L}} c_\ell \, x_\ell$$ Minimize cost

$$\text{s.t.} \sum_{\ell \in \mathcal{L}_{\delta(W)}} x_\ell \geq 1 \qquad \emptyset \neq W \cap T \neq V$$ Connect all OD nodes

$$x_\ell \in \{0, 1\}$$



Complexity and Approximation

Proposition

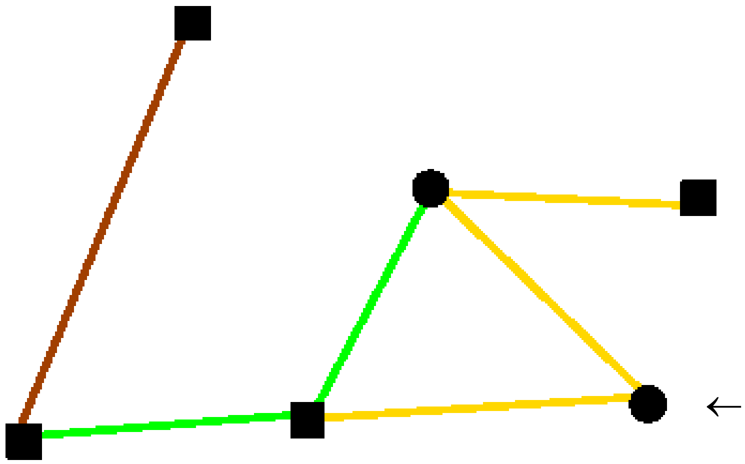
1. The Steiner connectivity problem (SCP) is NP-hard even for T=V.
2. The SCP is solvable in polynomial time for $|T| = r$, r fixed (T set of OD nodes).
3. The SCP is not approximable in the general case.

Theorem

There exists a $(k+1)$-approximation algorithm for the SCP, where

$$k = min\{\text{max. path length, max. number of terminals per path}\}.$$

Proof: Primal dual algorithm similar to Goemans, Williamson [1995] for the Steiner forest problem.



Basic property of minimal solutions (w.r.t. inclusion):
Proposition (B, Karbstein [2014]): The average path-degree (nr. of paths incident to a node) of a terminal node is $\leq k+1$.

For Steiner trees the node degree (nr. of edges) of non-terminal nodes is ≥ 2; the analogue is not true for minimal solutions of the SCP.

Cut Formulation

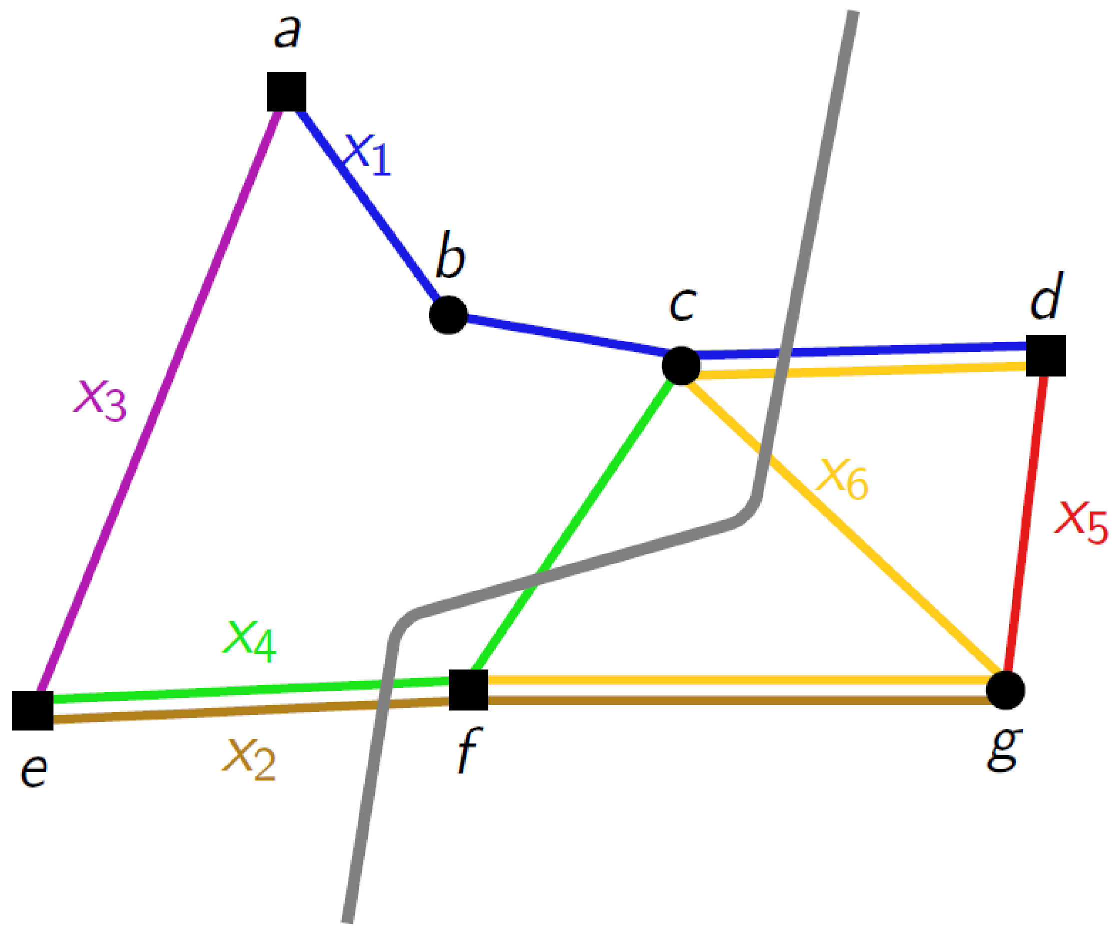
$$(\text{SCP}_{cut}) \qquad \min \sum_{p \in \mathcal{P}} c_p \, x_p$$

$$\sum_{p \in \mathcal{P}_{\delta(W)}} x_p \geq 1 \qquad \forall\, \emptyset \neq W \cap T \subsetneq T$$

$$x_p \in \{0,1\} \qquad \forall\, p \in \mathcal{P}$$

where $\mathcal{P}_{\delta(W)} = \{p \in \mathcal{P} : \mathcal{P}\delta(W) \cap p \neq \emptyset\}$ (Steiner path cuts) is the set of paths that cross a Steiner cut

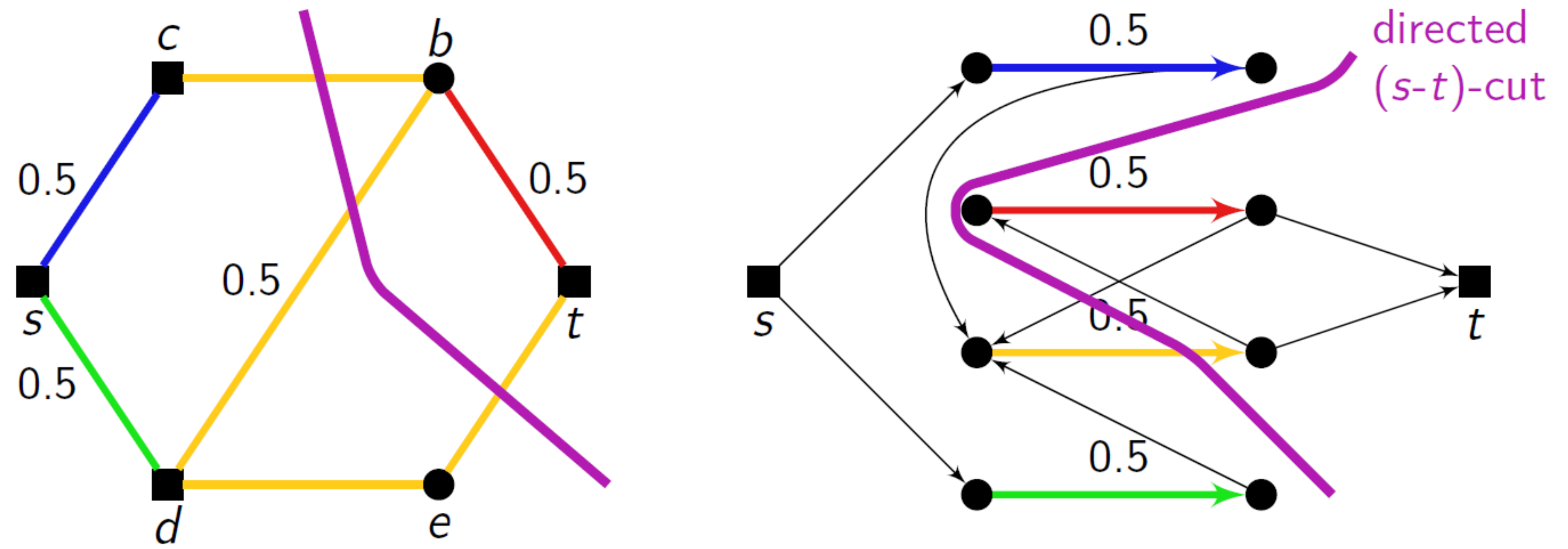
Steiner Path Cut



$W = \{d, f, g\}$ → $x_1 + x_2 + x_4 + x_6 \geq 1$

Weaker: $x_1 + x_2 + 2x_4 + x_6 \geq 1$

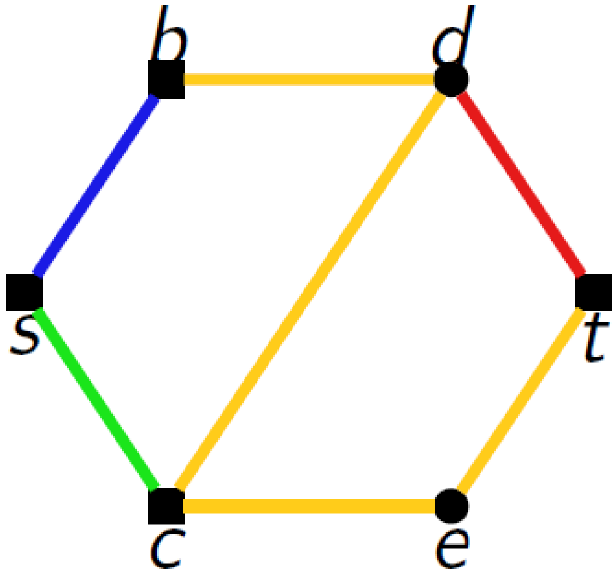
Separation of Steiner Path Constraints

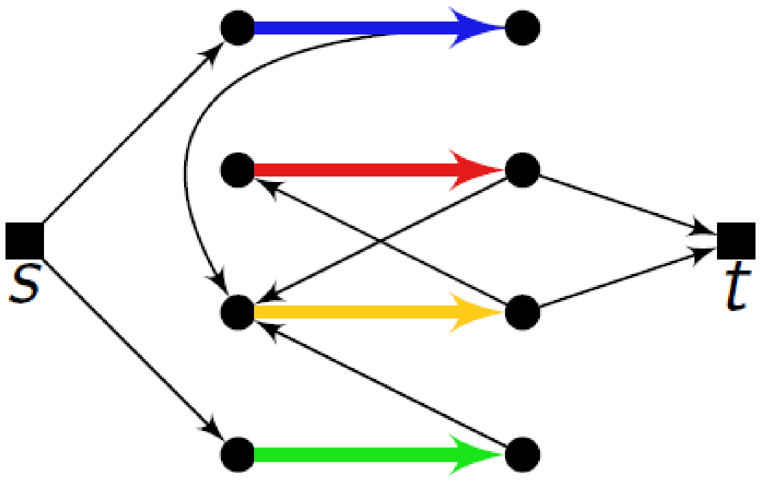


Construct Steiner connectivity digraph D' for $\hat{x} \in \mathbb{R}^{P}_{\geq 0}$:

- Consider cuts between s and t; $V' = \{s, t\} \cup \{v_p, w_p : p \in \mathcal{P}\}$
- Insert arcs $a = (v_p, w_p), p \in \mathcal{P}$ with capacity $\kappa_a = \hat{x}_p$
- Insert arcs $a = (s, v_p), (w_p, t),\ p \in \mathcal{P}$ with capacity $\kappa_a = \hat{x}_p$
- Insert arcs $a = (w_{\tilde{p}}, v_p), p, \tilde{p} \in \mathcal{P}$, $r \notin p, p \cap \tilde{p} \neq \emptyset$ with capacity $\kappa_a = \min\{\hat{x}_p, \hat{x}_{\tilde{p}}\}$

Separation of Cut Constraints





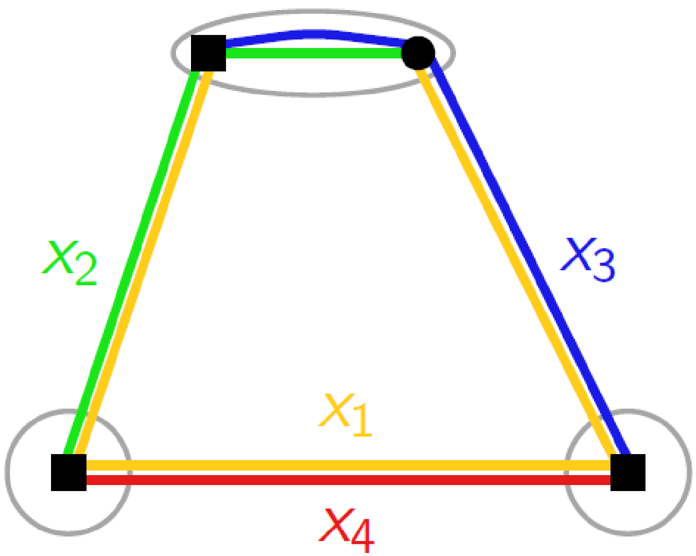
Proposition

There is a one-to-one correspondence between minimal directed (s,t)-cuts in D' and minimal (s,t)-Steiner path cuts in G, and the capacities are equal.

Proposition

The separation problem for Steiner path cut constraints can be solved in polynomial time.

Steiner Partition Inequality



$$x_1 + x_2 + x_3 \geq 1$$
$$x_1 + x_2 + x_4 \geq 1$$
$$x_1 + x_3 + x_4 \geq 1$$

$x_2 = x_3 = x_4 = 0.5$ satisfies all Steiner path cuts

$2x_1 + x_2 + x_3 + x_4 \geq 2$ separates this solution

Let $P = (V_1, V_2, \dots, V_k),\ V_i \cap T \neq \emptyset,\ i = 1, \dots, k,\ V = \bigcup_{i=1}^{k} V_i$.

The Steiner partition inequality is defined as

$$\sum_{p \in \mathcal{P}} a_p \cdot x_p \geq k - 1$$

where $a_p \coloneqq |\{i \in \{1, \dots, k\} : V_i \cap p \neq \emptyset\}| - 1.$

Facetness

Let $\overline{\mathcal{P}} := \{p \in \mathcal{P}: a_p = 0\}$. Generalization of a result of Grötschel, Monma & Stoer [1990]:

Proposition

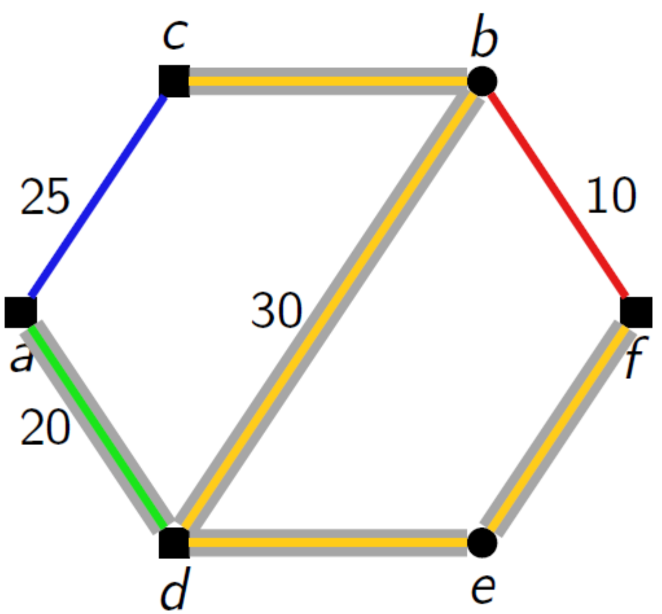
A Steiner partition inequality is facet defining if the following properties are satisfied.

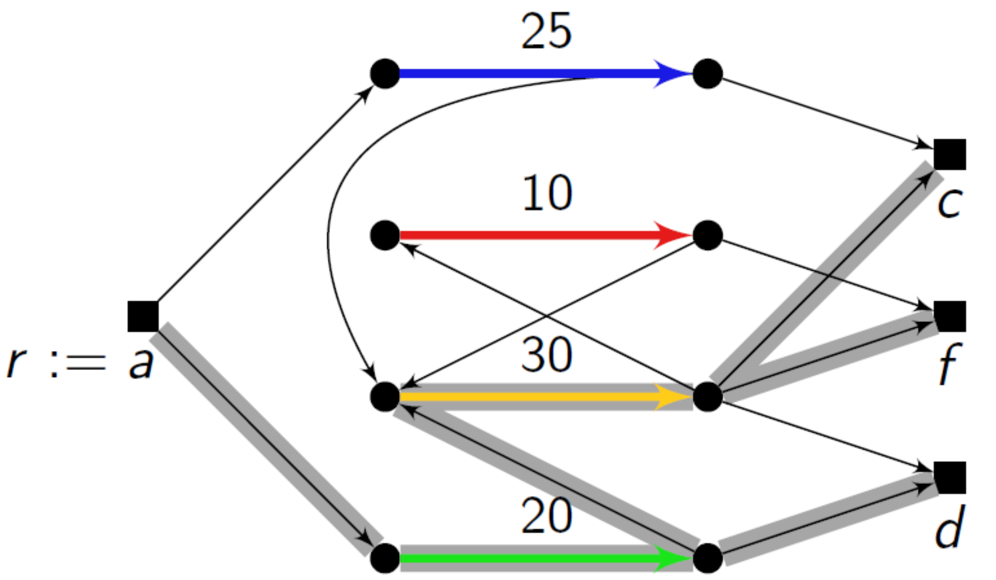
1. $G[V_i]$ is connected by $\overline{\mathcal{P}}$, $\forall i$
2. $G[V_i]$ contains no Steiner-path-bridge in $\overline{\mathcal{P}}$, $\forall i$
3. Each path is incident with at most two node sets, i.e., $a_p \in \{0,1\}$
4. The shrunk graph (each node set a single node) is 2-node-path-connected

Only property 4 is necessary!

Directed Cut Formulation

Can use Steiner connectivity digraph to get equivalent directed formulation:





All "path"-arcs get the cost of the corresponding path. All other arcs get 0 cost.

→ Directed Steiner tree problem in D'

Directed Cut Formulation

$$(\mathrm{SCP}'^{r}_{arc}) \quad \min \sum_{a \in A'} c'_a x_a$$
$$\text{s.t.} \sum_{a \in \delta^-(W)} x_a \geq 1 \qquad \forall\, W \subseteq V' \setminus \{r\},\ W \cap T \neq \emptyset$$
$$x_a \in \{0,1\} \quad \forall\, a \in A'.$$

Theorem

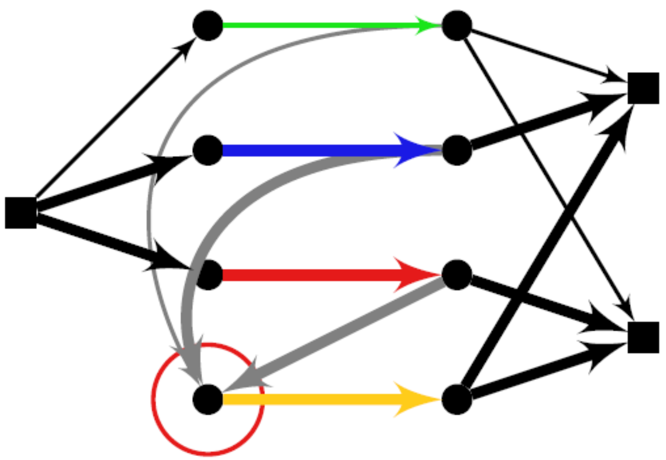
$$P_{LP}(\mathrm{SCP}_{\mathrm{cut}}) = P_{LP}(\mathrm{SCP}_{\mathrm{arc}})\Big|_{\mathcal{P}}$$

Consequence: objective value of (SCP^{r}_{arc}) is independent of r.

Flow Balance Constraints

Flow balance constraints:

$$\sum_{a \in \delta^+(v)} y_a \geq \sum_{a \in \delta^-(v)} y_a \quad \forall\, v \in V' \setminus T$$



$$(\text{SCP}^r_{arc^+}) \ \min \sum_{a \in A'} c'_a y_a$$

$$\text{s.t.} \sum_{a \in \delta^-(W)} y_a \geq 1 \qquad \forall\, W \subseteq V' \setminus \{r\},\ W \cap T \neq \emptyset$$

$$y_{v_p w_p} \geq \sum_{a \in \delta^-(v_p)} y_a \quad \forall\, v_p \in V' (p \in \mathcal{P} : r \notin p)$$

$$\sum_{a \in \delta^+(w_p)} y_a \geq y_{(v_p, w_p)} \qquad \forall\, w_p \in V' (p \in \mathcal{P} : t \notin p \forall t \in T)$$

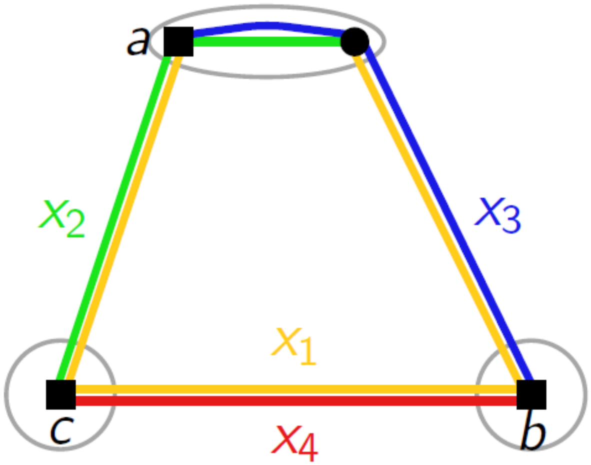
$$y_a \in \{0, 1\} \qquad \forall\, a \in A'$$

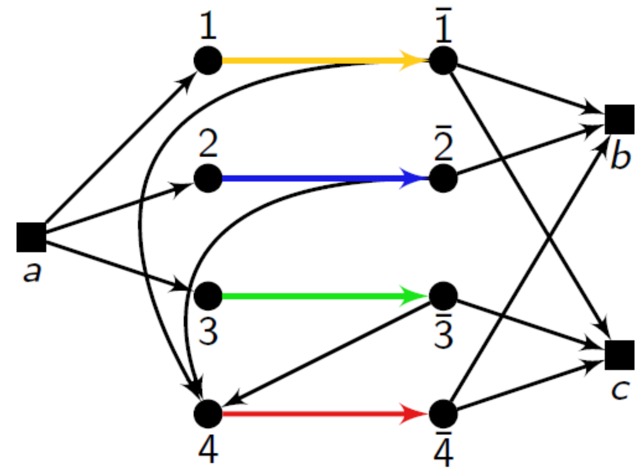
- Quality depends on root

Directed vs. Undirected Formulation

Theorem

$P_{LP}\left(\mathrm{SCP}^{\mathrm{r}}_{\mathrm{arc}^+}\right)|_{\mathcal{P}}$ satisfies all Steiner partition inequalities.





$$2x_1 + x_2 + x_3 + x_4 = 2x_{1\bar{1}} + x_{2\bar{2}} + x_{3\bar{3}} + x_{4\bar{4}}$$
$$\geq 2x_{a1} + x_{a2} + x_{a3} + x_{\bar{1}4} + x_{\bar{2}4} + x_{\bar{3}4}$$
$$\geq 1 + x_{a1} + x_{a3} + x_{\bar{1}4} + x_{\bar{2}4}$$
$$\geq 2 + x_{\bar{1}4}$$
$$\geq 2$$

The Case $|T| = 2$

$$(\mathrm{SCP}_{cut}) \qquad \min \sum_{p \in \mathcal{P}} c_p \, x_p$$

$$\sum_{p \in \mathcal{P}_{\delta(W)}} x_p \geq 1 \qquad \forall\, \emptyset \neq W \cap T \subsetneq T$$

$$x_p \in \{0,1\} \qquad \forall\, p \in \mathcal{P}$$

Theorem

$(\mathrm{SCP}_{\mathrm{cut}})$ is TDI for $T = \{s, t\}$. In particular, it is integral.

Totally dual integral (TDI): For each integral c for which the optimum is finite, the dual has an integral optimal solution.

Proof: Primal-dual algorithm.

The Case $|T| = 2$

$$(\mathrm{SCP}_{cut}) \qquad \min \sum_{p \in \mathcal{P}} c_p x_p$$

$$\sum_{p \in \mathcal{P}_{\delta(W)}} x_p \geq 1 \qquad \forall\, \emptyset \neq W \cap T \subsetneq T$$

$$x_p \in \{0, 1\} \qquad \forall\, p \in \mathcal{P}$$

$c \equiv 1$: A Menger companion theorem follows:

Theorem

The minimum cardinality of an (s, t)-connecting set is equal to the maximum number of path-disjoint (s, t)-disconnecting sets.

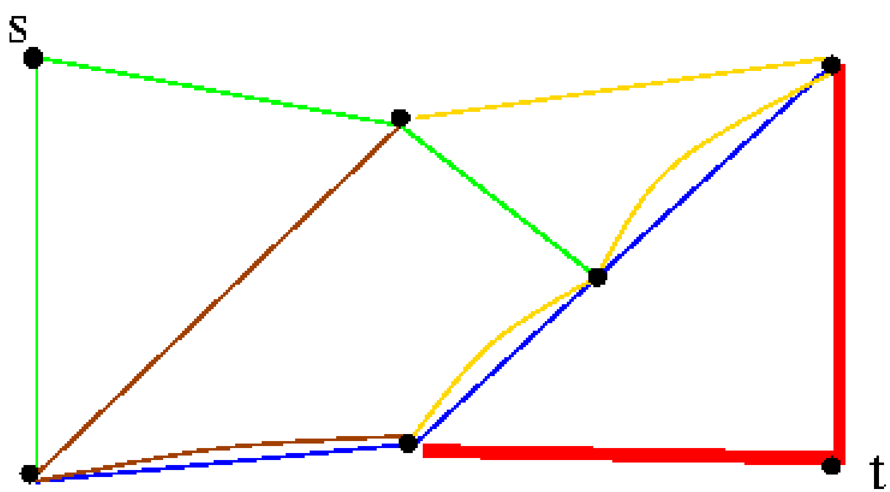
Min-Max Results

- An (s,t)-connected set of paths connects two nodes s and t ($\triangleq$ (s,t)-path).
- An (s,t)-disconnecting set of paths breaks all (s,t)-connected sets ($\triangleq$ (s,t)-cut).

Theorem (Menger and companion theorem for path-connectivity)

1. The minimum cardinality of an (s,t)-disconnecting set is equal to the maximum number of path-disjoint (s,t)-connected sets.
2. The minimum cardinality of an (s,t)-connected set is equal to the maximum number of path-disjoint (s,t)-disconnecting sets.

- 1. is hypergraph folklore, 2. is new.
- Generalizes class of blocking pairs of ideal incidence matrices of paths and cuts.



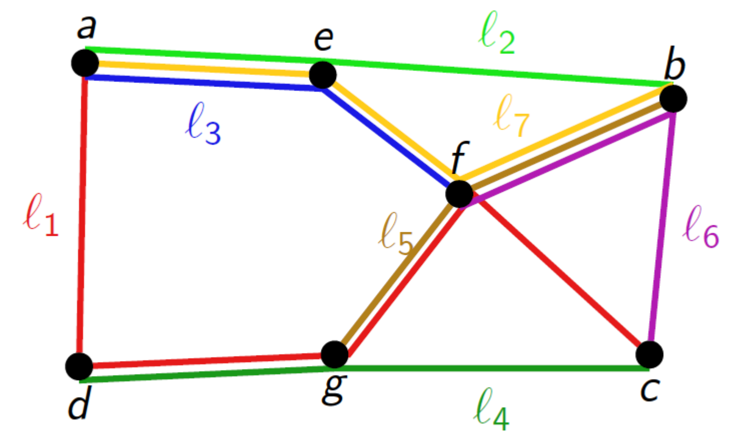
Example (2): Network with 5 paths

- (s,t)-connected:
  1. green, 2. yellow, 3. red
- (s,t)-disconnecting (path-disjoint):
  1. green
  2. yellow, blue, brown
  3. red

Relation to Line Planning

Observation for $c \equiv 1$:

- minimum number of paths connecting s and t ≙ lower bound on number of transfers−1 in line planning



Idea: use this lower bound in line planning

- associate with a passenger route r the minimum number of transfers k
- k depends on *all possible* lines
- include constraints to ensure direct connections, i.e., for $k_r = 0$
  - $y_{r,0}$ – variable for direct connection
  - $y_{r,1}$ – variable for connections with at least one transfer

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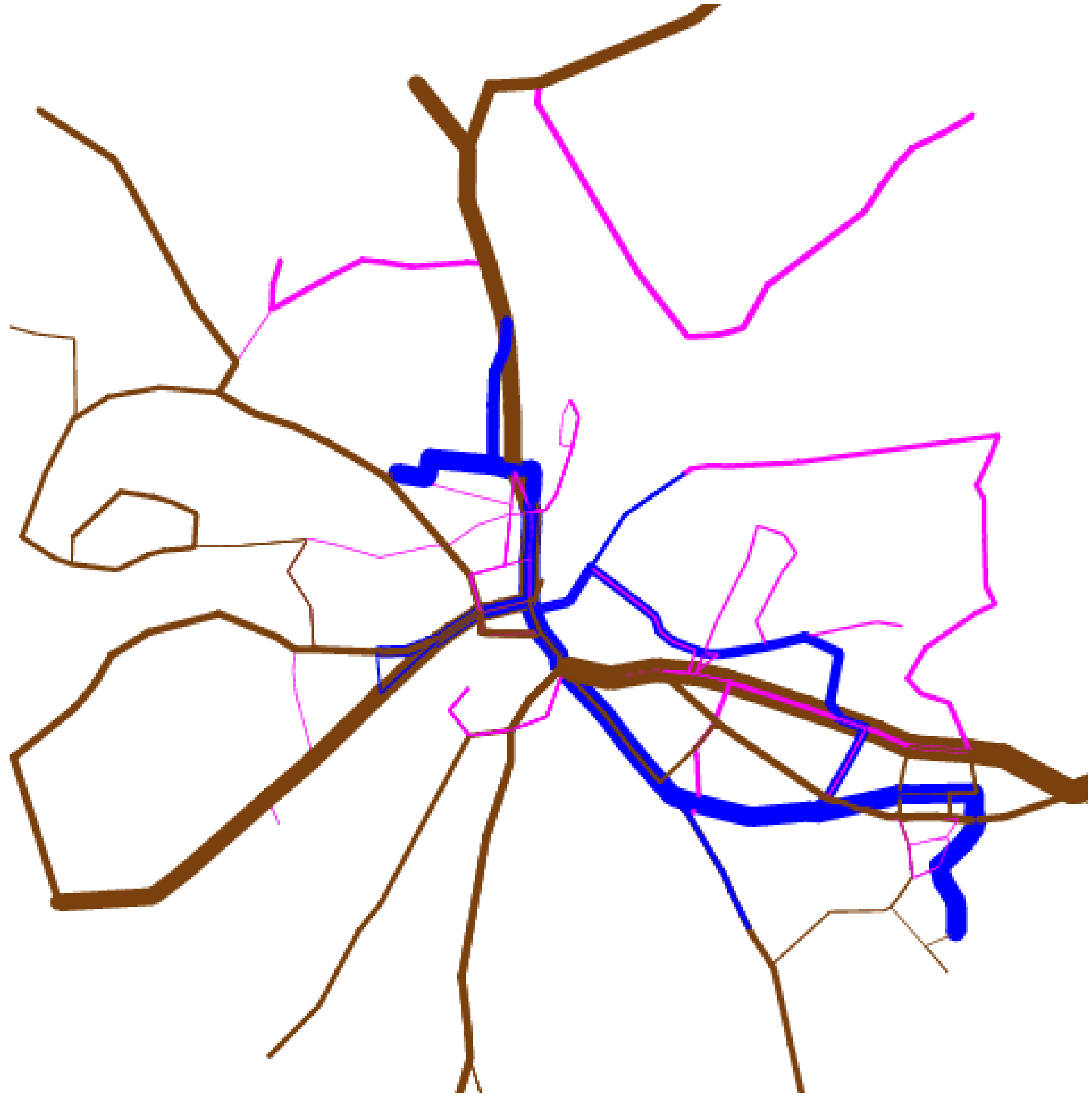
Find a cost minimal set of lines and associated frequencies, s.t. a given set of travel demands can be transported in minimal time.

Steiner Path Connectivity Problem

Find a cost minimal set of paths that provide enough capacity to route a fastest multi-commodity flow.

Features

- Bicriteria problem
  (cost vs. quality)
- Passenger behavior
  (transfers)



Graphics: JavaView, F4

Line Planning and Steiner Path Connectivity

Line Planning Problem

- Find lines and frequencies to satisfy a given demand

Objectives

- Minimize operation costs
- Minimize travel time

Input

- Public transport network
- OD-matrix of travel demands
- Operation costs and travel times

Output

- Set of lines and frequencies
- Passenger flow

Steiner Path Connectivity Problem

- Find paths to connect a set of terminal nodes

Objectives

- Minimize path costs

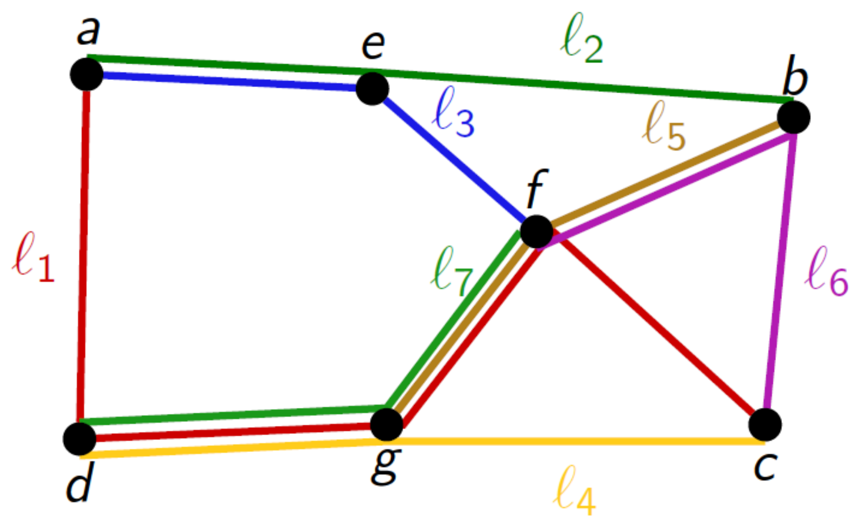
Input

- Undirected graph
- Set of terminal nodes
- Path costs

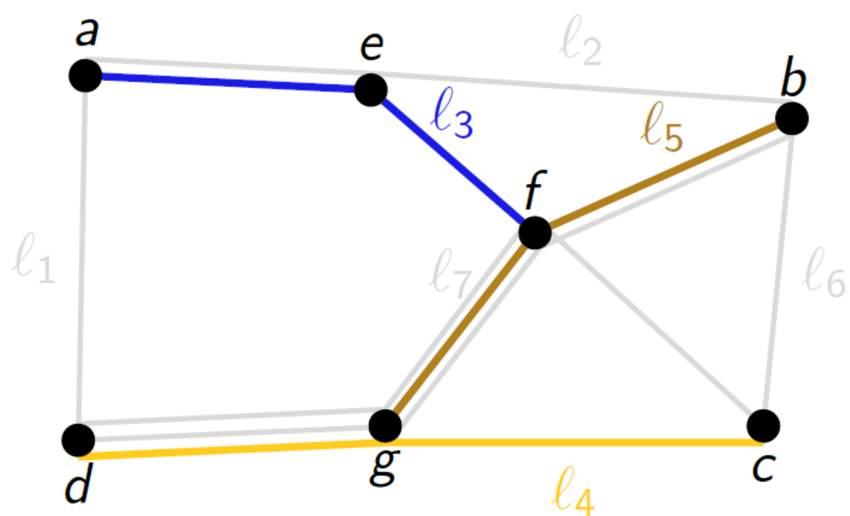
Output

- Connecting set of paths

Example



- Line capacities 50
- Demands
  $a \to f = 50, a \to b = 50$
  $d \to f = 20, d \to c = 80$



- Solution
  lines ℓ_3, ℓ_4 with frequency 2
  line ℓ_5 with frequency 1

Literature

- Minimize transfers/transfer time

  Scholl [2005], Schöbel & Scholl [2005], Schmidt [2012]

  - Advantage: detailed treatment of transfers
  - Disadvantage: *change-and-go-graph* on the basis of all possible lines; large scale model

- Maximize travel quality

  Nachtigall & Jerosch [2008]

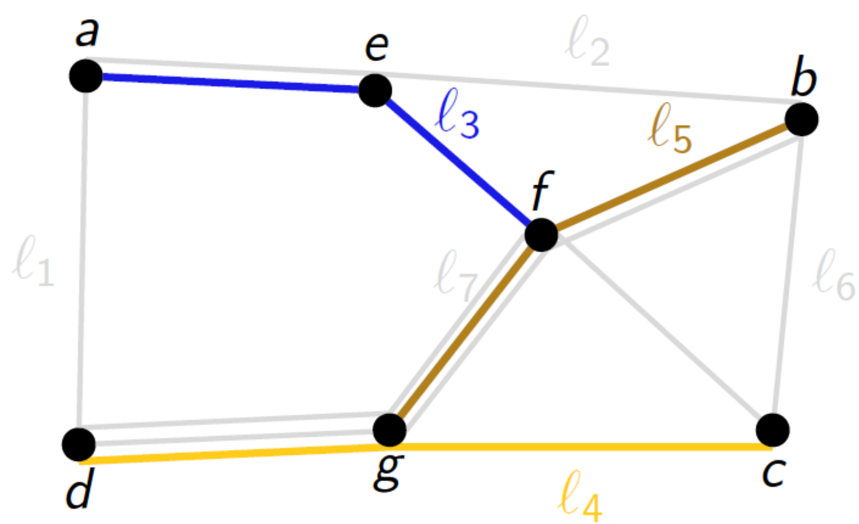
  - Advantage: utility for each path including all transfers
  - Disadvantage: capacity constraint for each partial route and line; large scale model

- Minimize pareto function of line cost and travel times

  B., Grötschel & Pfetsch[2005], B., Neumann & Pfetsch [2008]

  - Advantage: allows line pricing; computationally tractable
  - Disadvantage: ignores transfers within same transportation mode

Basic Line Planning Model (B)
(B., Grötschel & Pfetsch [2007])



- Line capacities 50
- Demands
  $a \to f = 50, a \to b = 50$
  $d \to f = 20, d \to c = 80$
- Solution
  lines ℓ_3, ℓ_4 with frequency 2
  line ℓ_5 with frequency 1

Travel time on path is sum of travel times on edges

$$p_1 = (a, e, f), \quad \tau_{p_1} = \tau_{ae} + \tau_{ef}$$
$$p_2 = (a, e, f, b), \quad \tau_{p_2} = \tau_{ae} + \tau_{ef} + \tau_{fb}$$
$$p_3 = (d, g, f), \quad \tau_{p_3} = \tau_{dg} + \tau_{gf}$$
$$p_4 = (d, g, c), \quad \tau_{p_4} = \tau_{dg} + \tau_{gc}$$

Note: direct connection are not distinguished from transfer connections, e.g., paths p_1 and p_3

Basic Line Planning Model (B)

(B., Grötschel & Pfetsch [2007])

$$\min \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_{\ell,f} x_{\ell,f} + (1-\lambda) \sum_{p \in \mathcal{P}} \tau_p y_p$$

Minimize cost and travel time

$$\sum_{p \in \mathcal{P}_{st}} y_p = d_{st} \qquad \forall\, s,t \in D$$

Transport all demand

$$\sum_{p \ni a} y_p \leq \sum_{\ell \in \mathcal{L} \ni a} \sum_{f \in \mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \qquad \forall\, a \in A$$

Capacity constraints

$$\sum_{f \in \mathcal{F}} x_{\ell,f} \leq 1 \qquad \forall\, \ell \in \mathcal{L}$$

One frequency per line

$$y_p \geq 0,\ x_{\ell,f} \in \{0,1\}$$

Variables: $x_{\ell,f} = 1$ if line $\ell \in \mathcal{L}$ is chosen with frequency $f \in \mathcal{F}$; $x_{\ell,f} = 0$ otherwise
$y_p \geq 0$ passengers on path $p \in \mathcal{P}$

Features

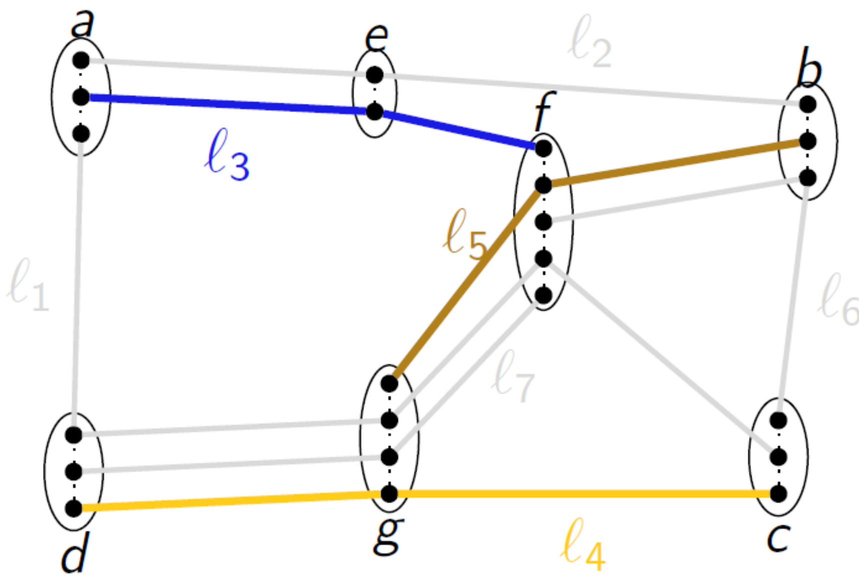
- Complete line pool
- Multi-criteria objective
- Integrated passenger routing

Disadvantage

- No transfers $\rightarrow$ Direct Connection Model (Metric Inequalities)

Change & Go Model (CG)

(Schöbel & Scholl [200])



- Copy every node number of lines containing node times, i.e., nodes
  $$(v,\ell)\ \forall v \in V, v \ni \ell \in \mathcal{L}$$
- Complete transfer graph for every node, i.e., edges
  $$\big((\mathrm{v},\ell),(v,\bar{\ell})\big)\ \forall \ell,\bar{\ell} \in \mathcal{L}$$

Travel time on path is sum of travel times on tavel + transfer edges

$$p_1 = (a,e,f), \quad \tau_{p_1} = \tau_{(a,\ell_3)(e,\ell_3)} + \tau_{(e,\ell_3)(f,\ell_3)}$$

$$p_2 = (a,e,f,b), \quad \tau_{p_2} = \tau_{(a,\ell_3)(e,\ell_3)} + \tau_{(e,\ell_3)(f,\ell_3)} + \tau_{(f,\ell_3)(f,\ell_5)} + \tau_{(f,\ell_5)(b,\ell_5)}$$

$$p_3 = (d,g,f), \quad \tau_{p_3} = \tau_{(d,\ell_4)(g,\ell_4)} + \tau_{(g,\ell_4)(g_{\ell_5})} + \tau_{(g,\ell_5)(f,\ell_5)}$$

$$p_4 = (d,g,c), \quad \tau_{p_4} = \tau_{(d,\ell_4)(g,\ell_4)} + \tau_{(g,\ell_4)(c,\ell_4)}$$

Note: all transfers are taken into account

Change & Go Model (CG)
(Schöbel & Scholl [200])

$$\min \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_{\ell,f} x_{\ell,f} + (1-\lambda) \sum_{p \in \mathcal{P}} \tau_p y_p$$ Minimize cost and travel time

$$\sum_{p \in \mathcal{P}_{st}} y_p = d_{st} \qquad \forall\, s,t \in D$$ Transport all demand

$$\sum_{p \ni a} y_p \leq \sum_{\ell \in \mathcal{L} \ni a} \sum_{f \in \mathcal{F}} \kappa_{\ell,f} x_{\ell,f} \qquad \forall (a,\ell) \in \mathcal{A}_{\mathcal{L}}$$ Capacity constraints

$$\sum_{f \in \mathcal{F}} x_{\ell,f} \leq 1 \qquad \forall\, \ell \in \mathcal{L}$$ One frequency per line

$$y_p \geq 0,\; x_{\ell,f} \in \{0,1\}$$

Variables: $x_{\ell,f} = 1$ if line $\ell \in \mathcal{L}$ is chosen with frequency $f \in \mathcal{F}$; $x_{\ell,f} = 0$ otherwise
$y_p \geq 0$ passengers on path $p \in \mathcal{P}$

Features
- Complete line pool
- Multi-criteria objective
- Integrated passenger routing
- Correct handling of transfers

Disadvantage
- Very large graph

Direct Line Connection Model (DLC)
(B., Karbstein [2012])

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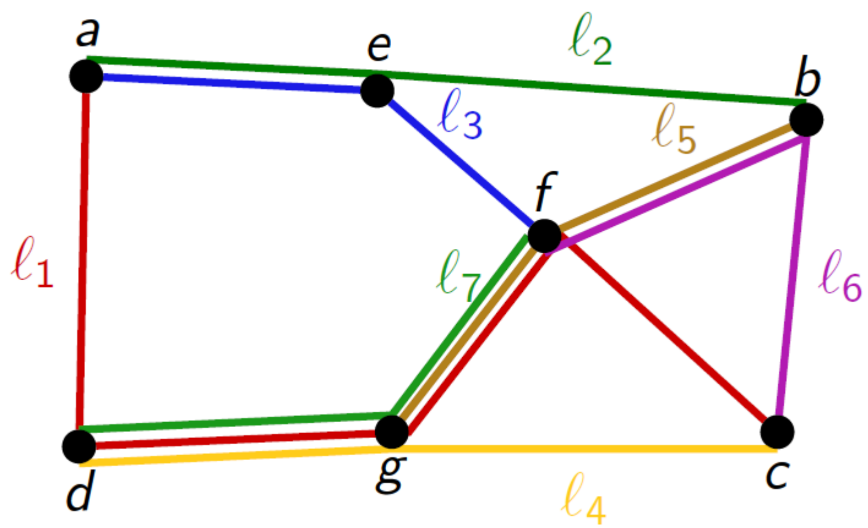


- Idea: associate with a pax path either a direct connection line or a transfer penalty:

  $y_{p,0}^{\ell}$ number of pax on p traveling directly with ℓ

  $y_{p,1}$ number of pax on p traveling with ≥ 1 transfer

Associate transfer penalty σ with non-direct connections

$p_1 = (a, e, f)$, $y_{p_1,0}^{\ell_3} = 50$, $\tau_{p_1} = \tau_{ae} + \tau_{ef}$

$p_2 = (a, e, f, b)$, $y_{p_2,1} = 50$, $\tau_{p_2} = \tau_{ae} + \tau_{ef} + \tau_{fb} + \sigma$

$p_3 = (d, g, f)$, $y_{p_3,1} = 20$, $\tau_{p_3} = \tau_{dg} + \tau_{gf} + \sigma$

$p_4 = (d, g, c)$, $y_{p_4,0}^{\ell_4} = 80$, $\tau_{p_4} = \tau_{dg} + \tau_{gc}$

Note: $y_{p_1,0}^{\ell_1}$=$y_{p_3,0}^{\ell_7}$=0, since ℓ_1, ℓ_7 not in solution

Direct Line Connection Model (DLC)

(B., Karbstein [2012])

Variables: $x_{\ell,f} \in \{0,1\}$ choose line ℓ with frequency f

$z^{\ell}_{p,0} \in \mathbb{R}_+$ passenger flow on direct connection (p,ℓ)

$y_{p,1} \in \mathbb{R}_+$ passenger flow on p with at least one transfer

Objective:

$$\min \lambda \overbrace{\sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_{\ell,f}\, x_{\ell,f}}^{\text{line cost}} + (1-\lambda)\left(\overbrace{\sum_{p \in \mathcal{P}^0} \sum_{\ell \in \mathcal{L}: p \in \mathcal{P}^{0,\ell}} \tau_{p,0}\, z^{\ell}_{p,0} + \sum_{p \in \mathcal{P}} \tau_{p,1}\, y_{p,1}}^{\text{travel times}} \right)$$

transport all passengers

$$\sum_{\ell \in \mathcal{L}} \sum_{p \in \mathcal{P}^{0,\ell}_{st}} z^{\ell}_{p,0} + \sum_{p \in \mathcal{P}_{st}} y_{p,1} = d_{st} \qquad \forall (s,t) \in D$$

capacity constraints

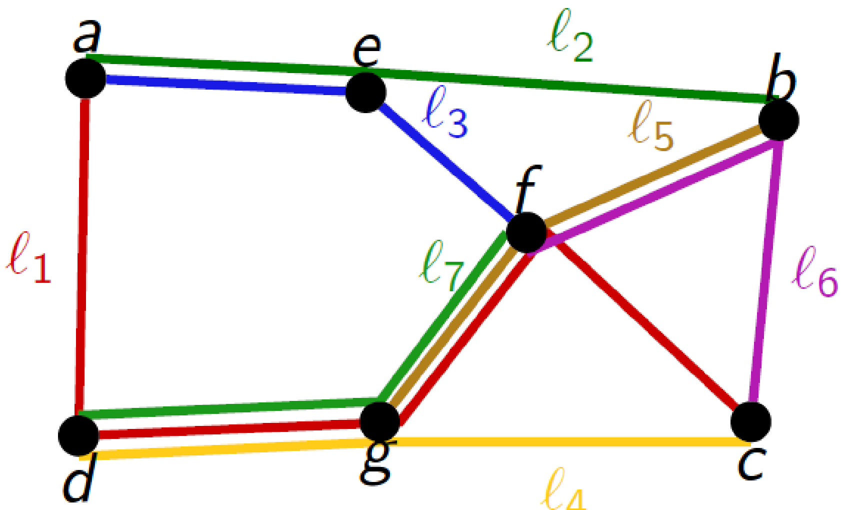
$$\sum_{\ell \in \mathcal{L}} \sum_{p \in \mathcal{P}^{0,\ell}: a \in p} z^{\ell}_{p,0} + \sum_{p \in \mathcal{P}: a \in p} y_{p,1} \leqslant \sum_{\ell \in \mathcal{L}: e(a) \in \ell} \sum_{f \in \mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \qquad \forall a \in A$$

direct line connection (capacity) constraints

$$\sum_{p \in \mathcal{P}^{0,\ell}: a \in p} z^{\ell}_{p,0} \leqslant \sum_{f \in \mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \qquad \forall \ell \in \mathcal{L},\ e(a) \in \ell$$

one frequency per line $\displaystyle\sum_{f \in \mathcal{F}} x_{\ell,f} \leqslant 1 \qquad \forall \ell \in \mathcal{L}$

Relaxation I – Line Aggregation



$\mathcal{L}^0_{df}((d,g)) = \{\ell_1, \ell_7\}$,
$\mathcal{L}^0_{af}((d,g)) = \{\ell_1\}$

$[d,f]_{(d,g)} = \{(s,t)\}$,
$[d,f]^{\leqslant}_{(d,g)} = \{(d,f),(a,f)\}$

$(d,g) : y_{adgf,0} + y_{dgf,0} \leqslant \sum_{f \in \mathcal{F}} \kappa_{\ell_1,f} x_{\ell_1,f} + \sum_{f \in \mathcal{F}} \kappa_{\ell_7,f} x_{\ell_7,f}$

$(a,d) : y_{adgf,0} \leqslant \sum_{f \in \mathcal{F}} \kappa_{\ell_1,f} x_{\ell_1,f}$

Idea: Aggregate $y_{p,0} = \sum_{\ell \in \mathcal{L}} y^{\ell}_{p,0}$

Problem: Replace dlc constraints

$$\sum_{p \in \mathcal{P}^{0,\ell} : a \in p} z^{\ell}_{p,0} \leqslant \sum_{f \in \mathcal{F}} \kappa_{\ell,f} x_{\ell,f}$$

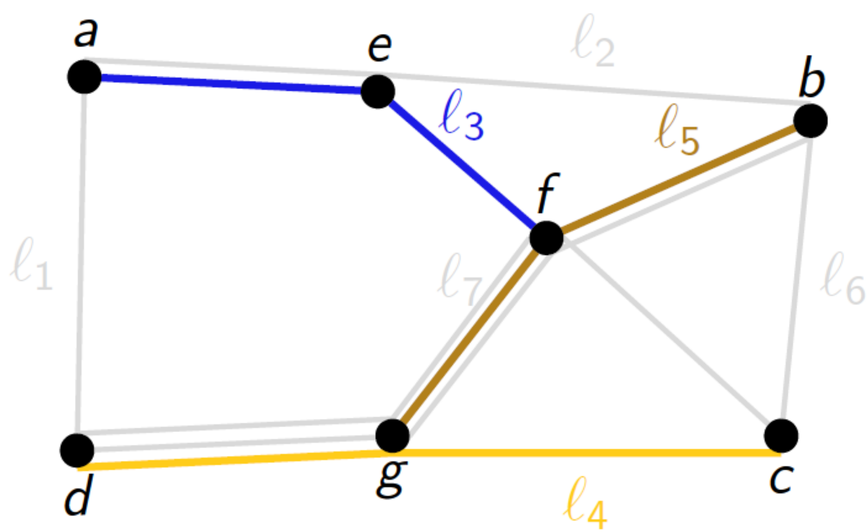
Idea: find set of paths that have the same direct connection lines

- $\mathcal{L}^0_{st}(a)$ = set of dc lines for (s,t) containing arc a
- $[s,t]_a = \{(u,v) \colon \mathcal{L}^0_{uv}(a) = \mathcal{L}^0_{st}(a)\}$
- $[s,t]^{\leqslant}_a = \{(u,v) \colon \mathcal{L}^0_{uv}(a) \subseteq \mathcal{L}^0_{st}(a)\}$

$$\sum_{(u,v) \in [s,t]^{\leqslant}_a} \sum_{p \in \mathcal{P}^{0+}_{uv}(a)} y_{p,0} \leqslant \sum_{\ell \in \mathcal{L}^0_{st}(a)} \sum_{f \in \mathcal{F}} \kappa_{\ell,f} x_{\ell,f} \qquad \forall a \in A, \forall [s,t]_a$$

Direct Connection Model (DC)

(B., Karbstein [2012])



$p_1 = (a, e, f)$, $y_{p_1,0} = 50$, $\tau_{p_1} = \tau_{ae} + \tau_{ef}$

$p_2 = (a, e, f, b)$, $y_{p_2,1} = 50$, $\tau_{p_2} = \tau_{ae} + \tau_{ef} + \tau_{fb} + \sigma$

$p_3 = (d, g, f)$, $y_{p_3,1} = 20$, $\tau_{p_3} = \tau_{dg} + \tau_{gf} + \sigma$

$p_4 = (d, g, c)$, $y_{p_4,0} = 80$, $\tau_{p_4} = \tau_{dg} + \tau_{gc}$

Here: same travel times as in DLC

Direct Connection Model (DC)

(B., Karbstein [2012])

Variables:

$x_{\ell,f} \in \{0,1\}$ choose line ℓ with frequency f

$y_{p,0} \in \mathbb{R}_+$ passenger flow on a direct connection path

$y_{p,1} \in \mathbb{R}_+$ passenger flow on a path with at least one transfer

Objective:

$$\min \lambda \sum_{\ell \in \mathcal{L}} \sum_{f \in \mathcal{F}} c_{\ell,f}\, x_{\ell,f} + (1-\lambda) \left(\sum_{p \in \mathcal{P}^{0+}} \tau_{p,0}\, y_{p,0} + \sum_{p \in \mathcal{P}} \tau_{p,1}\, y_{p,1} \right)$$

transport all passengers

$$\sum_{p \in \mathcal{P}^{0+}_{st}} y_{p,0} + \sum_{p \in \mathcal{P}_{st}} y_{p,1} = d_{st} \qquad \forall\, (s,t) \in D$$

capacity constraints

$$\sum_{p \in \mathcal{P}^{0+} : a \in p} y_{p,0} + \sum_{p \in \mathcal{P} : a \in p} y_{p,1} \leqslant \sum_{\ell \in \mathcal{L} : e(a) \in \ell} \sum_{f \in \mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \qquad \forall\, a \in A$$

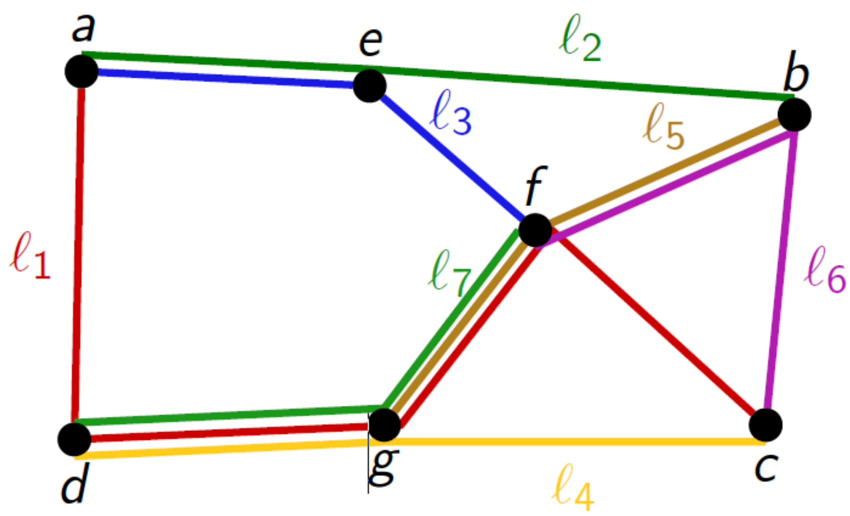
direct connection (capacity) constraints

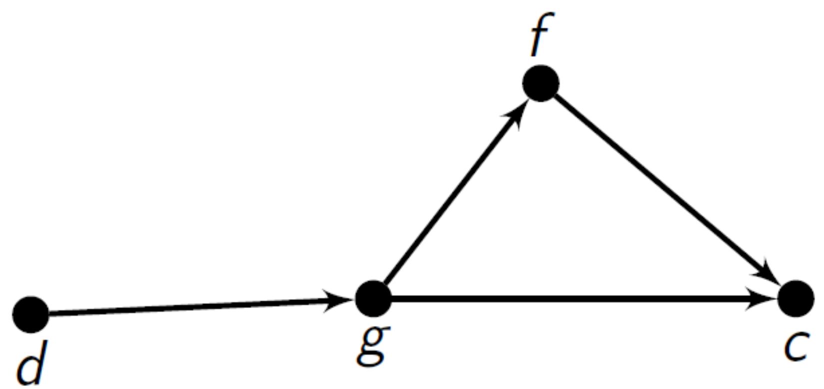
$$\sum_{(u,v) \in [s,t]_a^{\leqslant}} \sum_{p \in \mathcal{P}^{0+}_{uv}(a)} y_{p,0} \leqslant \sum_{\ell \in \mathcal{L}^0_{st}(a)} \sum_{f \in \mathcal{F}} \kappa_{\ell,f}\, x_{\ell,f} \qquad \forall\, a \in A, [s,t]_a \in D(a)$$

one frequency per line

$$\sum_{f \in \mathcal{F}} x_{\ell,f} \leqslant 1 \qquad \forall\, \ell \in \mathcal{L}$$

Relaxation II – DC Path Pricing





dc-passenger routing digraph

Problem: How to identify direct connection paths, i.e., what is $\mathcal{P}^{0+}$?

Idea: For OD pair (s,t) consider *direct connection st-passenger routing digraph* G_{st} induced by all direct connection lines for (s,t)

$\mathcal{P}^{0+}_{st}$ = set of all paths in G_{st}, $\mathcal{P}^{0+} = \bigcup_{s,t} \mathcal{P}^{0+}_{st}$

Relaxation II – DC Path Pricing

Proposition

- The pricing problem for the passenger paths variables $y_{p,0}$ for OD pair (s,t) is a shortest path problem in G_{st}.
- G_{st} can be constructed in polynomial time.
- The pricing problem for the passenger paths variables $y_{p,1}$ is a shortest path problem in the passenger routing graph induced by all lines.

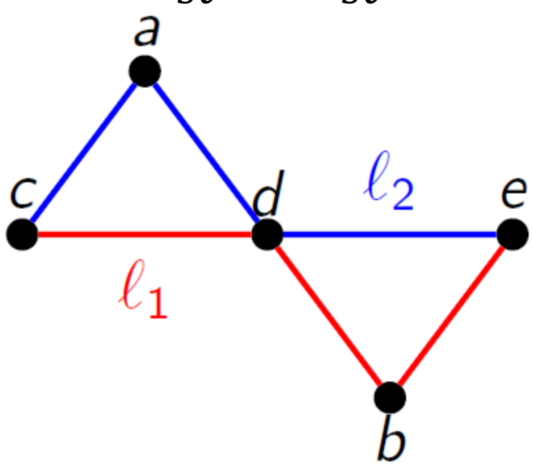
Problem: How to identify direct connection paths, i.e., what is $\mathcal{P}^{0+}$?

Idea: For OD pair (s,t) consider *direct connection st-passenger routing digraph* G_{st} induced by all direct connection lines for (s,t)

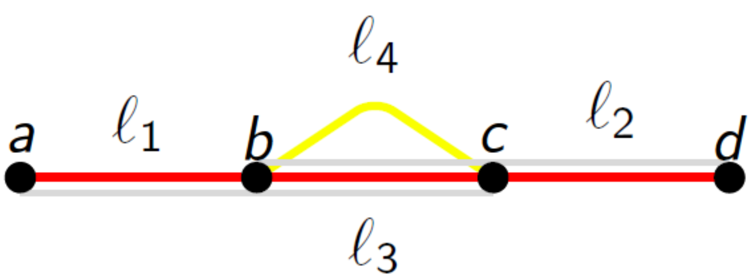
$\mathcal{P}^{0+}_{st}$ = set of all paths in G_{st}, $\mathcal{P}^{0+} = \bigcup_{s,t} \mathcal{P}^{0+}_{st}$

Example

$$\mathcal{P}^0_{st} \subseteq \mathcal{P}^{0+}_{st}$$



dc constraints



- Line capacity = 50
- Demands
  $c \to a = 50, \qquad d \to b = 50$
  $c \to e = 50$
- Path (c, d, e) is considered as a direct connection since it is a path in G_{ce}
- Line capacity = 50
- Demands
  $c \to a = 50, \qquad d \to b = 50$
- DC-constraints on (b, c)
  $y_{abc,0} \leqslant \sum_{f \in \mathcal{F}} \kappa_{\ell_1,f} x_{\ell_1,f} + \kappa_{\ell_3,f} x_{\ell_3,f}$
  $y_{bcd,0} \leqslant \sum_{f \in \mathcal{F}} \kappa_{\ell_1,f} x_{\ell_1,f} + \kappa_{\ell_2,f} x_{\ell_2,f}$
- $y_{abc,0} = y_{bcd,0} = 50$, but either (a, c) or (b, d) must transfer

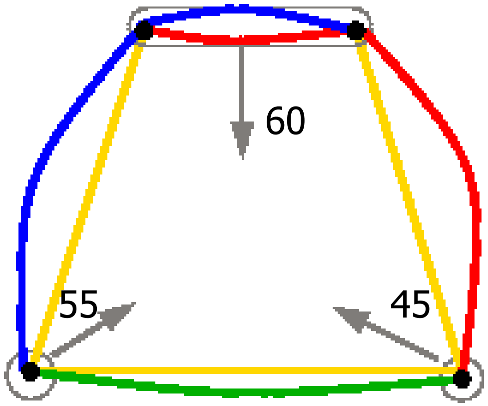
DC Model Discussion

- The direct line connection model (DLC) accounts exactly for the number of direct travelers in a system optimum. It is a first order approximation of model (CG).
- Model (DC) is a relaxation of the projection of model (DLC) onto the space of the direct connection path variables, i.e., (DC) approximates the number of direct travelers.
- The relaxation is due to
  - a larger set of direct connection paths $\mathcal{P}^0_{st} \subseteq \mathcal{P}^{0+}_{st}$,
  - the direct connection constraints, a small, combinatorial set of all projected direct line connection constraints.
- (DC) is algorithmically tractable:
  - passenger paths variables not dependent on lines;
  - pricing passenger path variables is a shortest path problem.
- Model (DC) can be seen as transfer improvement of model (B)

Further Polyhedral Results

Proposition

1. Uncapacitated case (B., G., Pfetsch [2009]): The fundamental classes of Steiner cut and Steiner partition inequalities can be generalized to the Steiner connectivity problem and hence to the line planning problem.
2. Capacitated case (K. [recently]): The fundamental classes of band inequalities and Steiner partition band inequalities can be generalized to the line planning problem.



Example (2): Network with 4 lines

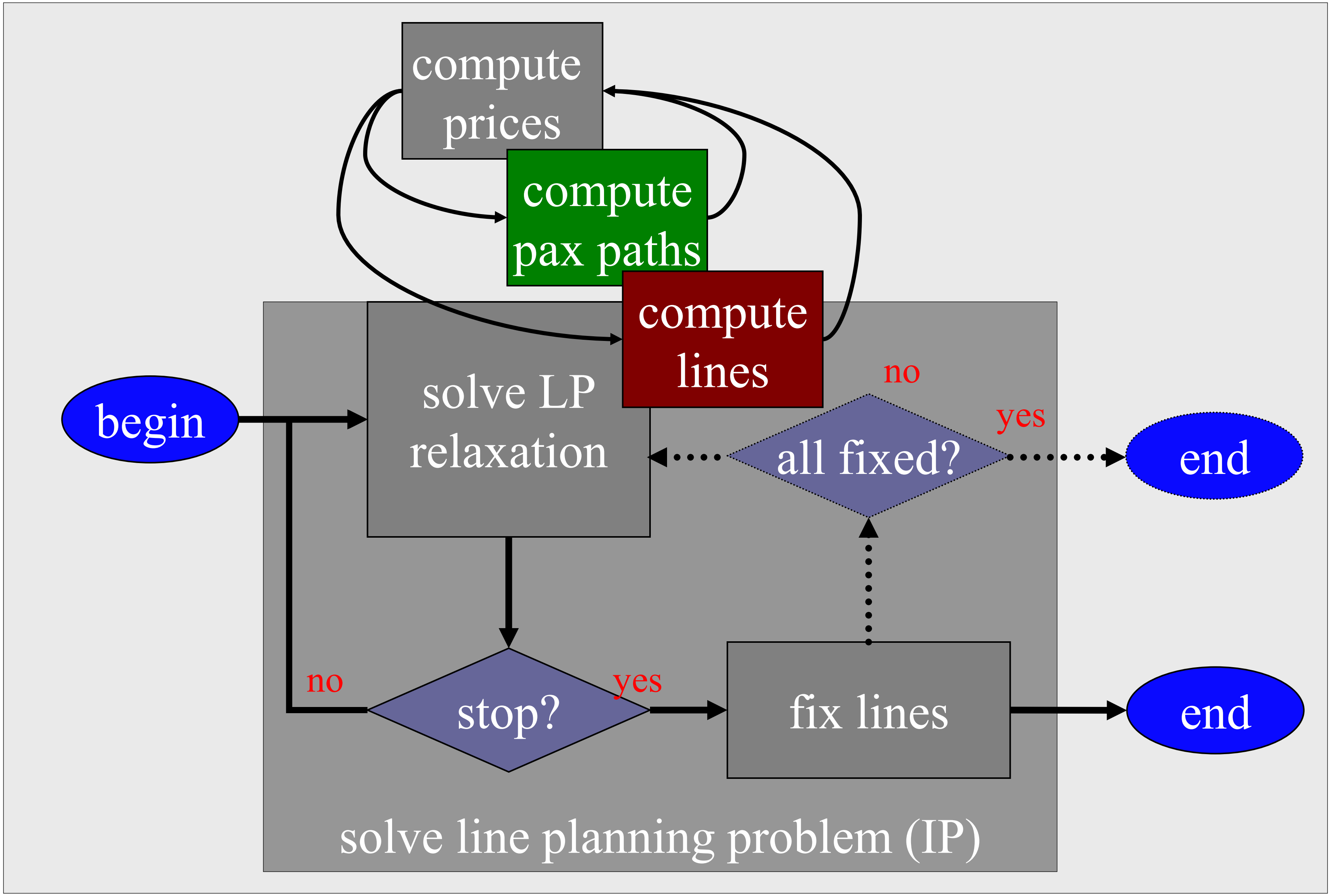
- Capacity $\kappa_\ell = 10$
- Frequency $\mathcal{F} = \{1,2\}$
- Steiner partition band inequality

$$2x_{1,2} + x_{2,2} + x_{3,2} + x_{4,2} \geq 2$$

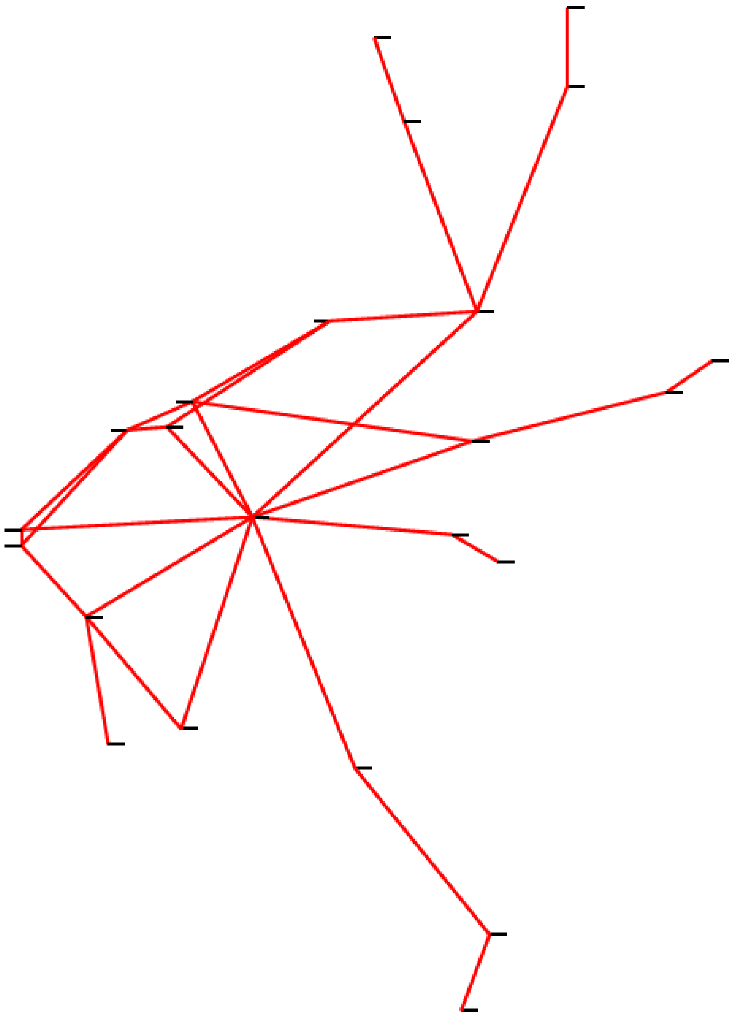
- The yellow or two other lines with frequency 2 have to be chosen.

Public transport network partitioned into 3 components
Numbers give max. number of passengers leaving/entering component

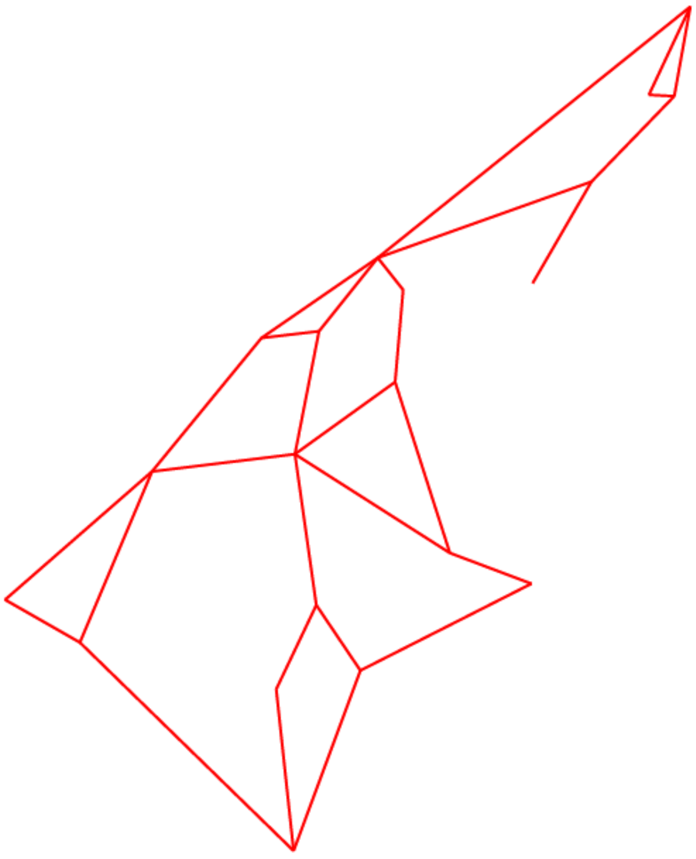
Column Generation Approach



Test Instances

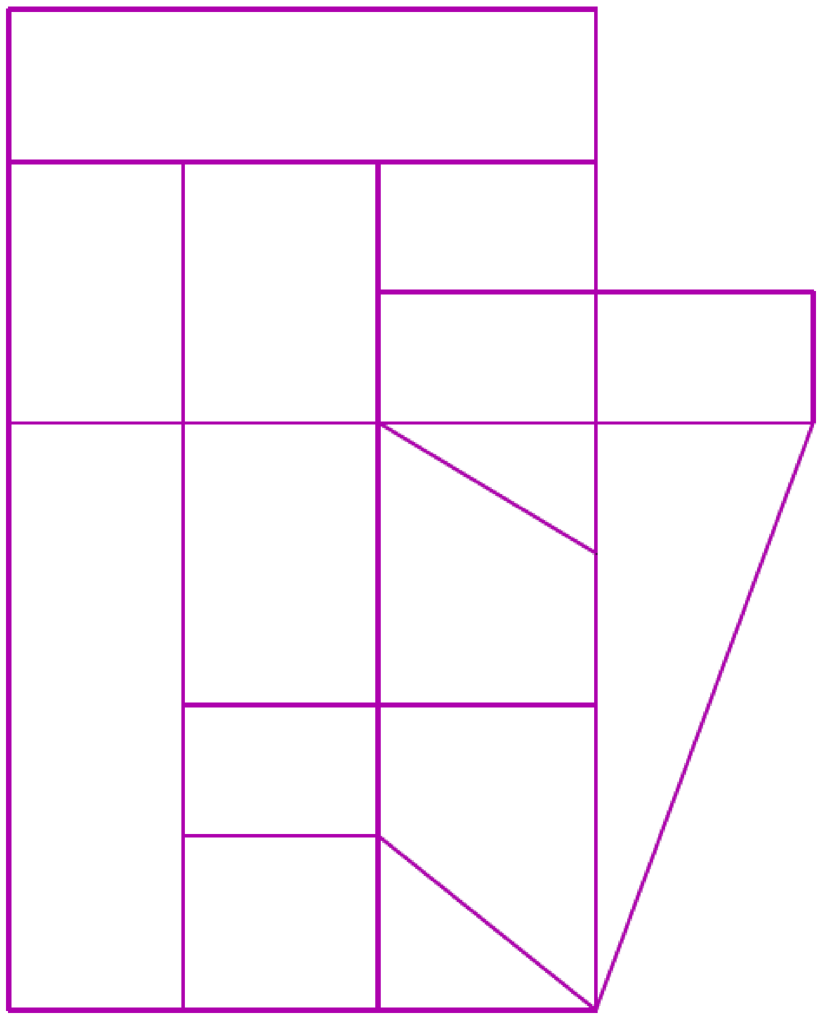


Dutch



China

Test Instances



Sioux Falls

Potsdam

Test Instances

problem	$\lvert D\rvert$	$\lvert V_O\rvert$	$\lvert V\rvert$	$\lvert A\rvert$	$\lvert \mathcal{L}\rvert$	x-vars	dc-cons	cons
Dutch1	420	23	23	106	402	1 608	1 832	1 080
Dutch2	420	23	23	106	2 679	10 716	7 544	3 341
Dutch3	420	23	23	106	7 302	29 208	9 736	7 945
China1	379	20	20	98	474	1 896	2 754	1 178
China2	379	20	20	98	4 871	19 484	8 162	5 457
China3	379	20	20	98	19 355	77 420	12 443	19 931
SiouxFalls1	528	24	24	124	866	3 464	4 400	1 779
SiouxFalls2	528	24	24	124	9 397	37 588	16 844	10 197
SiouxFalls3	528	24	24	124	15 365	61 460	21 220	16 145
Potsdam98a	7 734	107	344	2 746	207	776	3 538	9 970
Potsdam98b	7 734	107	344	2 746	1 907	7 572	60 902	11 991
Potsdam98c	7 734	107	344	2 746	4 342	17 313	76 640	14 366
Potsdam2009	4 443	236	851	5 542	3 433	14 140	30 780	12 006

- Frequencis: 3,6,9,18 ($\triangleq$ cycle time of 60,30,20,10 min in 3 h)
- Could not solve CG root LP for red instances in 10 h

DC vs. B

problem	(DC) time	(DC) nodes	(DC) gap	(B) time	(B) nodes	(B) gap
Dutch1	15s	329	opt.	10h	5 940 327	0.03%
Dutch2	$<$1h	11 532	opt.	10h	815 966	0.04%
Dutch3	10h	57 273	0.05%	10h	151 053	0.08%
China1	10h	814 964	0.32%	10h	3 754 582	0.11%
China2	10h	5 366	0.53%	10h	129 217	0.15%
China3	10h	997	0.47%	10h	37 519	0.18%
SiouxFalls1	10h	458 379	0.10%	$<$1h	347 999	opt.
SiouxFalls2	10h	13 868	0.09%	10h	110 836	0.01%
SiouxFalls3	10h	3 230	0.10%	10h	44 713	0.00%
Potsdam98a	10h	7 357	0.09%	10h	6 266	0.12%
Potsdam98b	10h	62	0.28%	10h	2 491	0.26%
Potsdam98c	10h	10	0.27%	10h	661	0.25%
Potsdam2010	10h	2	0.81%	10h	2123	0.41%

- Line costs proportional to line lengths plus fixed cost
- Objective weighing parameter $\lambda = 0.8$, transfer penalty $\sigma = 15$ min

Verifying Direct Travelers

problem	travel time	cost	obj.	dir. trav.[1]	dir. trav.[2]
Dutch1 (DC)	$1.279 \cdot 10^7$	68 900	2613305	179 496	179 496
Dutch1 (B)	$1.333 \cdot 10^7$	57 800	2711770	183 582	148 924
Dutch2 (DC)	$1.279 \cdot 10^7$	66 900	2612122	180 484	179 384
Dutch2 (B)	$1.319 \cdot 10^7$	57 500	2683071	183 582	156 251
Dutch3 (DC)	$1.279 \cdot 10^7$	66 900	2612122	180 484	179 384
Dutch3 (B)	$1.319 \cdot 10^7$	57 500	2683071	183 582	156 251
China1 (DC)	$1.259 \cdot 10^7$	267 937	2732445	749 736	716 040
China1 (B)	$1.559 \cdot 10^7$	233 268	3304432	759 950	509 526
China2 (DC)	$1.258 \cdot 10^7$	247 241	2714438	759 936	709 145
China2 (B)	$1.559 \cdot 10^7$	233 268	3304432	759 950	509 526
China3 (DC)	$1.245 \cdot 10^7$	244 361	2684860	759 950	714 728
China3 (B)	$1.559 \cdot 10^7$	233 268	3304432	759 950	509 526

[1] direct travelers computed by model

[2] direct travelers computed a posteriori at system optimum

Verifying Direct Travelers

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problem	travel time	cost	obj.	dir. trav.	dir. trav.
SiouxFalls1 (DC)	$3.267 \cdot 10^6$	9 205	660675	360 600	358 888
SiouxFalls1 (B)	$3.633 \cdot 10^6$	8 295	733288	360 600	335 355
SiouxFalls2 (DC)	$3.392 \cdot 10^6$	5 787	682996	360 600	360 178
SiouxFalls2 (B)	$3.776 \cdot 10^6$	5 178	759365	360 600	326 625
SiouxFalls3 (DC)	$3.431 \cdot 10^6$	4 899	690200	360 600	355 068
SiouxFalls3 (B)	$3.695 \cdot 10^6$	4 283	742397	360 600	334 052
Potsdam98a (DC)	$5.076 \cdot 10^6$	27 044	1036865	70 513	71 075
Potsdam98a (B)	$5.102 \cdot 10^6$	29 018	1043617	83 702	68 900
Potsdam98b (DC)	$4.836 \cdot 10^6$	33 484	993938	78 745	79 511
Potsdam98b (B)	$4.970 \cdot 10^6$	28 302	1016610	84 879	73 983
Potsdam98c (DC)	$4.829 \cdot 10^6$	32 544	991772	79 694	79 576
Potsdam98c (B)	$4.952 \cdot 10^6$	29 320	1013779	84 979	74 356
Potsdam2010 (DC)	$1.032 \cdot 10^6$	9 314	213769	38 152	38 001
Potsdam2010 (B)	$1.073 \cdot 10^6$	8 734	221549	41 052	35 285

Verifying Direct Travelers

Deviation of direct travelers from a posteriori system optimum:

DC	B
Max: 7%	Max: 49%
Min: 0%	Min: 7.5%
Avg: 1.9%	Avg: 22.8%

Project Stadt+

Goal

- Rearrange line plan, minimize travel time, no cost increase

Data

- Public transport network including lengths and times of 2009
- Demand estimation (OD data) of 2007
- Vehicle capacity bus 125, tram 170

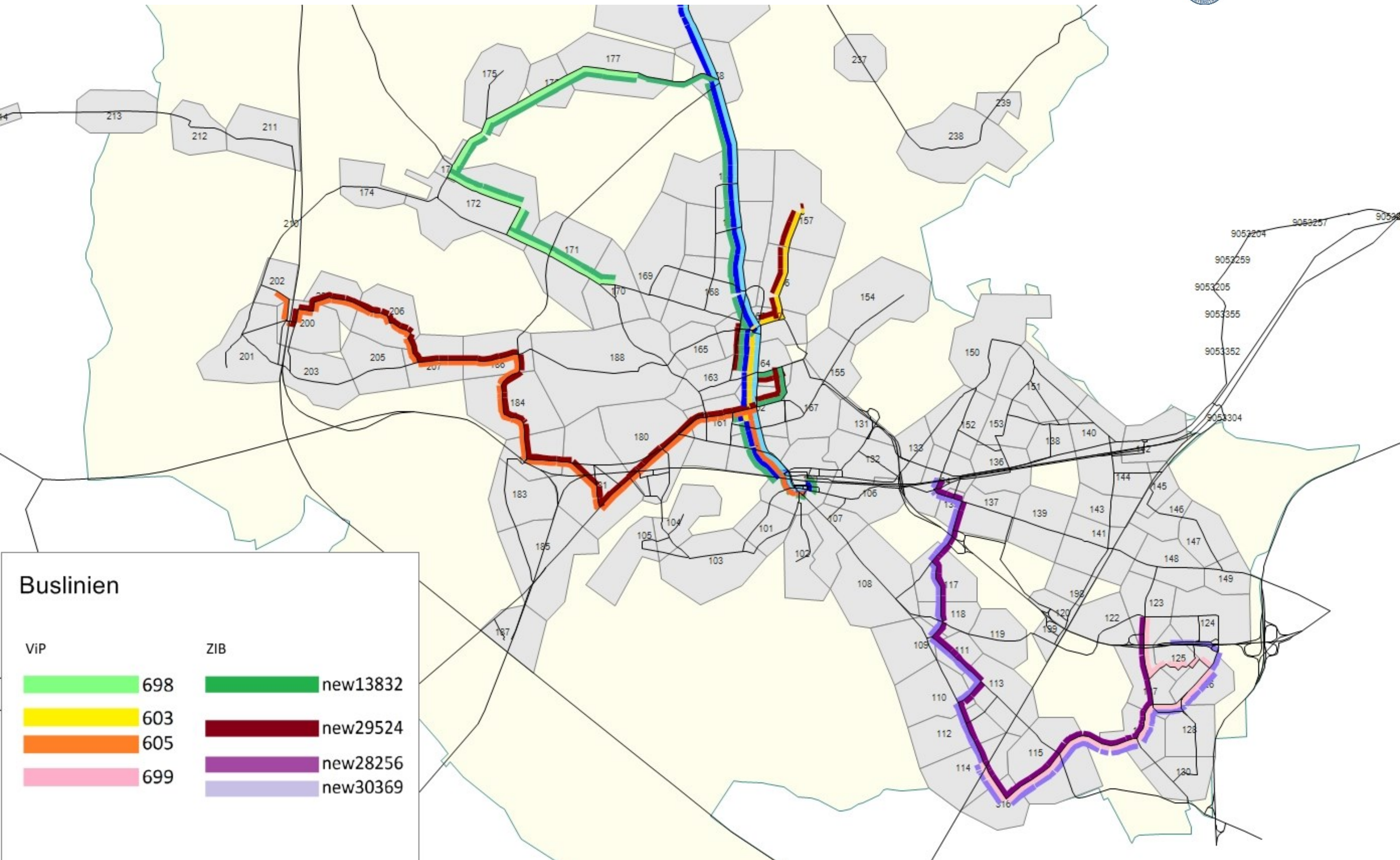
Planning Requirements

- Cycle time of (10,) 20, 30, 60 minutes; tram at least 20 minutes
- Minimum frequency 20, 60 (one line), 10, 30 (two lines) minutes
- Maximum travel time for each line $\leq$ 45 minutes
- Definition of endpoints for lines, important and forbidden stations
- No parallel traffic of bus and tram
- Fixed BVG and HVG lines

Objectives

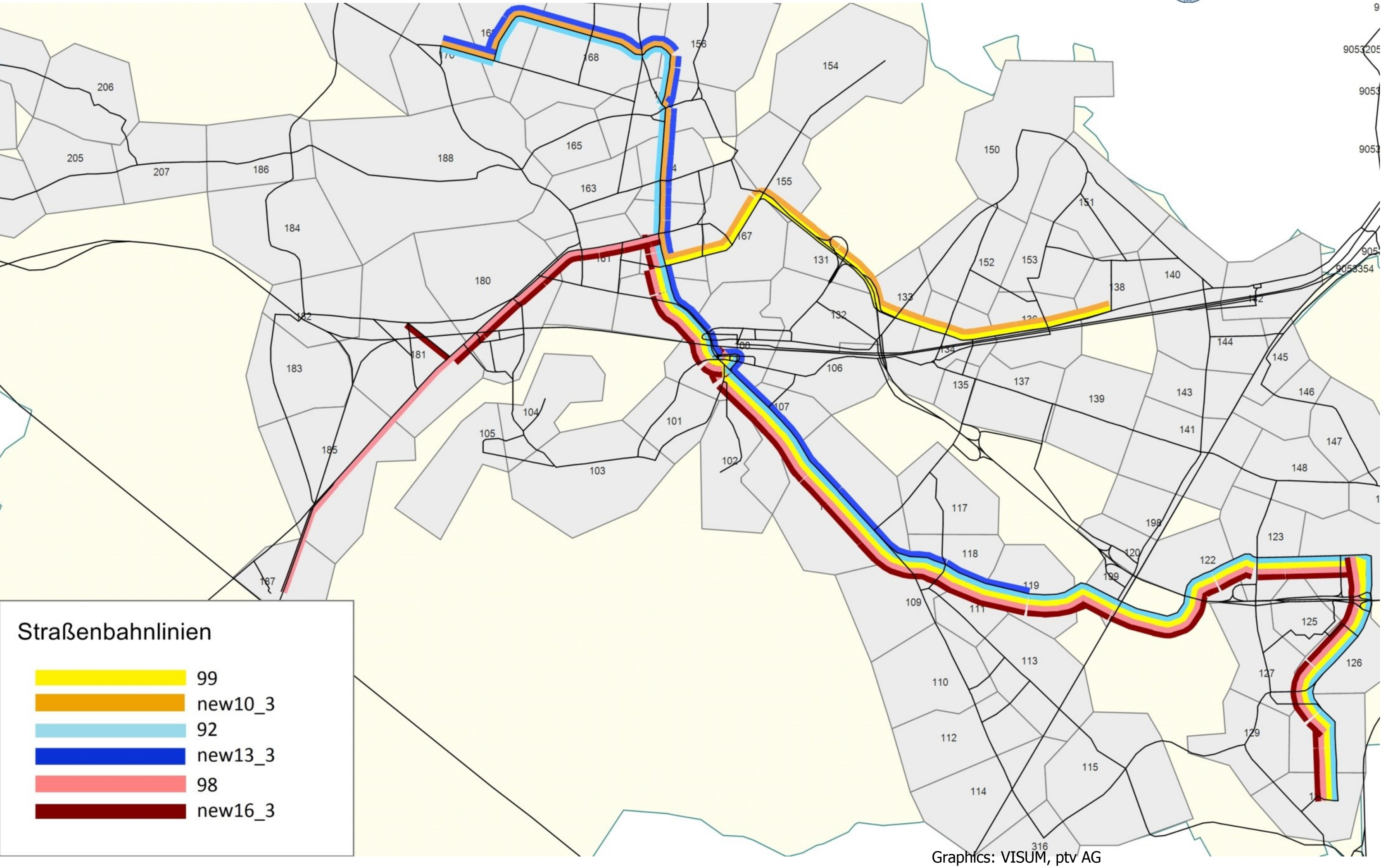
- Cost per kilometer for bus and tram
- Penalty of 15 minutes for each transfer

Bus: ViP vs. ZIB



Graphics: VISUM, ptv AG

Tram: ViP vs. ZIB



Graphics: VISUM, ptv AG

Comparing Cost and Travel Times

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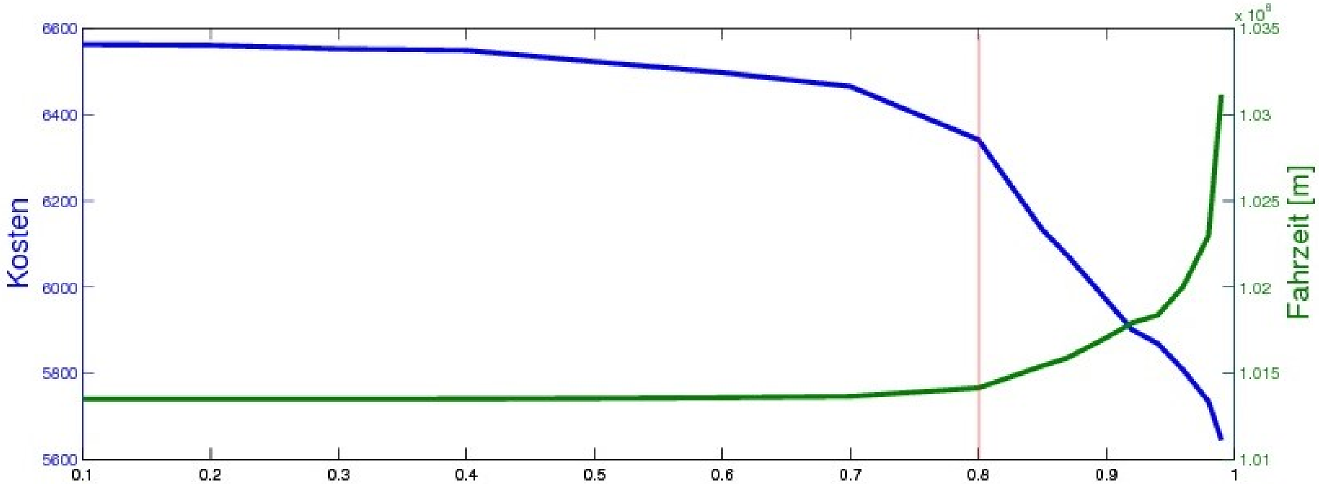
from 6:00 to 9:00 am	*ViP*	*ZIB*	*min cost*	*min time*
bus lines	16	16	17	120
tram lines	7	7	6	18
bus km	2392	2497	1644	10565
tram km	1440	1207	1054	3156
cost	8057	7717	5688[1]	28091[2]
time[3] with transfer penalty[4]	6.2484	6.1927	6.7344	6.0812
time[3] without transfer penalty	5.1641	5.1550	5.2889	5.1422

[1] optimal solution
[2] 0.4% duality gap
[3] in 10^7 sec
[4] transfer penalty 15 min



Comparing Travel Times with VISUM

	ZIB	*ViP*
Average travel time	36 min 3 s	36 min 39 s
Average carriage time	16 min 8 s	17 min 42 s
Average time in vehicle	13 min 8 s	14 min 36 s
Average transfer waiting time	1 in 30 s	1 min 29 s
Average walking time	1 min 38 s	1 min 37 s
Average perceived travel time	26 min	27 min 37 s
Total number transfers	10595	11141
0 transfers	37338	36851
1 transfer	10088	10503
2 transfers	243	306
>2 transfers	7	9

Comparing Transfers and Loads

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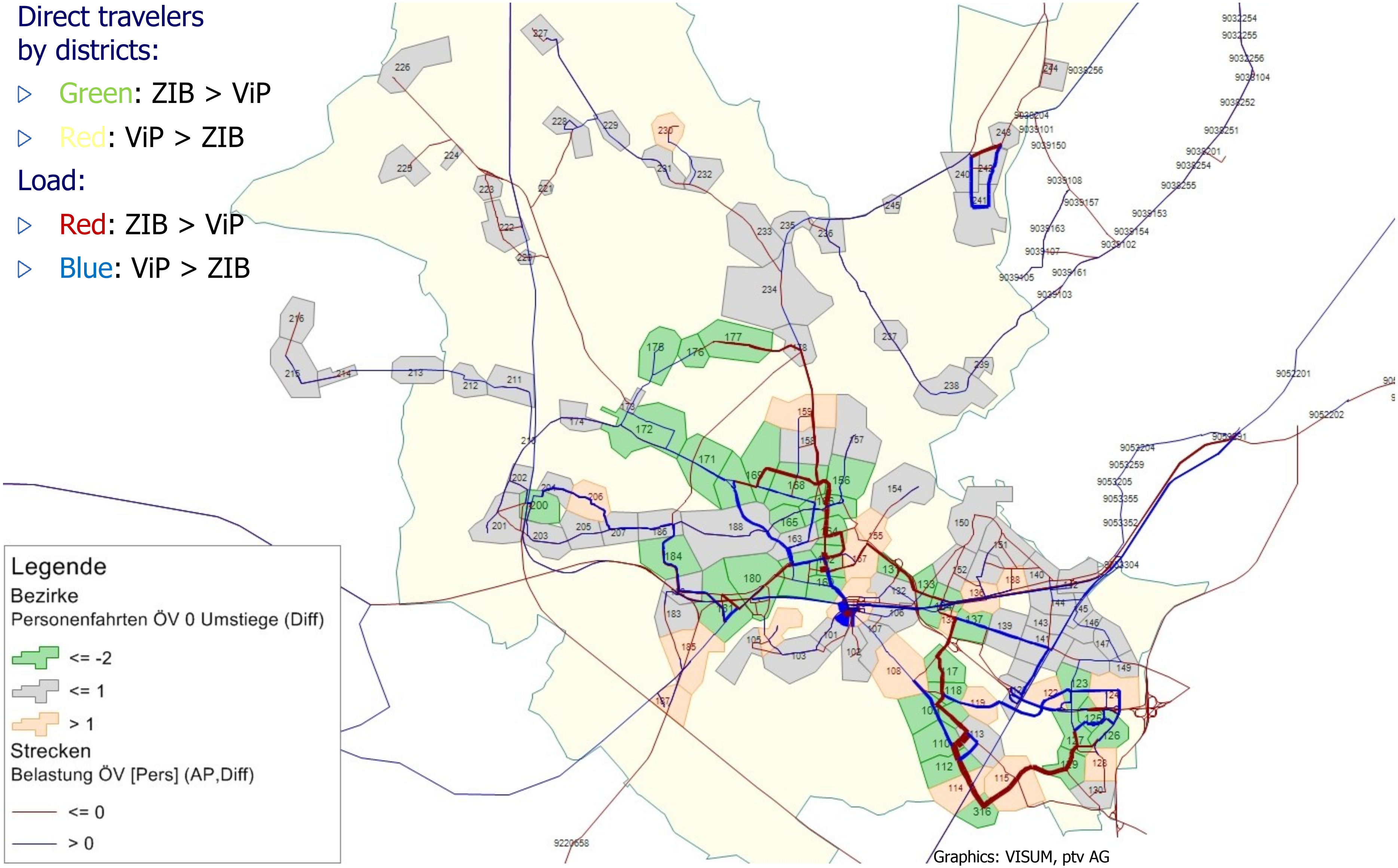
Direct travelers
by districts:

- Green: ZIB > ViP
- Red: ViP > ZIB

Load:

- Red: ZIB > ViP
- Blue: ViP > ZIB



Graphics: VISUM, ptv AG

Conclusion

- The ViP line plan achieves a good compromise between minimizing travel times and costs.
- The ZIB line plan achieve additional improvements by shortening the tram network.
- The guidelines (for means of transportation, minimum service frequencies) strongly influence the result.
- Even if the network is changed only slightly, passenger routes can differ strongly.
- The current practice of data acquisition, computation of statistics, and evaluation of results is unsatisfactory.
- Given suitable data and parameterization, optimization tools can compute line plans which are at least on a par with plans computed by experienced planners.

Line Plan Potsdam 2010

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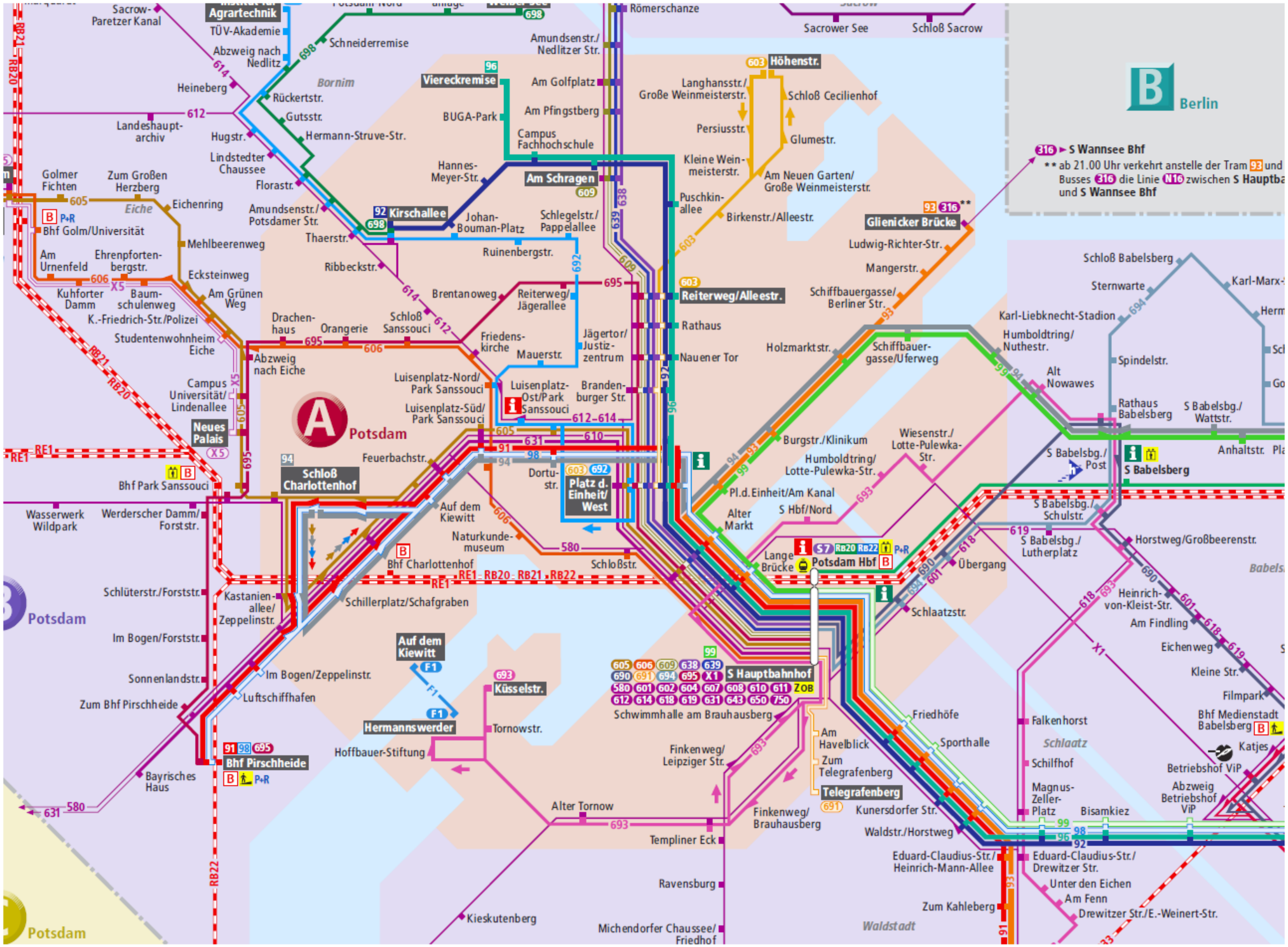

Thank you for your attention

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