Vehicle Rotation Planning and Hyperassignments

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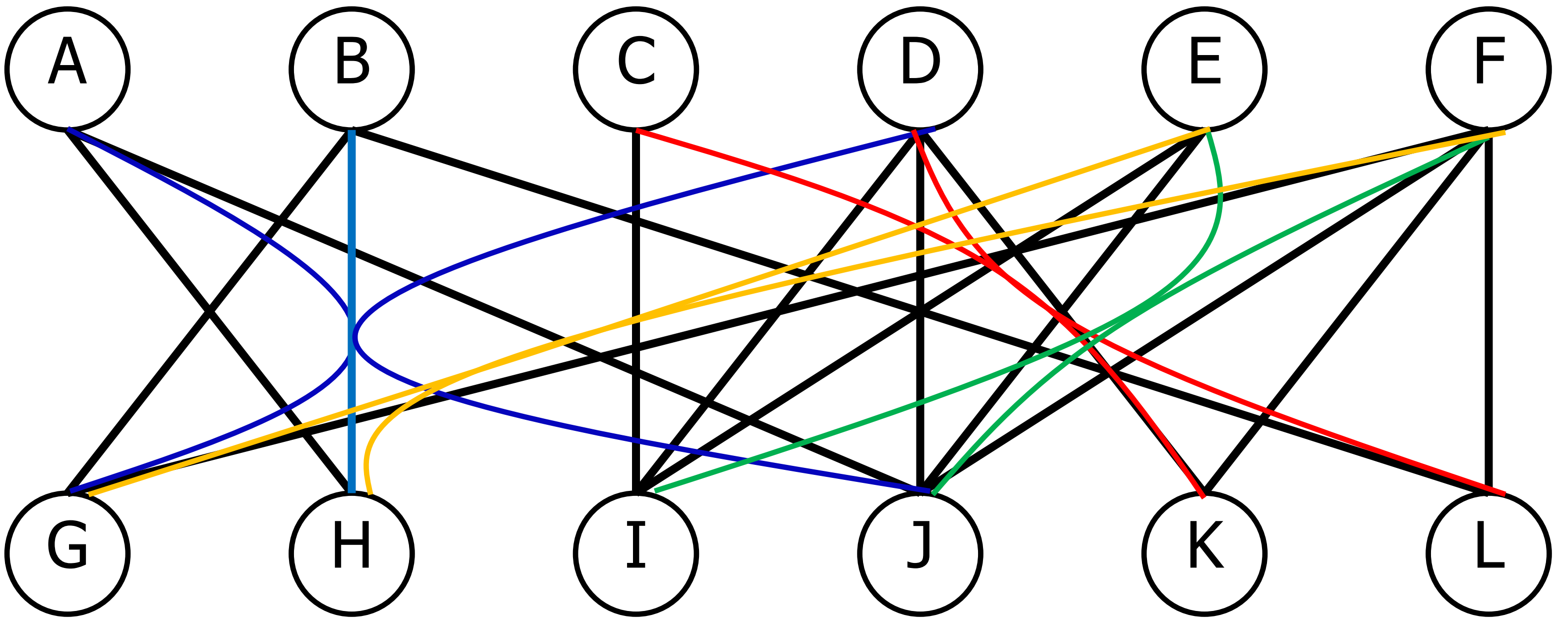
A Set Partitioning Example

What is the minimum cost of covering the letters A–L with a subset of the following sets s.t. no two sets intersect?

{A, B, D, G, H, J},	-100	{D, K},	0.81
{A, H},	0.24	{E, F, G, H},	0.71
{A, J},	0.43	{E, F, I, J},	0.62
{B, G},	0.13	{E, I},	0.04
{B, L},	0.02	{E, J},	0.06
{C, D, K, L},	0.02	{F, G},	0.14
{C, I},	0.19	{F, J},	0.53
{D, I},	0.05	{F, K},	0.08
{D, J},	0.11	{F, L}.	0.04

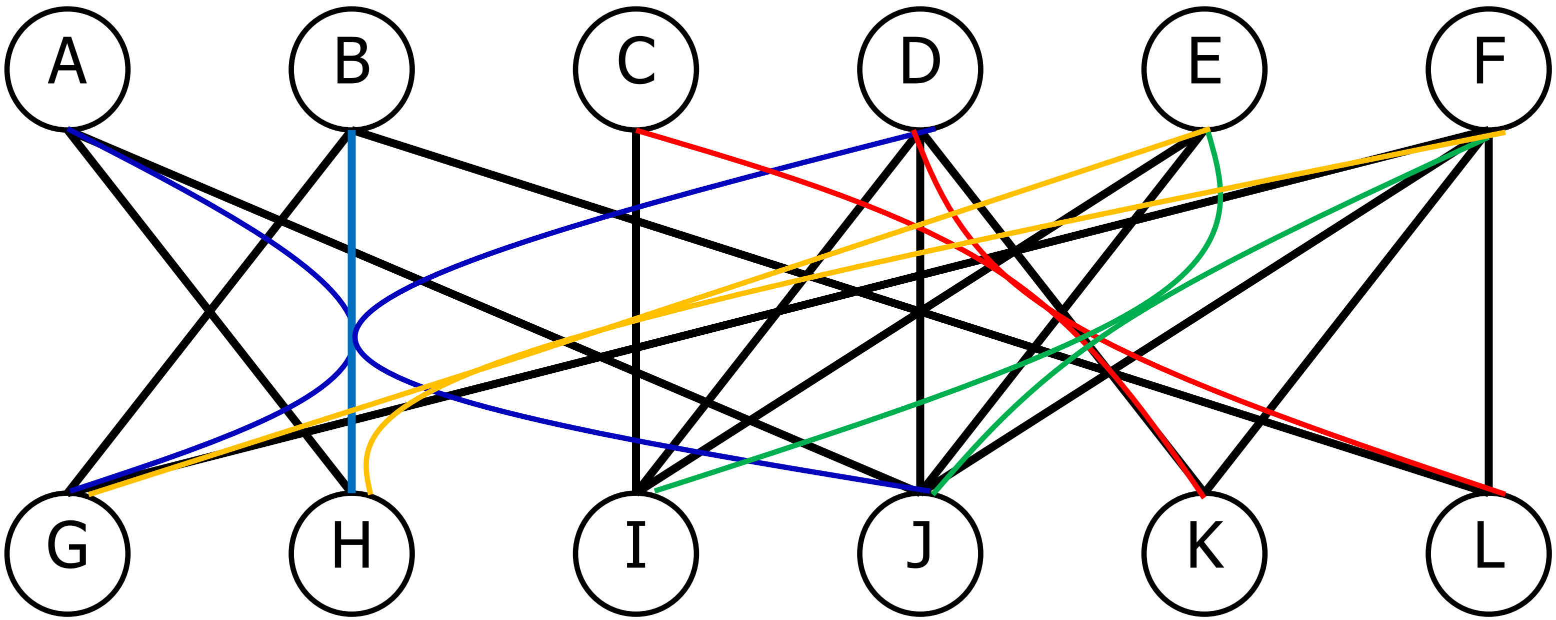
A Set Partitioning Example

{A, B, D, G, H, J}, {A, H}, {A, J}, {B, G}, {B, L}, {C, D, K, L}, {C, I}, {D, I}, {D, J}, {D, K}, {E, F, G, H}, {E, F, I, J}, {E, I}, {E, J}, {F, G}, {F, J}, {F;K}, {F; L}.



A Set Partitioning Example

{A, B, D, G, H, J}, {A, H}, {A, J}, {B, G}, {B, L}, {C, D, K, L}, {C, I}, {D, I}, {D, J}, {D, K}, {E, F, G, H}, {E, F, I, J}, {E, I}, {E, J}, {F, G}, {F, J}, {F;K}, {F; L}. All hyperedges that do not intersect {A, B, D, G, H, J}:



Hall's Theorem implies that there is no solution of cost < 0.

Overview

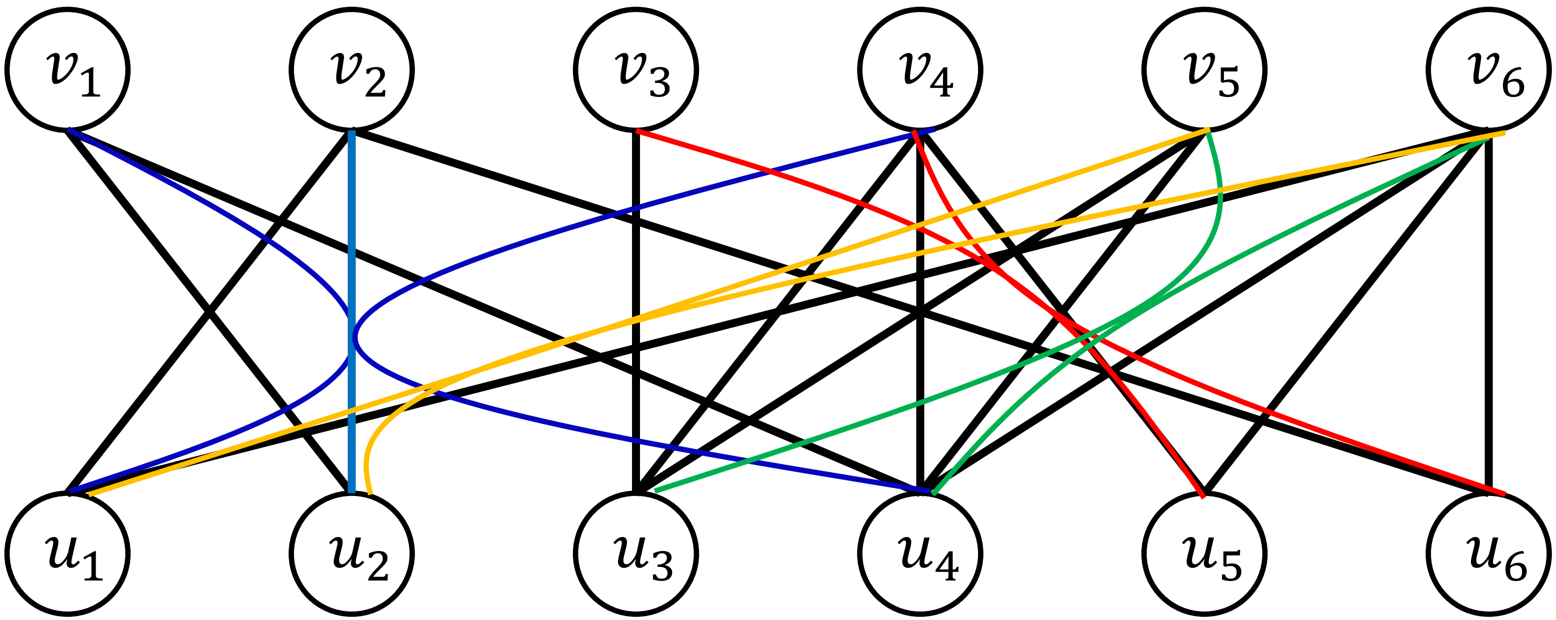
- The Hypergraph Assignment Problem
- Random Hyperassignments
- Complexity Results
- Partitioned Hypergraphs
- Polyhedral Results
- A Local Search Heuristic
- Regular ICE Rotations
- Vehicle Rotation Planning
- Coarse-to-Fine Method

Bipartite Hypergraphs

A hypergraph G is called bipartite if

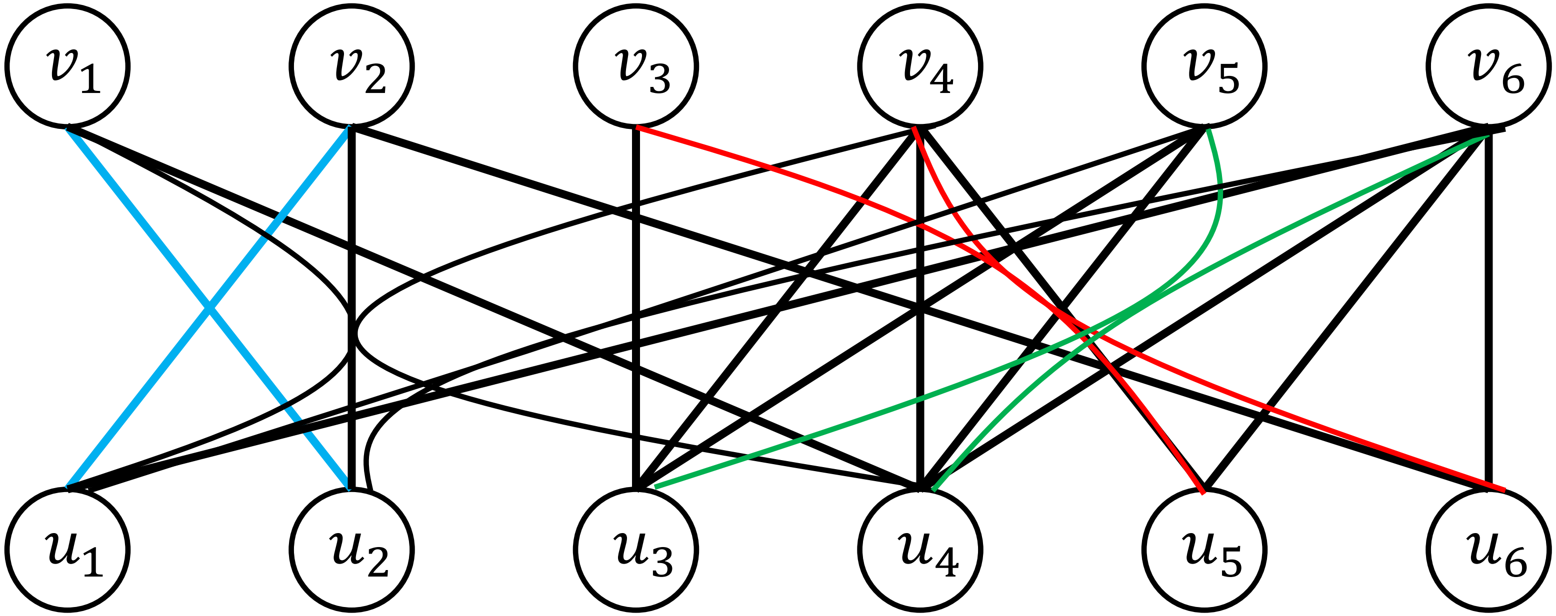
- its vertex set can be written as the disjoint union of two vertex sets U and V with the same size $|U| = |V|$, and
- every hyperedge $e \in E$ has the same number $|e \cap U| = |e \cap V|$ of vertices in U and V.

We then represent G as a triple $G = (U, V, E)$.



Hyperassignments

A hyperassignment is a subset H of E such that there is exactly one incident hyperedge for every vertex.



The Hyperassignment Problem

Freie Universität Berlin

Definition (Hyperassignment Problem)

Input: A bipartite hypergraph $G = (U, V, E)$ with edge costs $c_e \in \mathbb{R}$.
Output: A minimum cost hyperassignment H^* in G, i.e., a hyper-assignment H^* s.t.

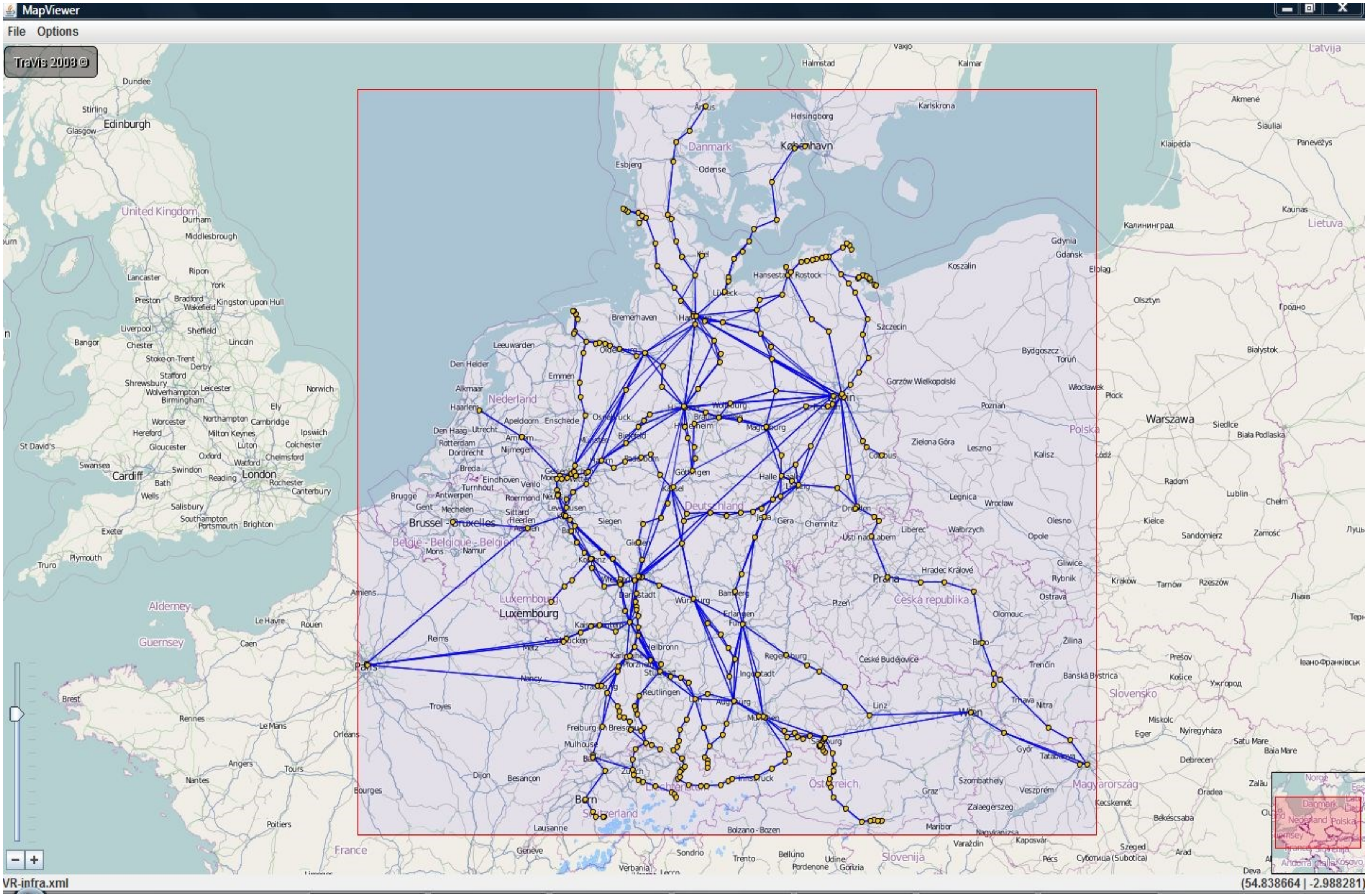
$$c(H^*) = \min\{c(H), H \text{ is a hyperassignment in } G\}$$

or the statement that no hyperassignment exists.

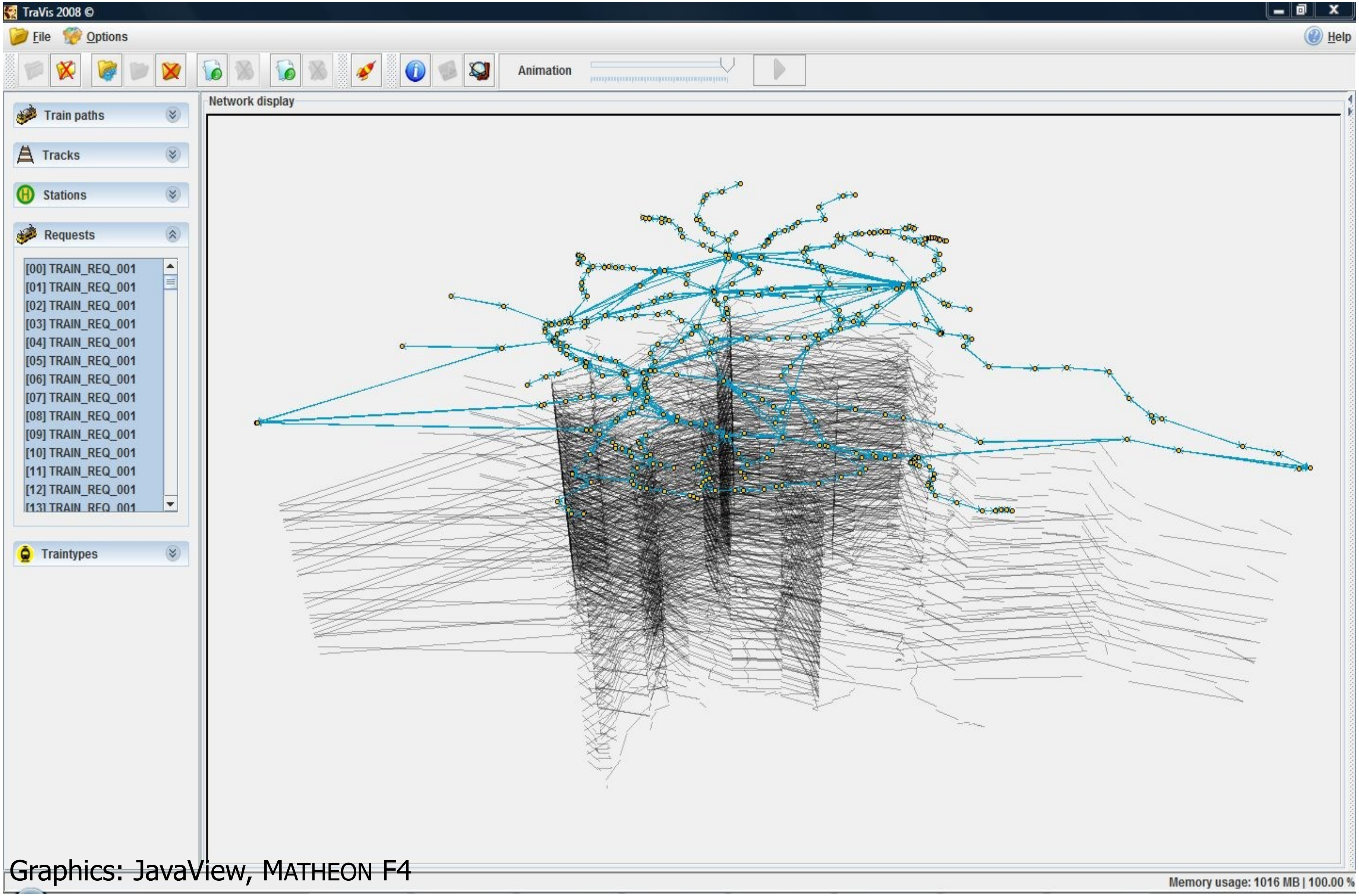
$$\begin{aligned} \min \quad & c^T x & & \\ x(\delta^+(v)) &= 1 & & \forall v \in U \cup V \\ x(\delta^-(v)) &= 1 & & \forall v \in U \cup V \\ x &\in \{0,1\}^E & & \end{aligned}$$

The HAP is a special type of set partitioning problem.

Motivation: ICE Connections

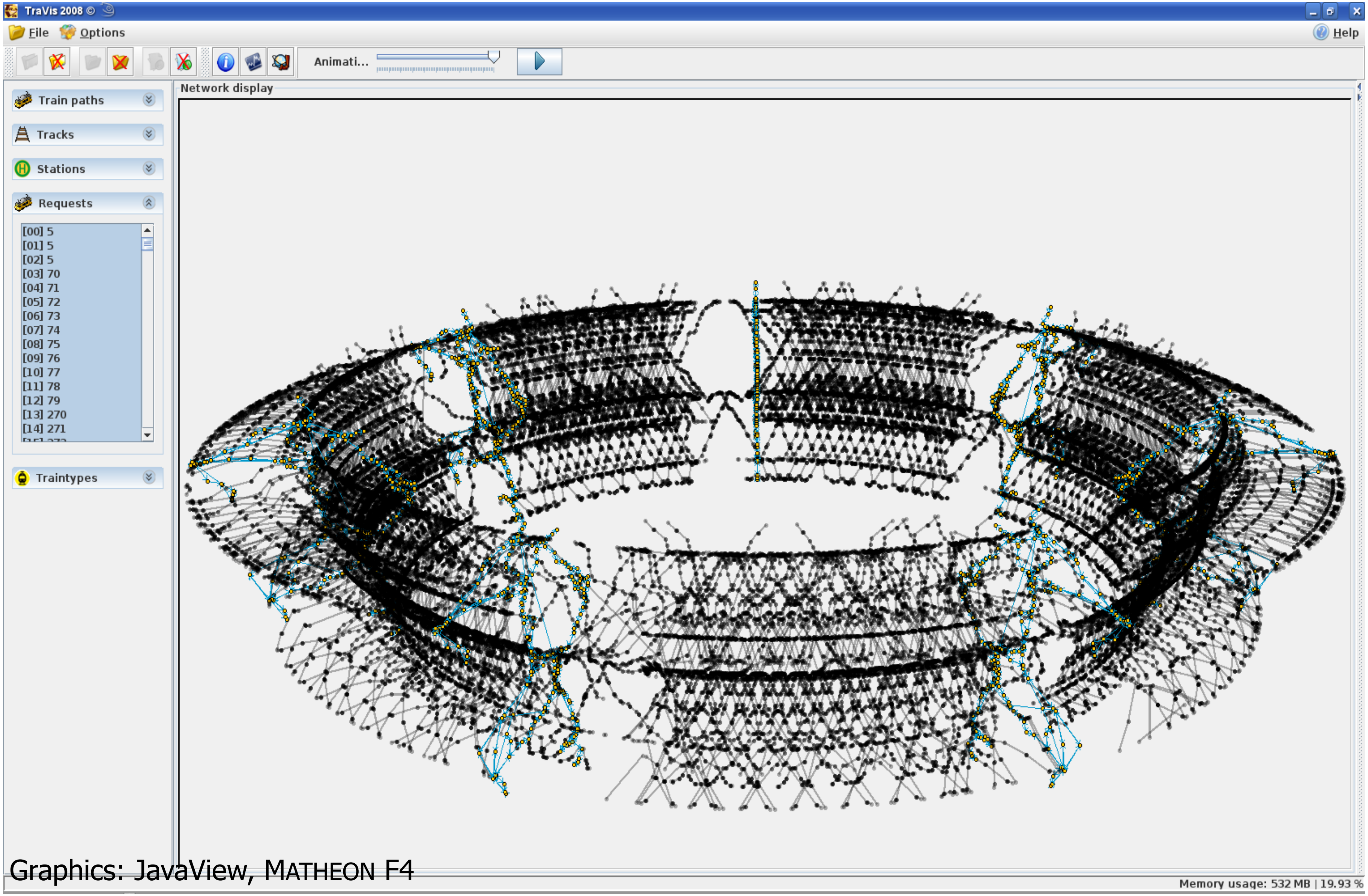


Timetabled Trips: 1 Day



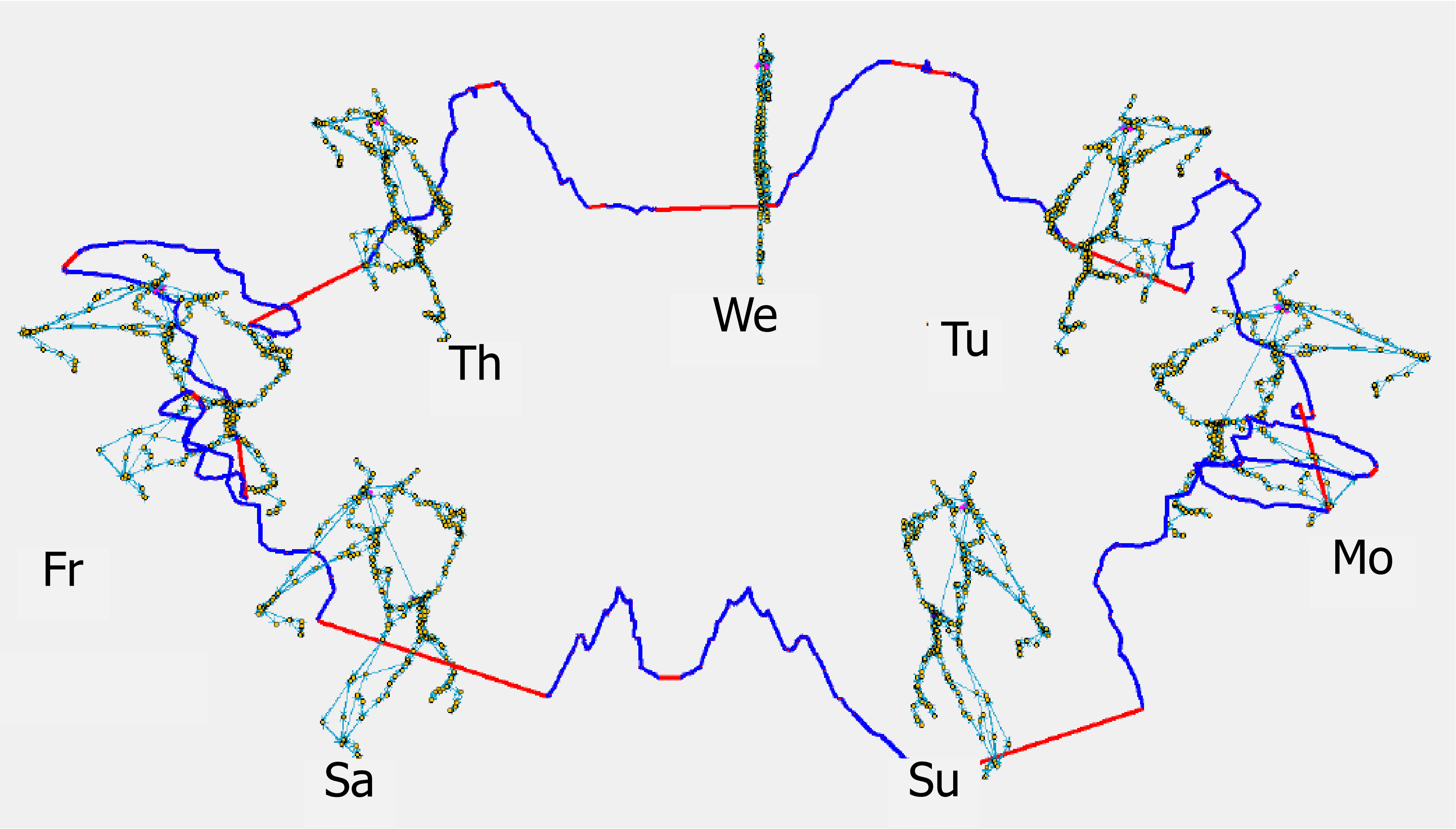
Graphics: JavaView, MATHEON F4

Timetabled Trips: Standard Week



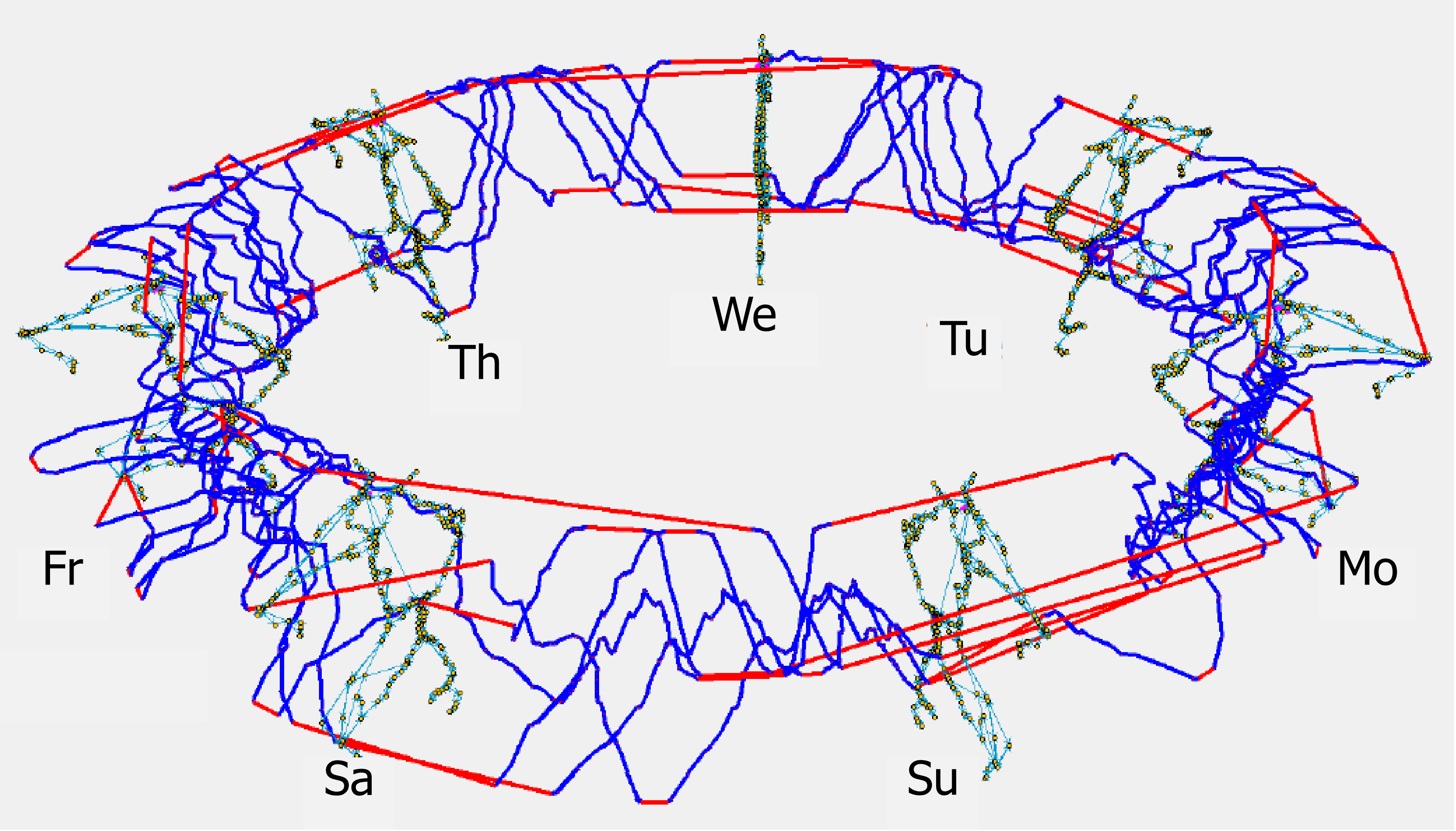
Graphics: JavaView, MATHEON F4

Vehicle Rotation (1 Week)



Graphics: JavaView, MATHEON F4

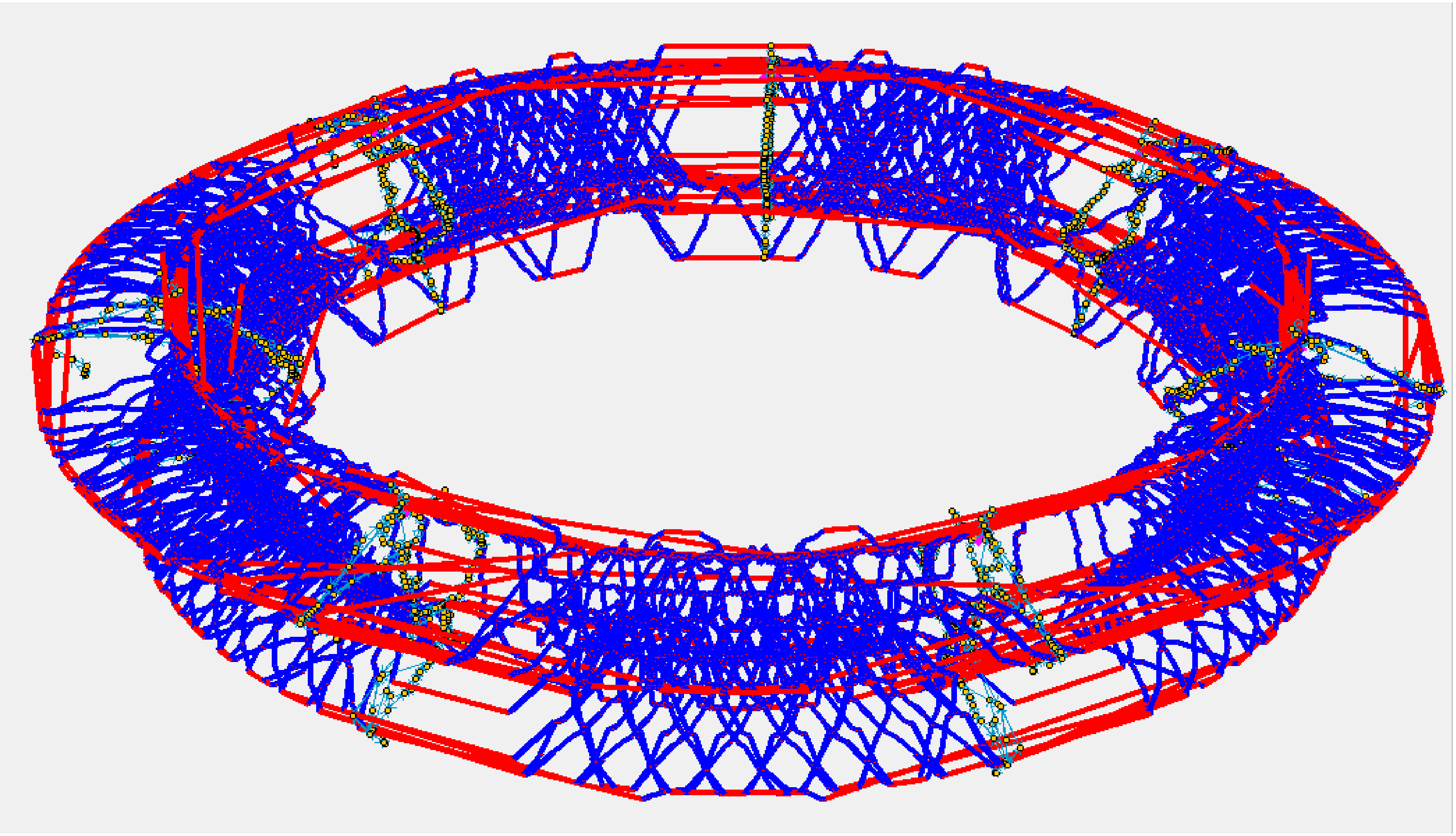
Vehicle Rotation (5 Weeks)



Graphics: JavaView, MATHEON F4

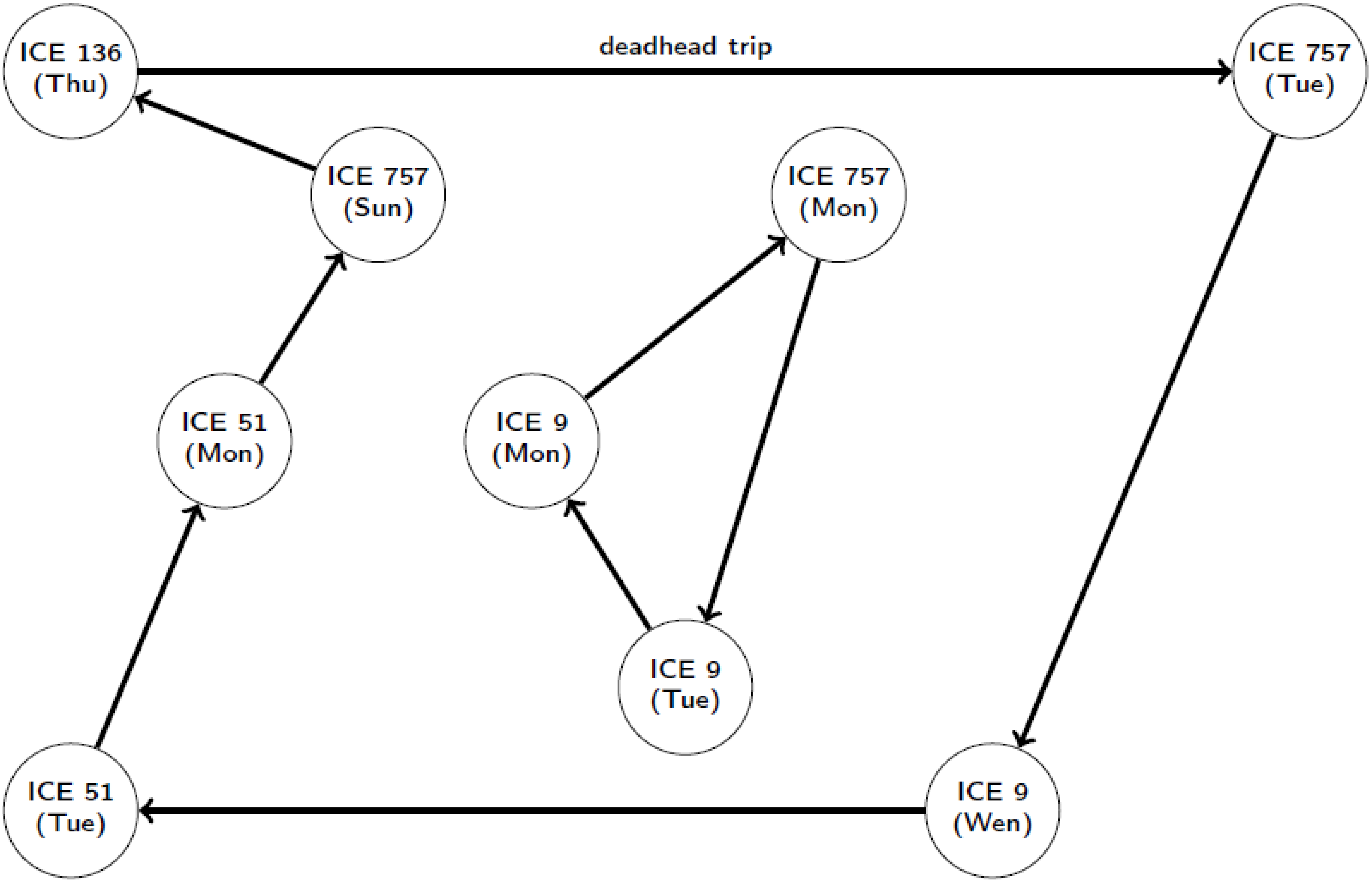
Rotation Plan: Follow-on Trip Assignment

(Blue: Timetabled Trips, Red: Deadhead Trips)

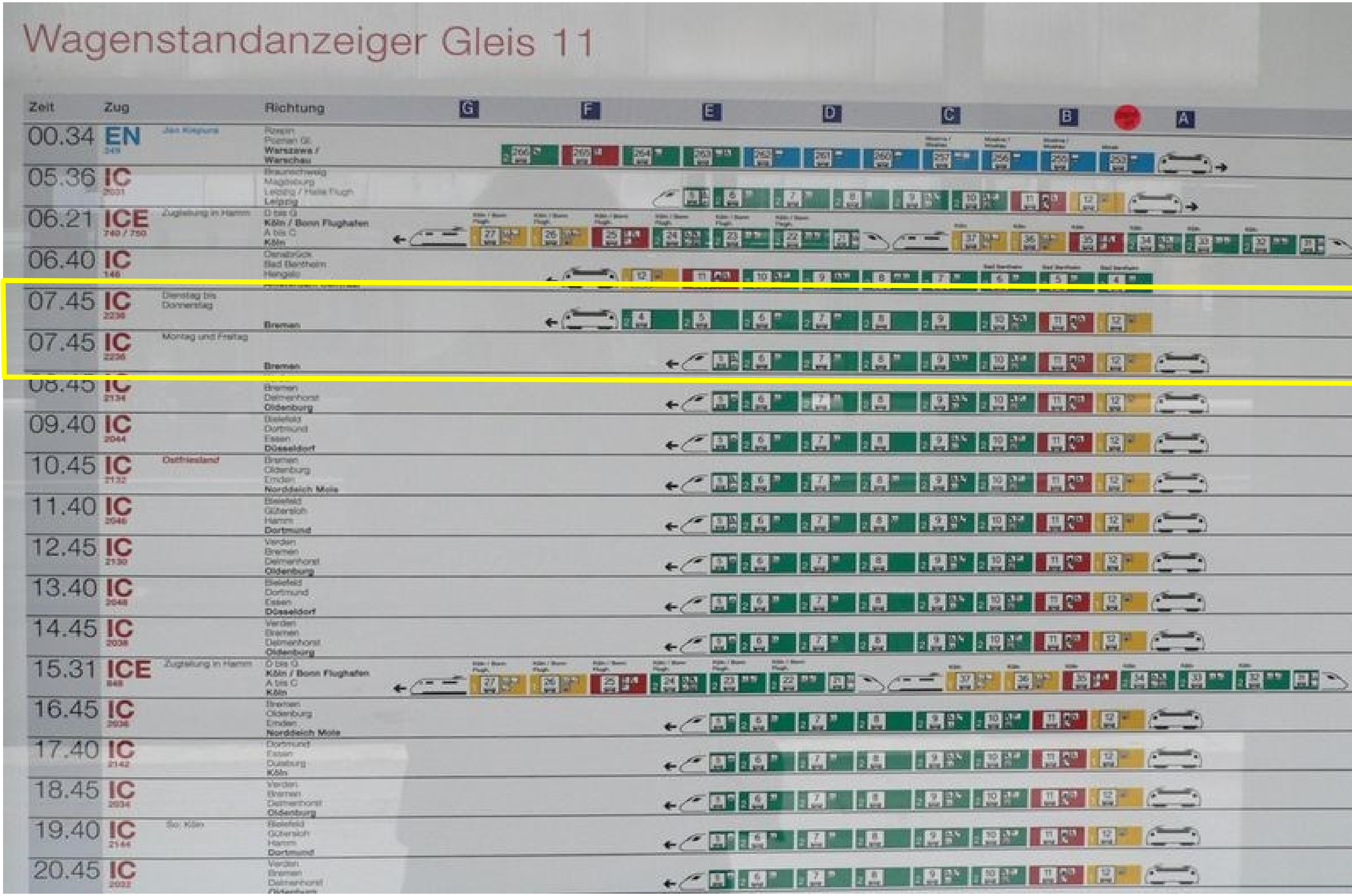


Graphics: JavaView, MATHEON F4

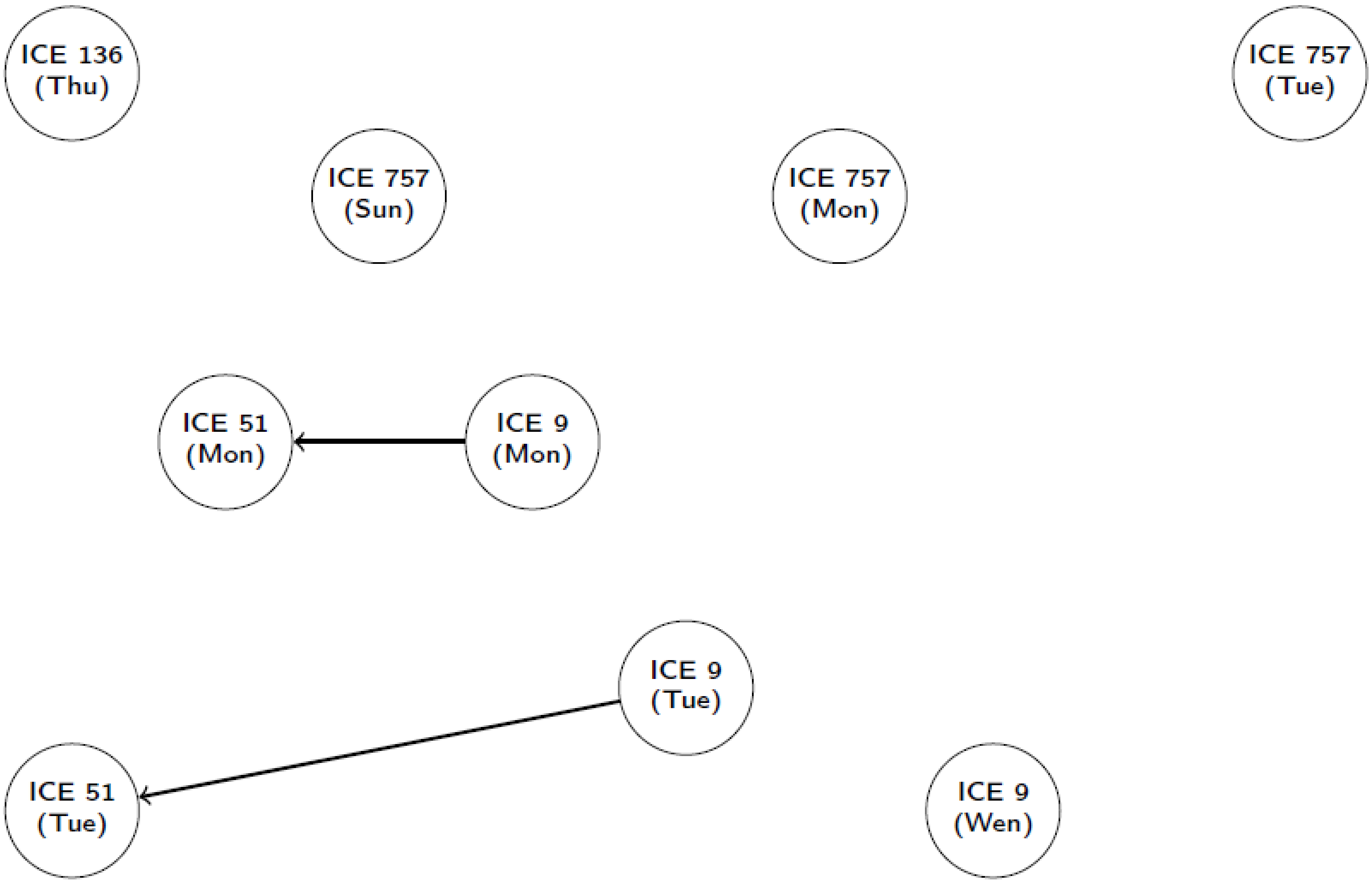
Assignment Solution



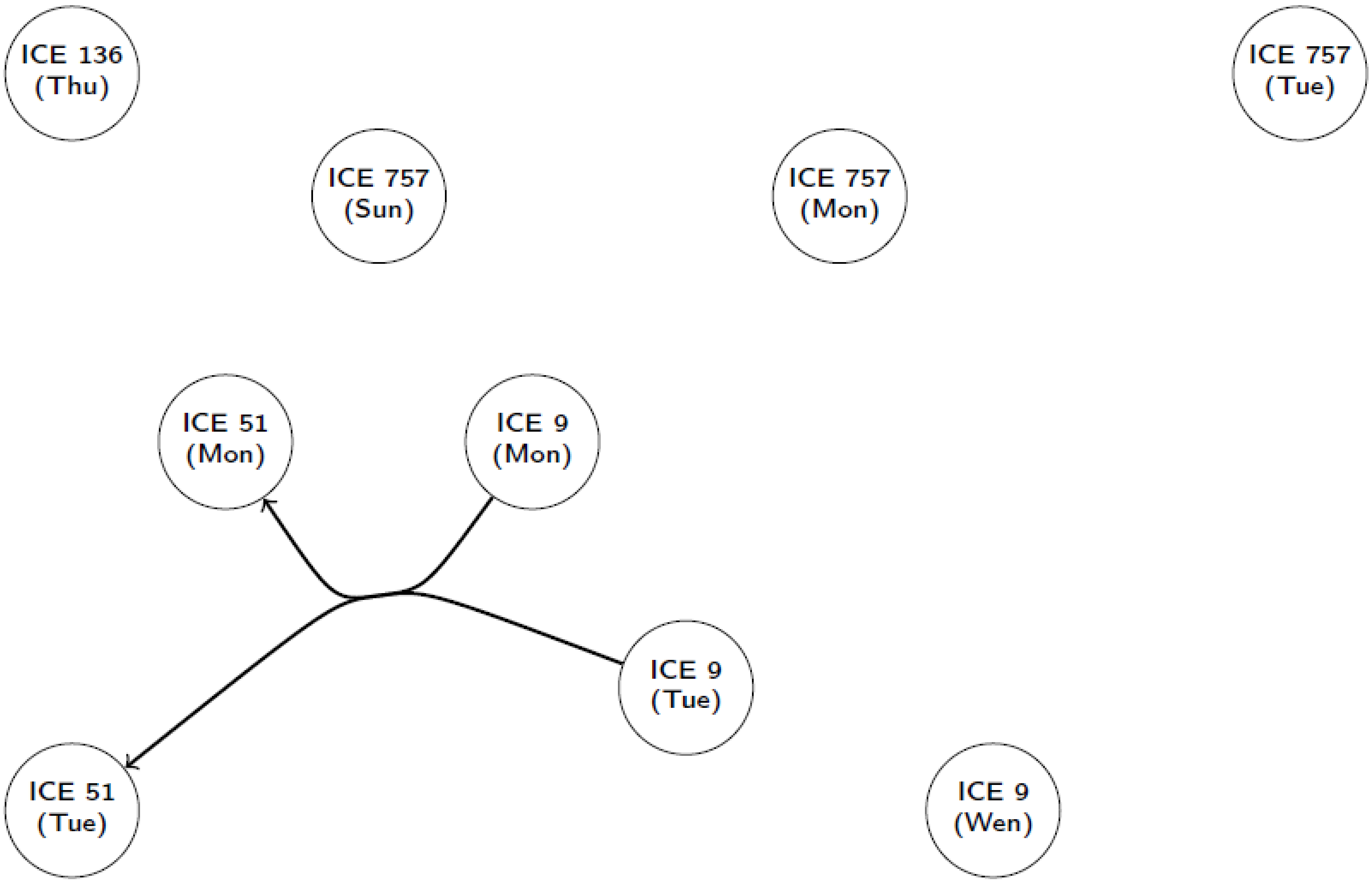
Timetable Regularity



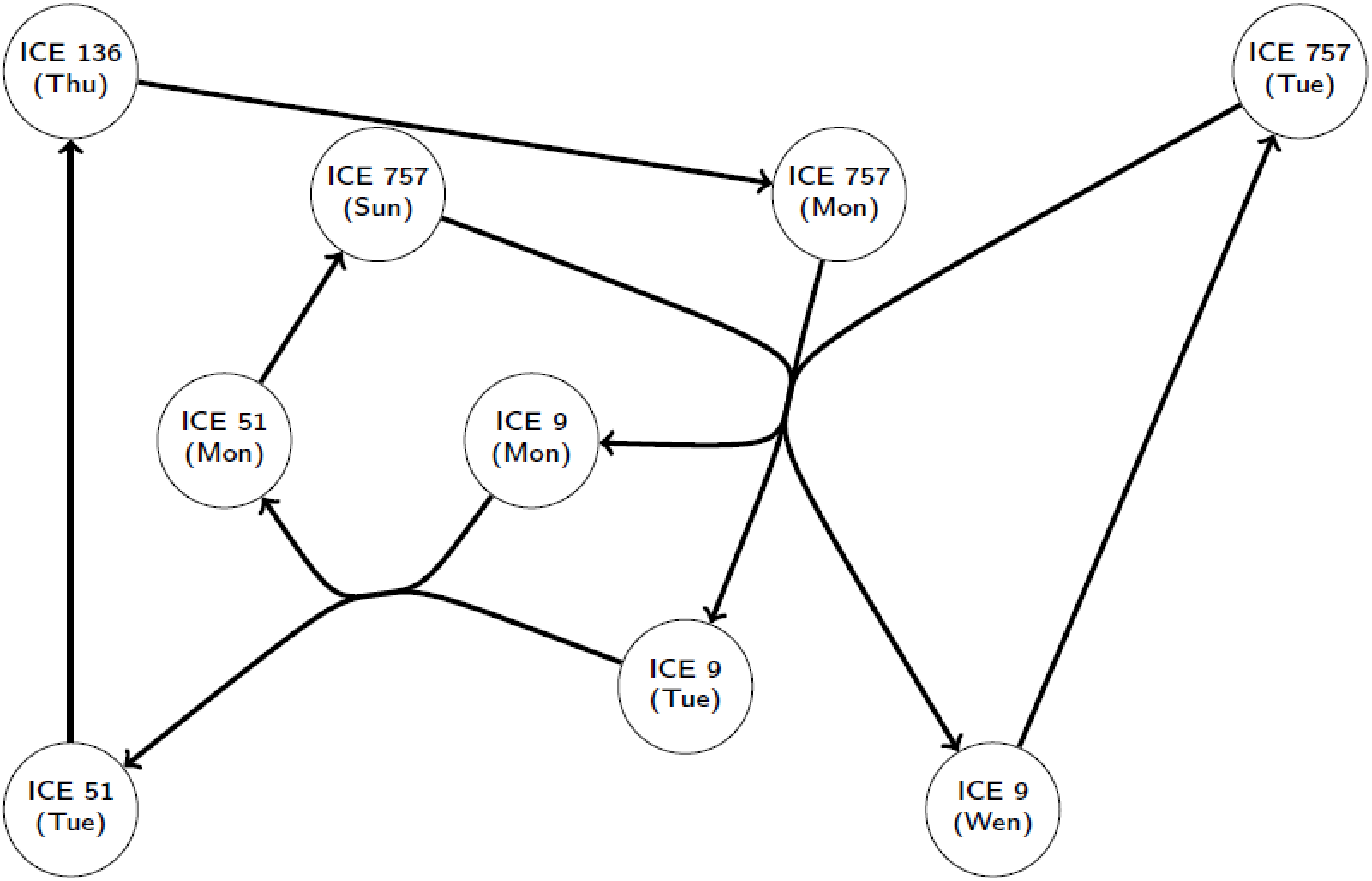
Rotation Regularity



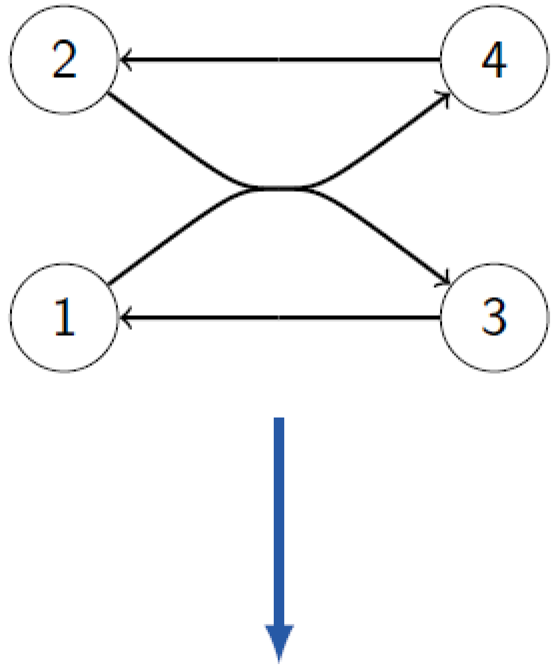
Modeling Regularity via Hyperedges

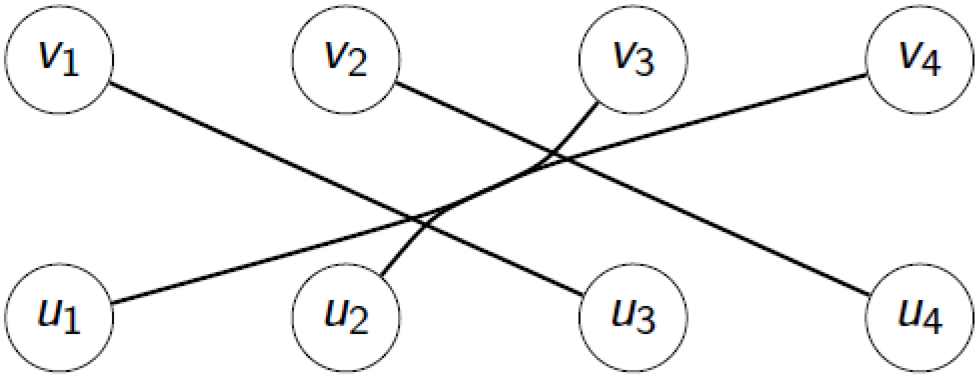


Hyperassignment Solution



Bipartite Hypergraph Model





Random Hypergraph Assignments

Theorem (Mézard, Parisi [1985], Aldous [1992, 2001])

The expected optimal objective value E of the assignment problem for random instances on complete bipartite graphs and uniformly i. i. d. edge costs in [0,1] or exponentially i. i. d. edge costs with mean 1 converges to

$$\mathrm{E} = \frac{\pi^2}{6} = 1.6449 \ldots$$

if the number of vertices tends to infinity.

Random Hypergraph Assignment Problems

(1 000 runs with exponentially i. i. d. edge costs with mean 1)

$G_{2,n}$	E	σ(E)	# e: |e|>2	σ(#e: |e|>2)
10	1.019	0.206	5.3	2.0
20	1.039	0.141	10.4	2.8
30	1.049	0.117	15.3	3.4
40	1.045	0.097	20.5	3.9
50	1.054	0.085	25.4	4.3
60	1.050	0.080	30.6	4.7
70	1.053	0.079	35.6	5.1
80	1.054	0.069	40.6	5.4
90	1.053	0.066	45.9	5.8
100	1.057	0.063	50.6	6.3
110	1.054	0.060	56.1	6.4
120	1.052	0.056	61.1	6.7
130	1.054	0.053	66.3	6.9
140	1.053	0.051	71.3	7.1
150	1.051	0.050	76.2	7.5
160	1.054	0.048	81.2	7.6

Random Hypergraph Assignments

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$$\mathrm{E} = \frac{\pi^2}{6} = 1.6449\ \ldots$$

if the number of vertices tends to infinity.

Theorem (B., Heismann [2014])

The expected optimal objective value E of the hyperassignment problem for random instances on complete bipartite hypergraphs $H_{2,n}$ with exactly n proper hyperedges and exponentially i. i. d. edge costs with mean 1 converges to

$$0.3718 \ < \ \mathrm{E} \ < \ 1.8310$$

as the number of vertices tends to infinity.

Relations to Hyperflow Problems

- Cambini, Gallo, Scutellà [1992]
  Minimum cost flows on hypergraphs; solves only LP relaxation
- Jeroslow, Martin, Rarding, Wang [1992]
  Gainfree Leontief substitution flows; does not hold for HAP

$$
\begin{array}{lrcll}
\min & c^T f & & & \\
 & f(\delta^+(n)) - f(\delta^-(n)) & = & b_n & \forall n \in N \\
 & f & \geq & 0 & \\
 & f & \leq & 1 &
\end{array}
$$

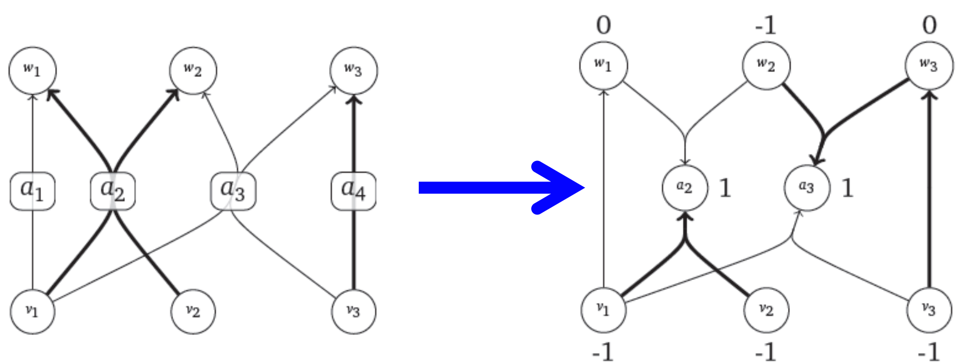
minimize cost

demand

nonnegativity

capacity

hyperassignment problem

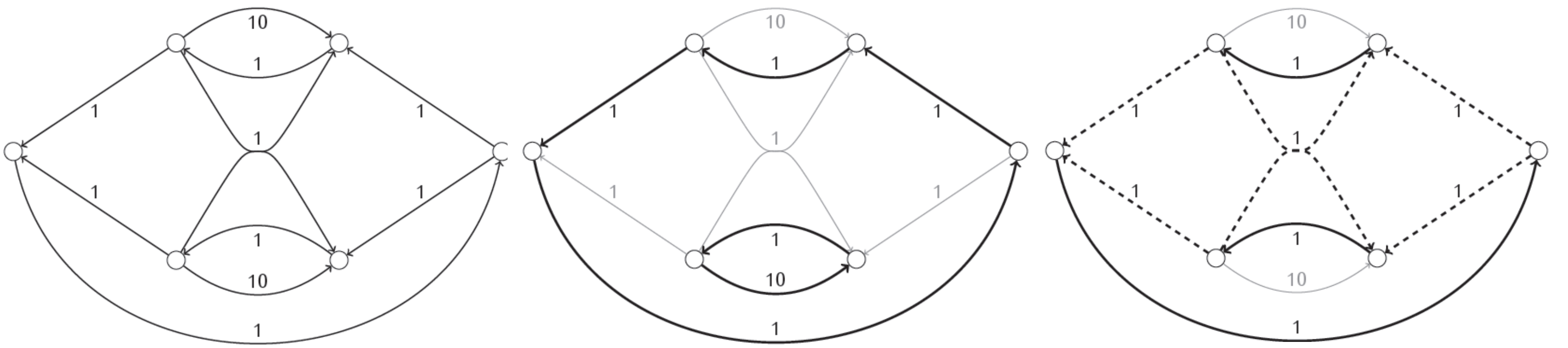


hyperflow problem: numbers next to the vertices are the demands

Complexity Results

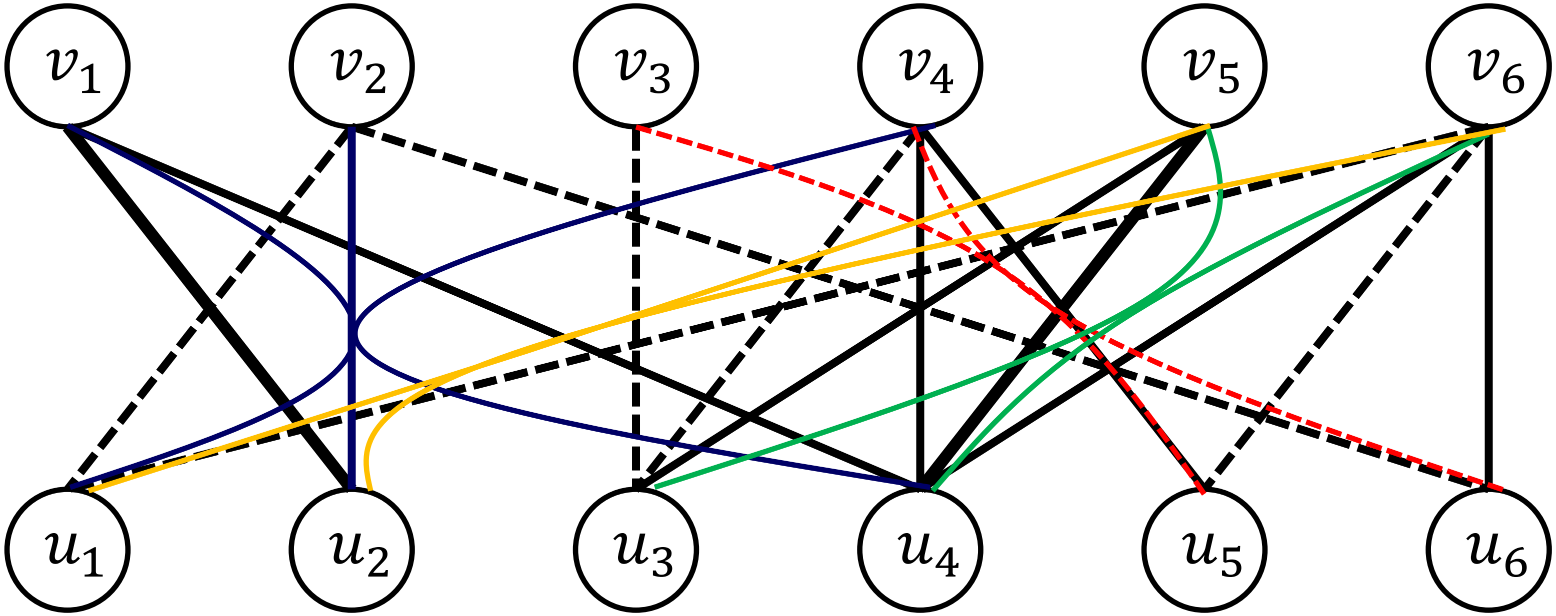
Theorem (B., Heismann [2011], Heismann [2014])

1. The HAP is NP-hard and APX-hard, even for bipartite hypergraphs with maximum hyperedge size 4.
2. The set packing/covering relaxations of the HAP are NP-hard, even for bipartite hypergraphs with maximum hyperedge size 6.
3. The LP/IP gap can be arbitrarily large.
4. The determinants of basis matrices can be arbitrarily large.



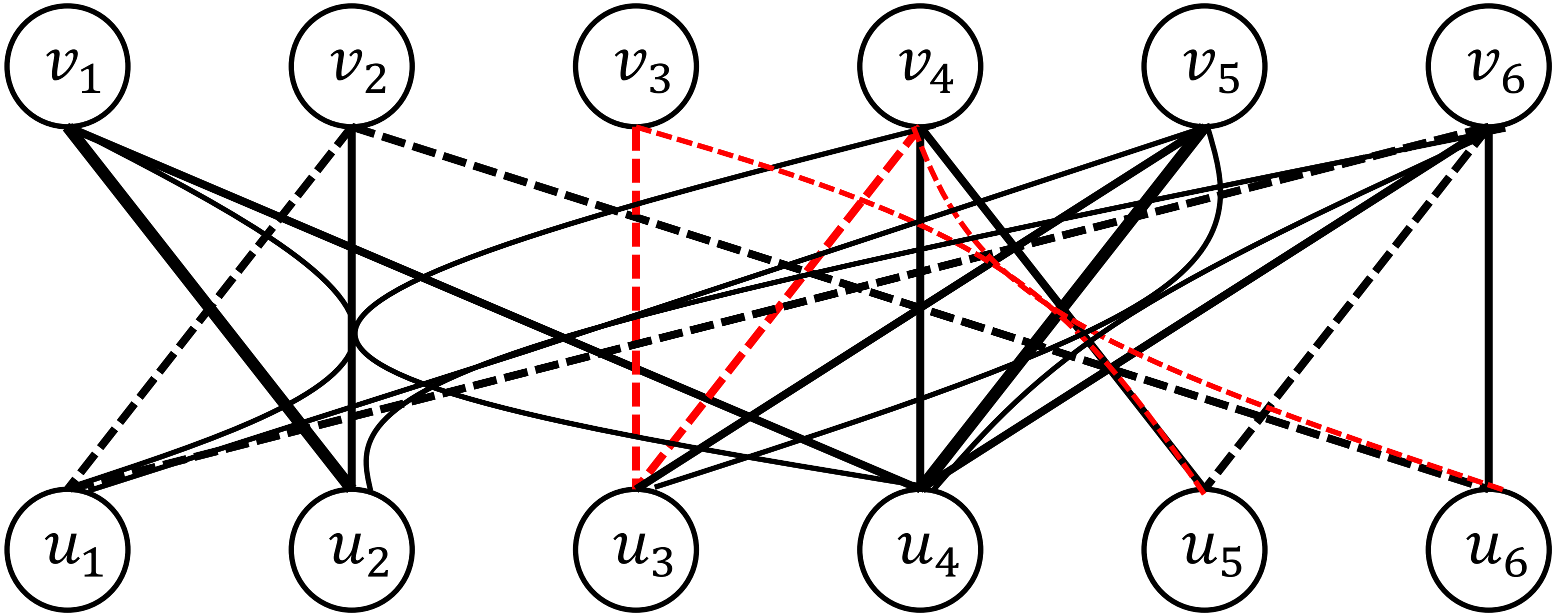
Solution of the LP Relaxation

- Fractional solution, cost = 0.615.

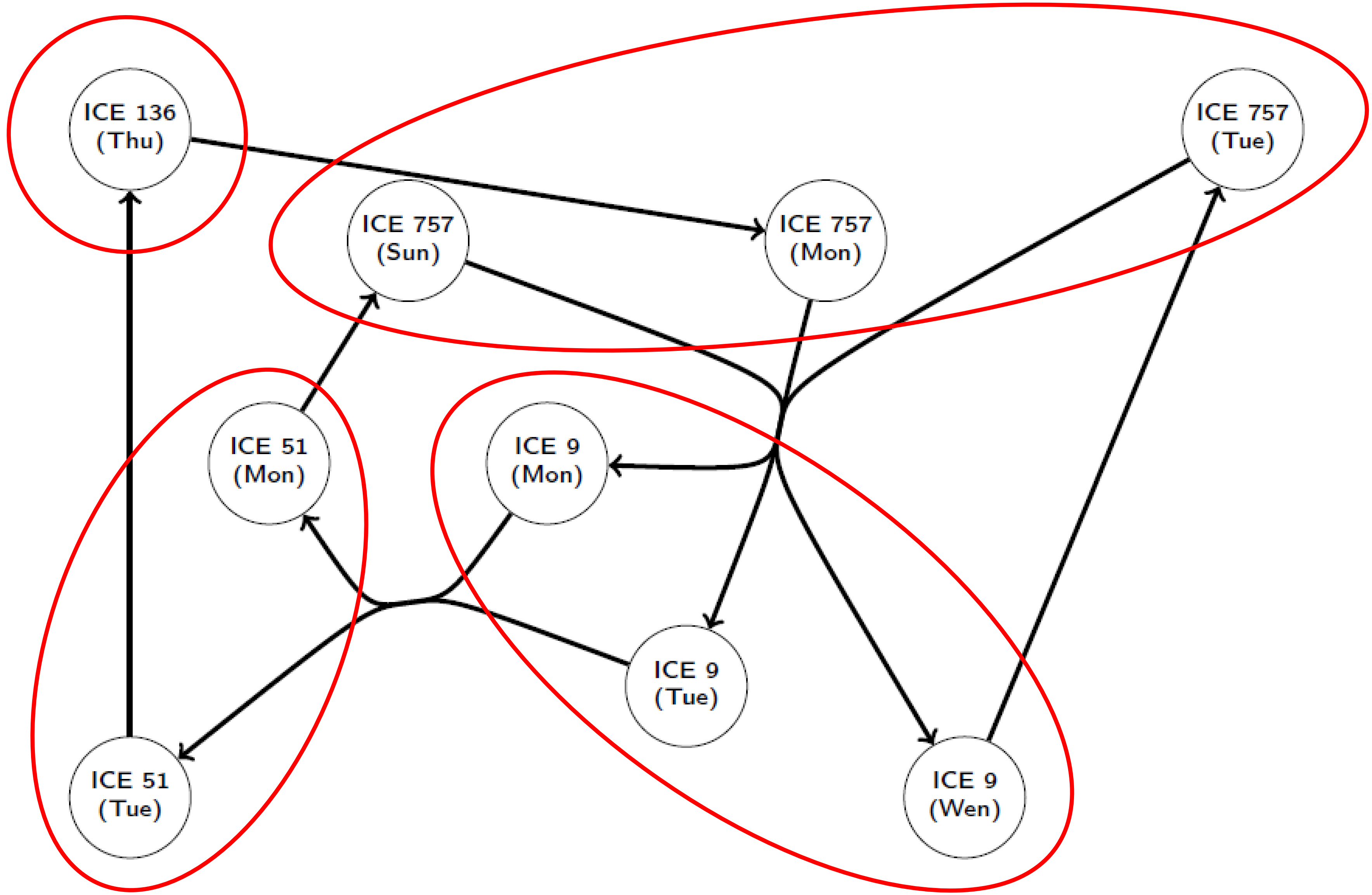


Solution of the LP Relaxation

- Fractional solution, cost = 0.615.
- The red hyperedge clique inequality separates this solution.
- Cliques can be separated efficiently by exploiting a "partitioning structure".

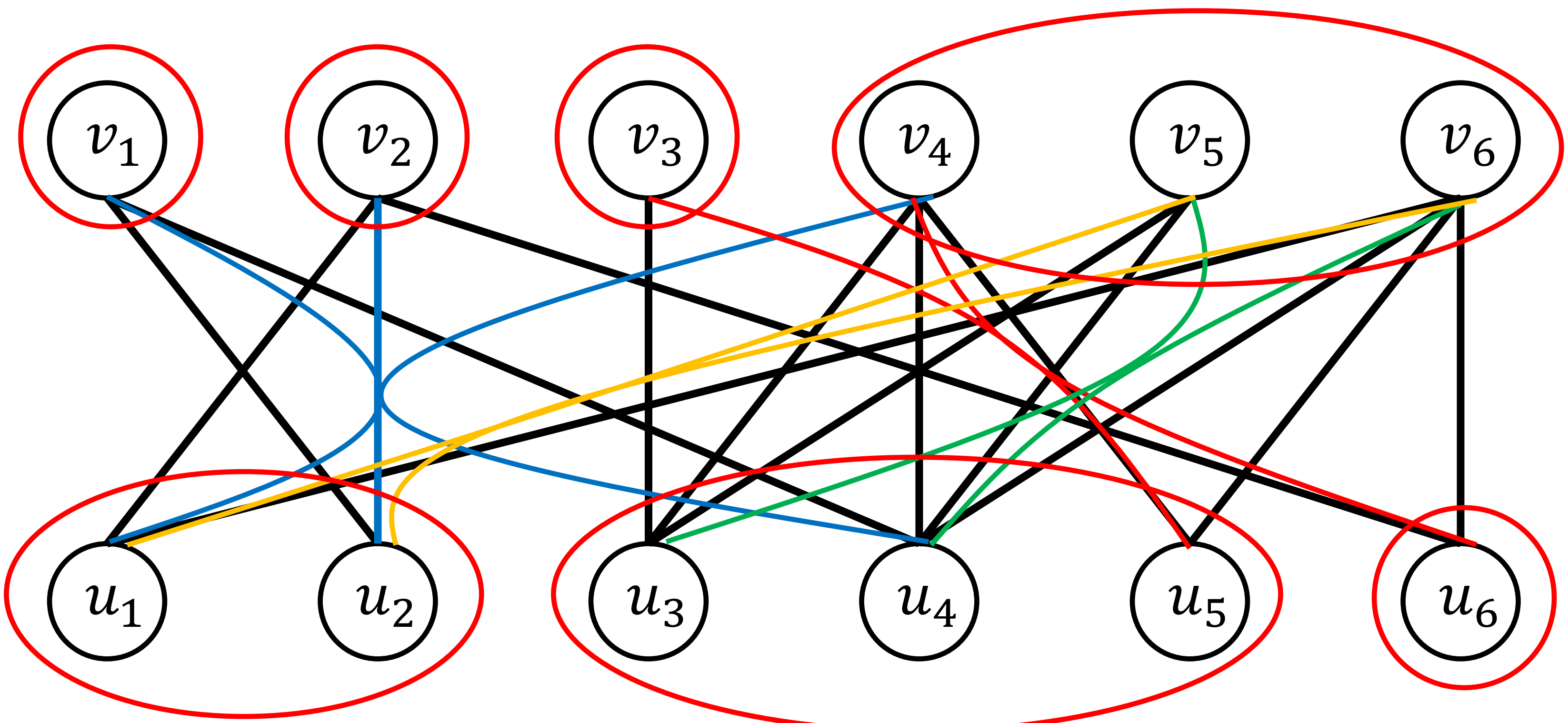


Partitioned Hypergraphs



Partitioned Hypergraphs

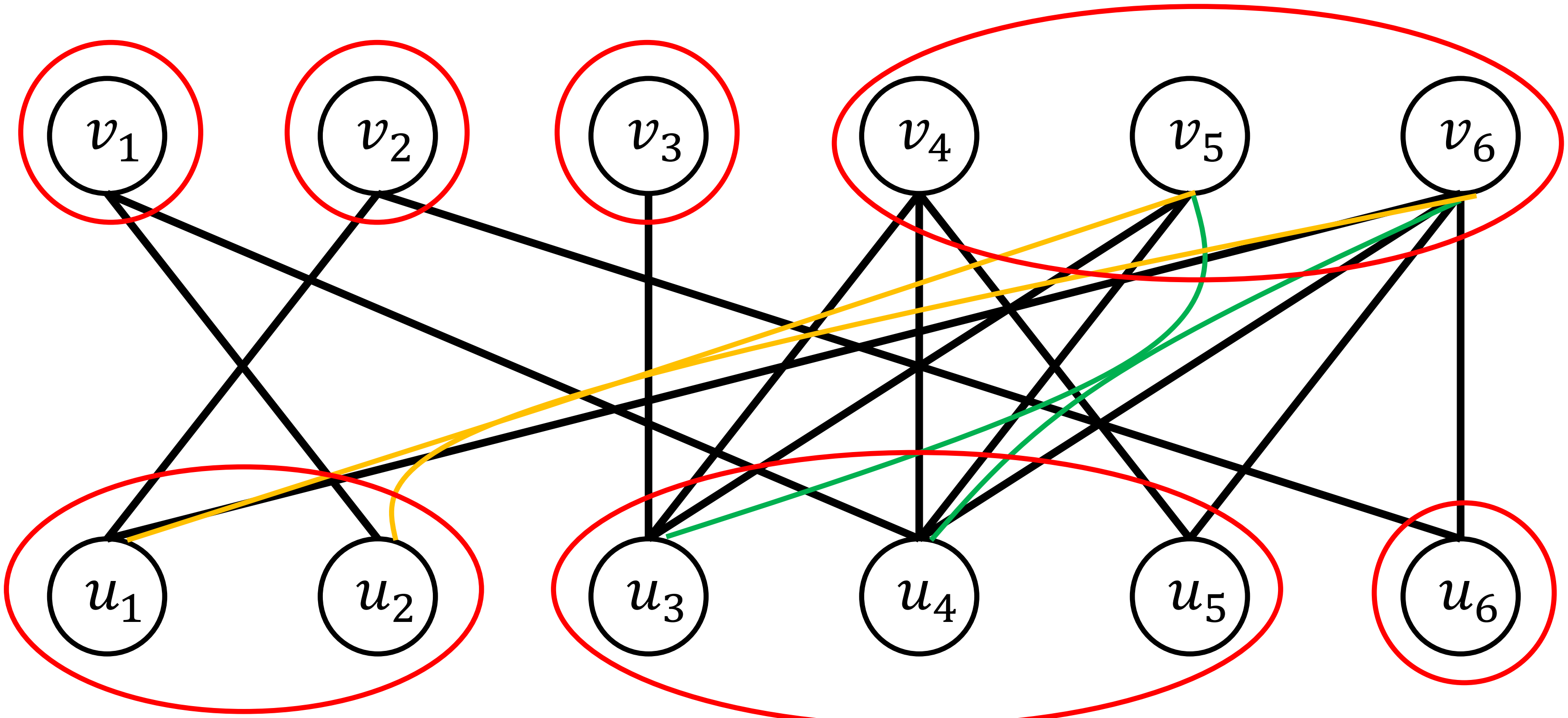
- A bipartite hypergraph $G = (U, V, E)$ is called partitioned with maximum part size $d \in \mathbb{N}$ if there exist pairwise disjoint $\leq d$-element sets $U_1, \ldots, U_p$ and $V_1, \ldots, V_q$ called the parts of G s.t. every hyperedge intersects exactly one part in U and one part in V.



Partitioned Hypergraphs

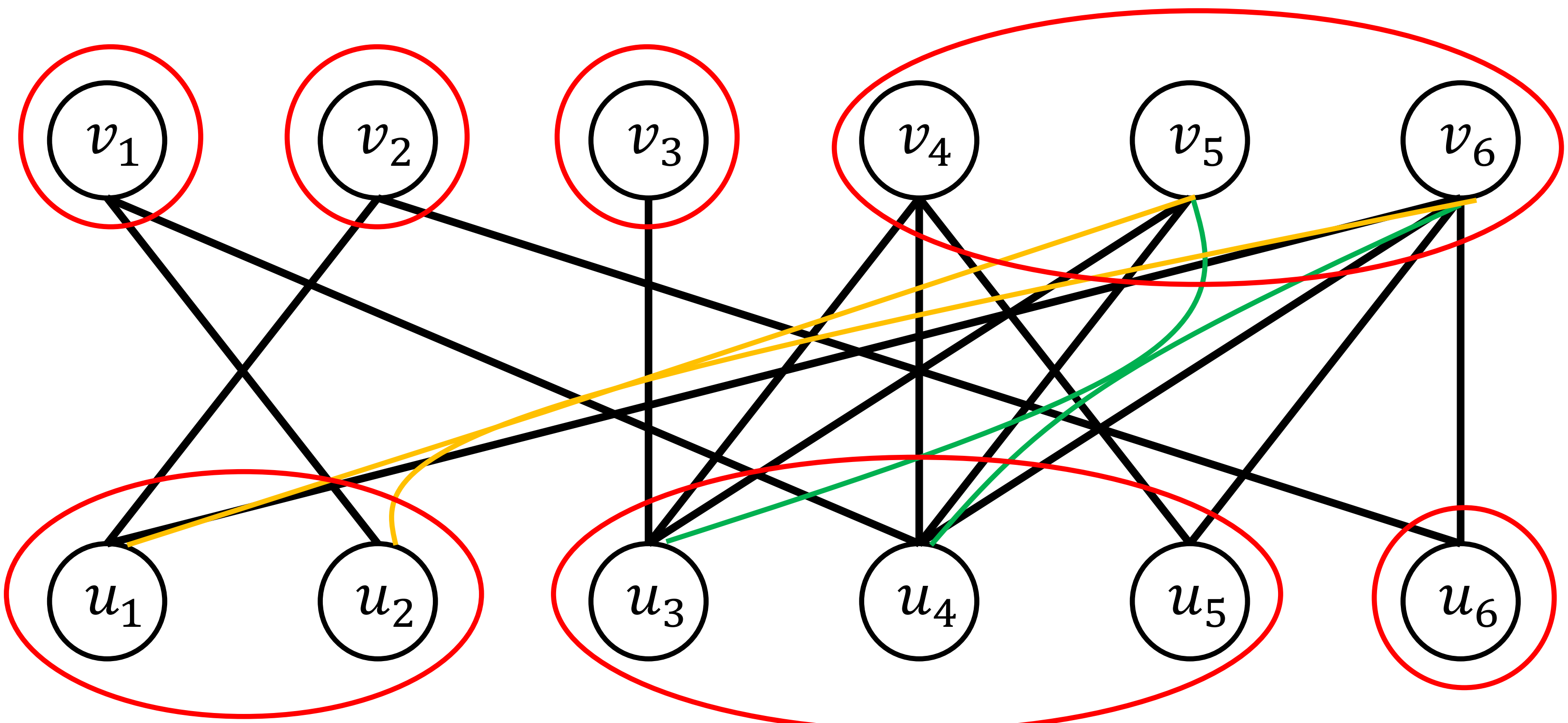
Theorem (B., Heismann [2012])

Every HAP in a bipartite hypergraph $G = (U, V, E)$ can be polynomially transformed into a HAP in a partitioned hypergraph with $d = 0.5 \max_{e \in E} |e|$.



Theorem (B., Heismann [2011])

- ▷ Every (hyperedge) clique in a partitioned hypergraph is a subset of the incident hyperedges $\delta(P)$ of some part P.
- ▷ The (hyperedge) conflict graph contains no holes of any size and no antiholes of size < 7.

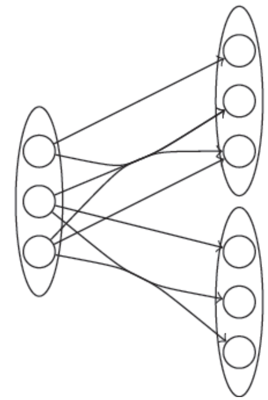


Extended Formulation

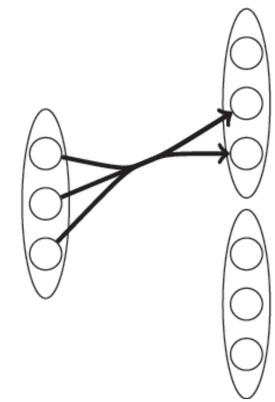
Theorem (B., Heismann [2011])

There exists an extended formulation with $O(|U|^{d+1})$ variables that implies all clique inequalities.

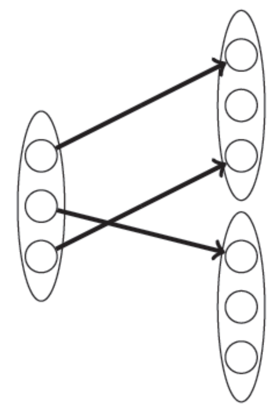
$$
\begin{aligned}
\min \quad & c^T x \\
x(\delta^+(v)) = & \quad 1 \quad \forall v \in U \cup V \\
x(\delta^-(v)) = & \quad 1 \quad \forall v \in U \cup V \\
x \quad & \in \{0,1\}^A
\end{aligned}
$$



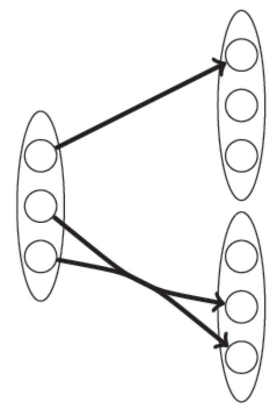
partitioned hypergraph



configuration 1



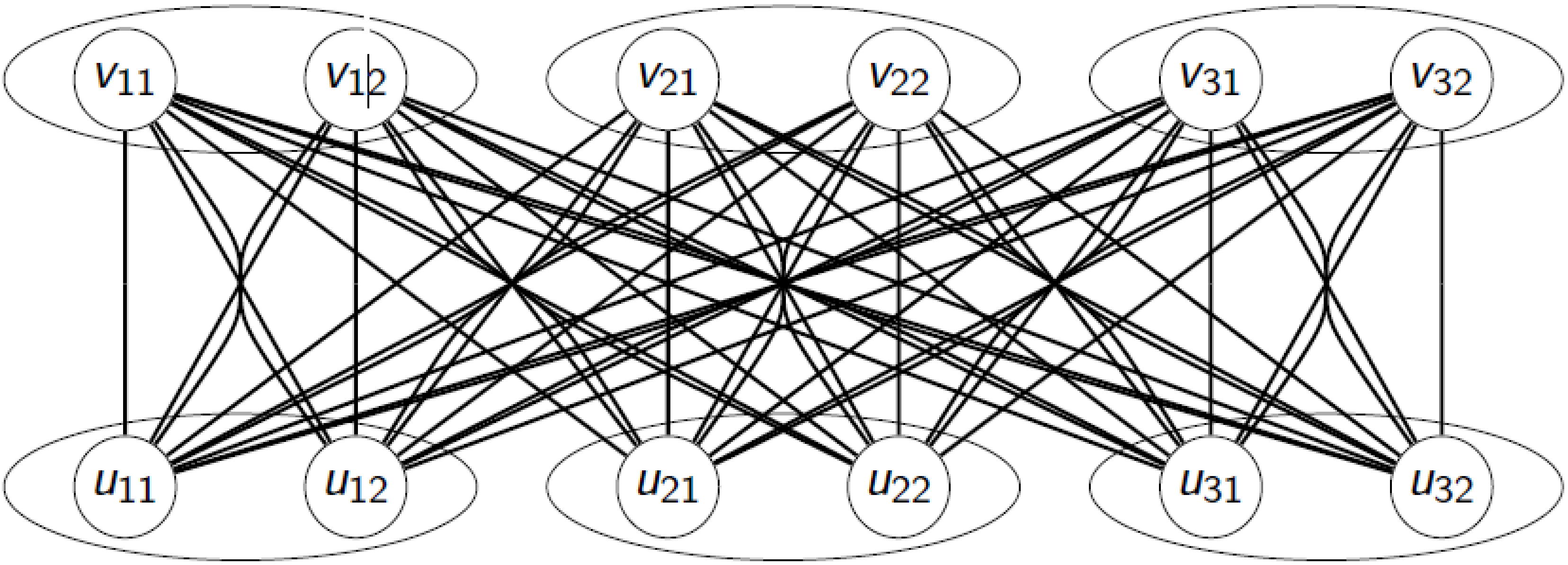
configuration 2



configuration 3

Further Polyhedral Results

- Let $G_{2,n}$ be the complete partitioned directed hypergraph on $2n$ nodes with n parts of size 2.
- Let $\mathrm{P}(G_{2,n})$ be the HAP polytope associated with $G_{2,n}$.
- $\mathrm{P}(G_{2,6})$ is completely described by 14 049 facets.



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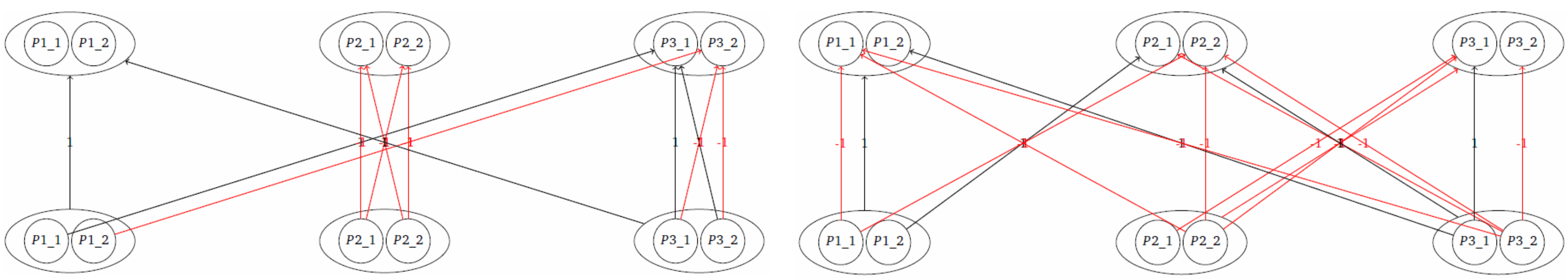
- Every facet of $\mathrm{P}(G_{2,6})$ can be described in many different ways, in particular, in the form

$$\sum_{e \in E_1} x_e - \sum_{e \in E_{-1}} x_e \leq 1.$$



Example of two facets with coefficients -1 and +1 only

- ▷ Red: coefficient -1
- ▷ Black: coefficient 1

Hyperedges are drawn as connections between the surroundings of the corresponding vertices.

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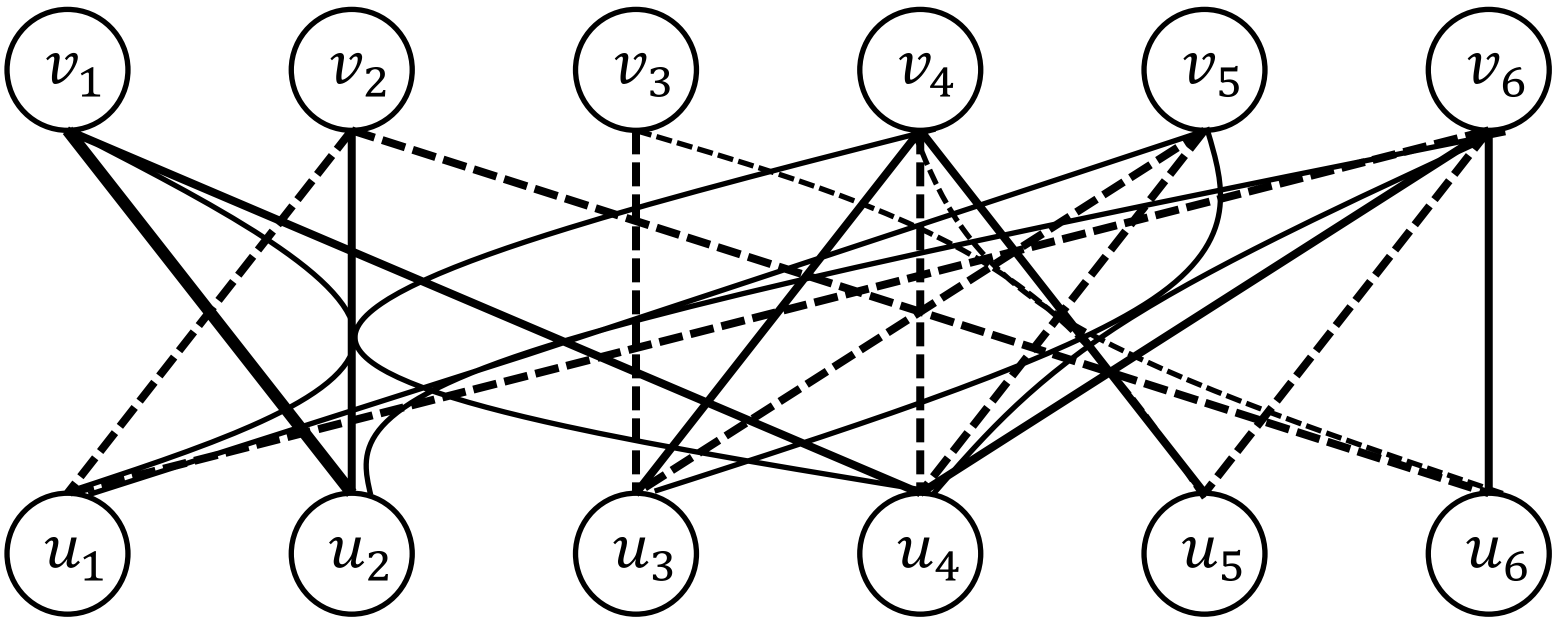
- The 14 049 facets fall into 30 symmetry classes, generated by swapping U and V, permuting parts inside U and V, permuting vertices inside of parts, resulting in 4 608 vertex and hyperedge permutations.
- 16 symmetry classes are understood, including nonnegativity constraints, clique inequalities, and odd clique set inequalities.
- Facet classification done by HUHFA program available at http://comopt.ifi.uni-heidelberg.de/people/hildenbrandt/HUHFA/ .

Solution of the LP Relaxation

- Fractional solution, cost = 0.635.
- Consider the 7=2·3+1 cliques associated with the vertices $v_1, v_3, v_4, u_2, u_3, u_4$ and the clique $\{v_5, v_6, u_3, u_4\}, \{v_5, u_3\}, \{v_5, u_4\}$.



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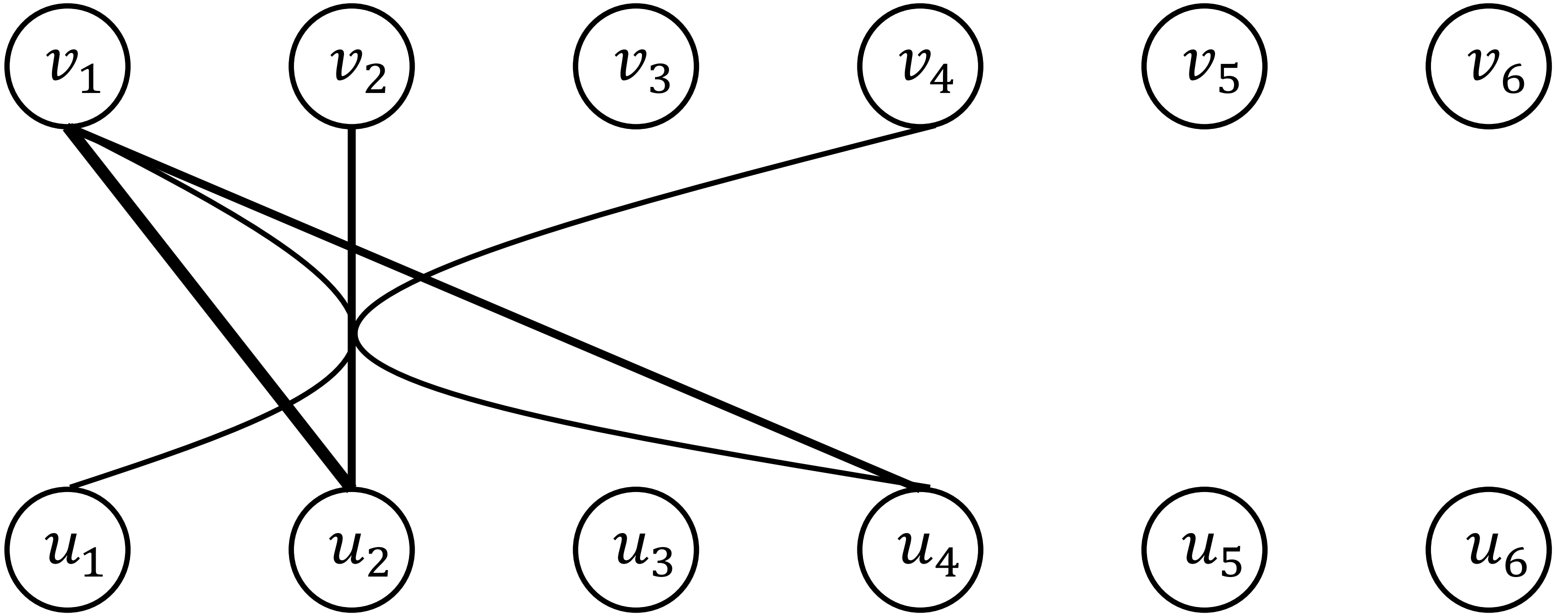

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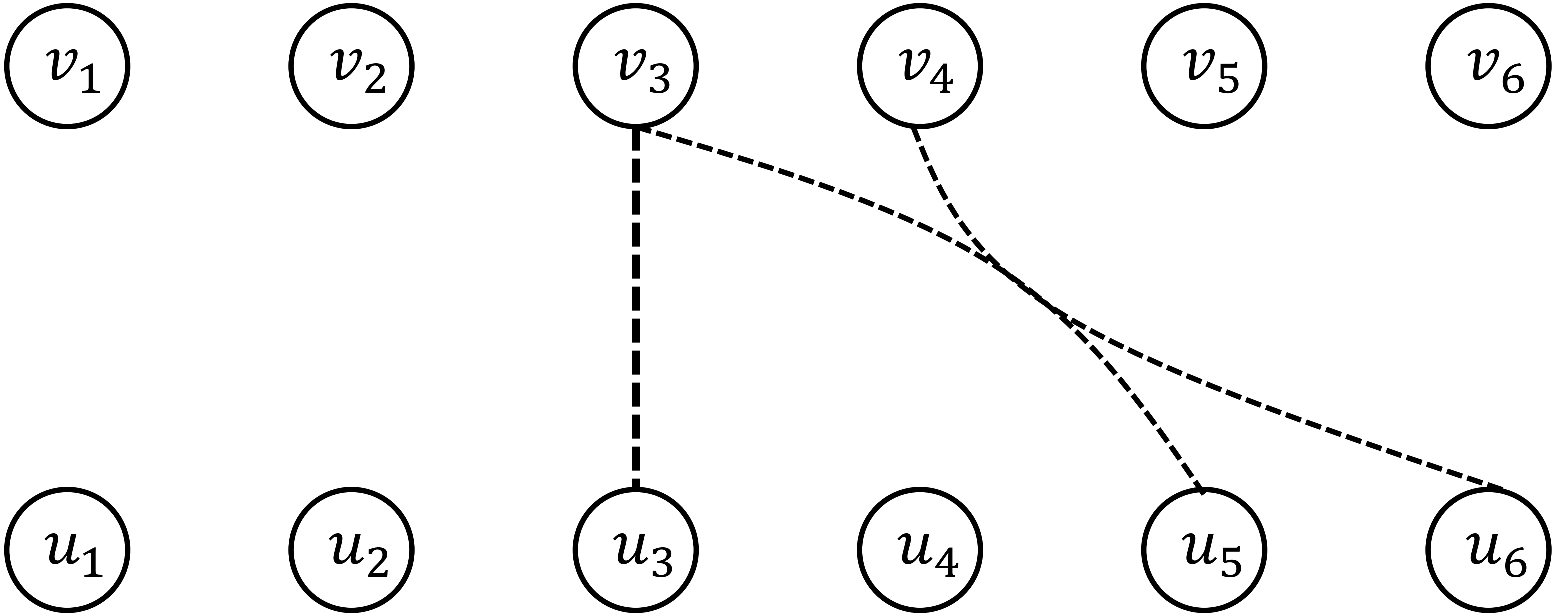

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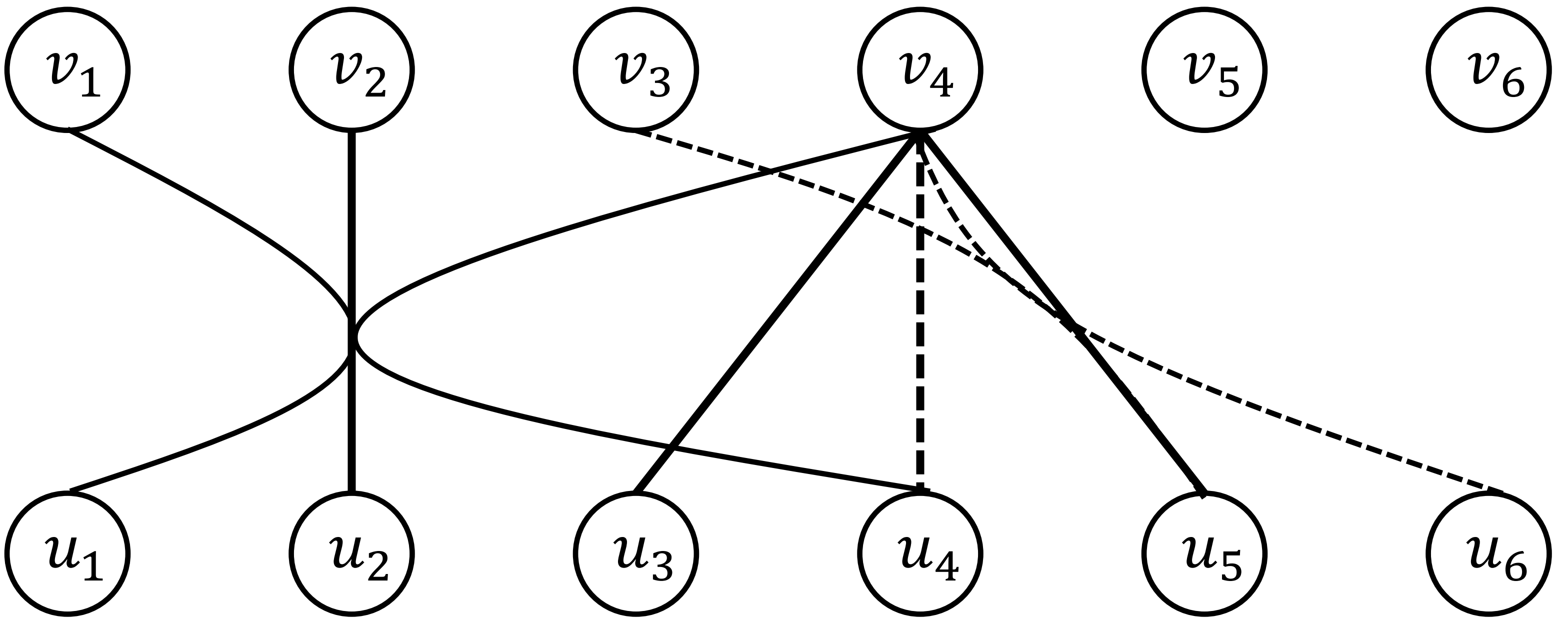

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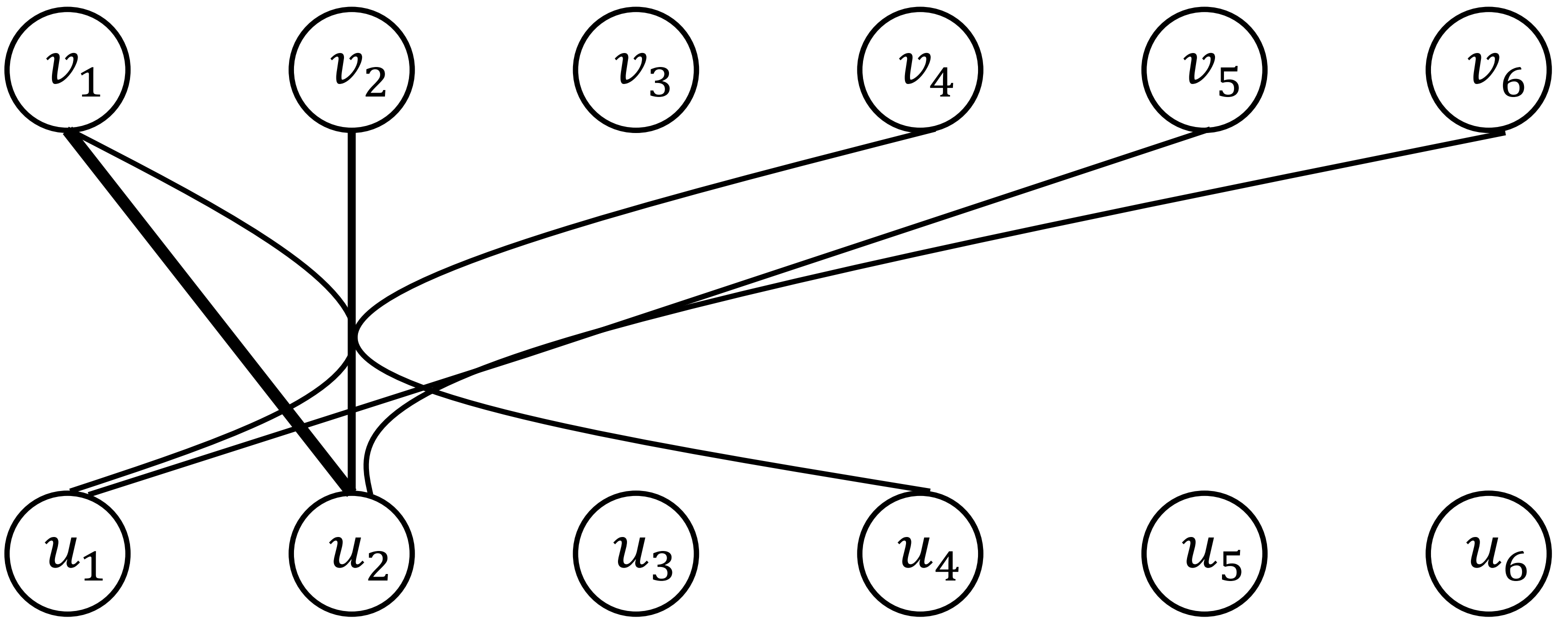

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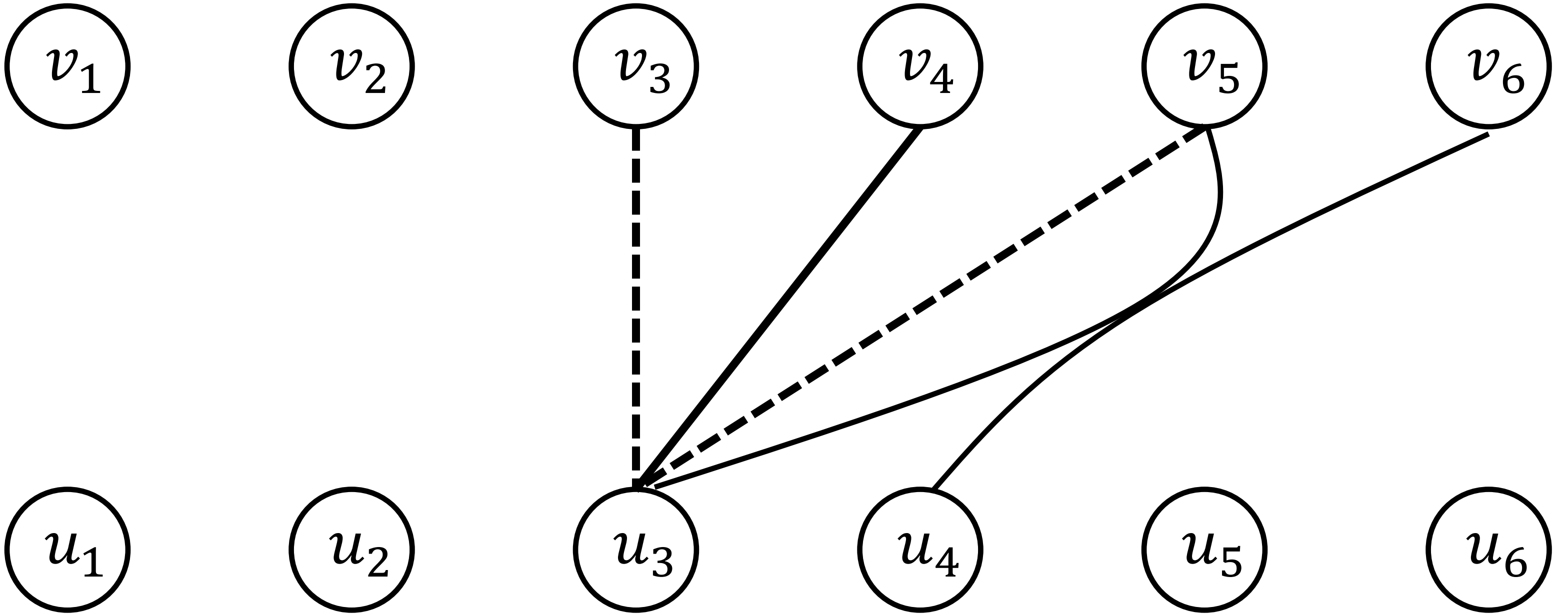

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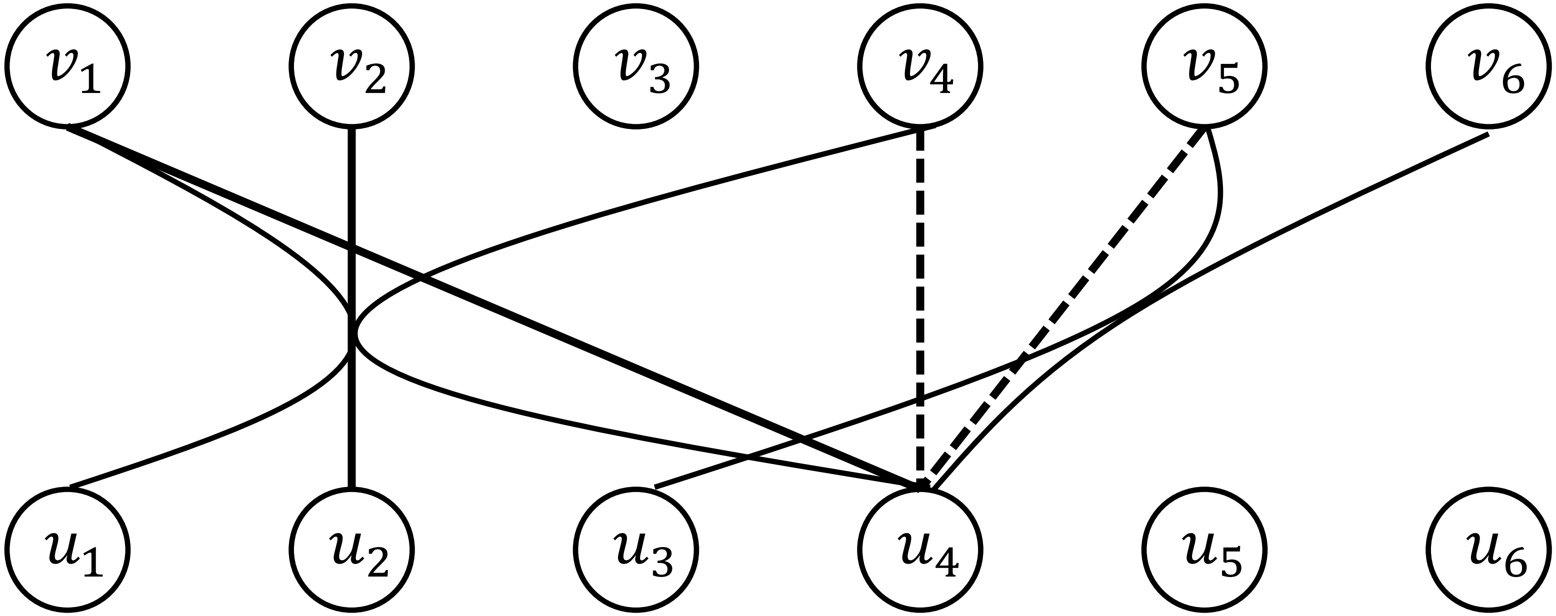

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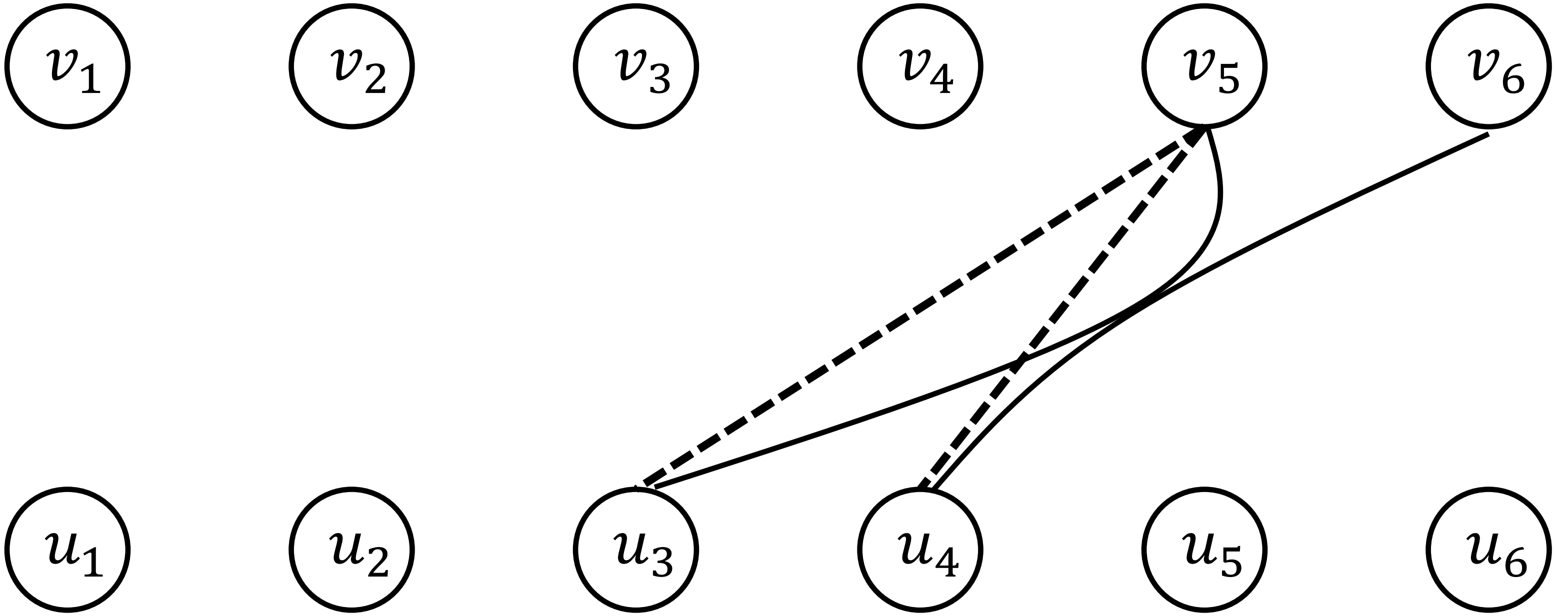
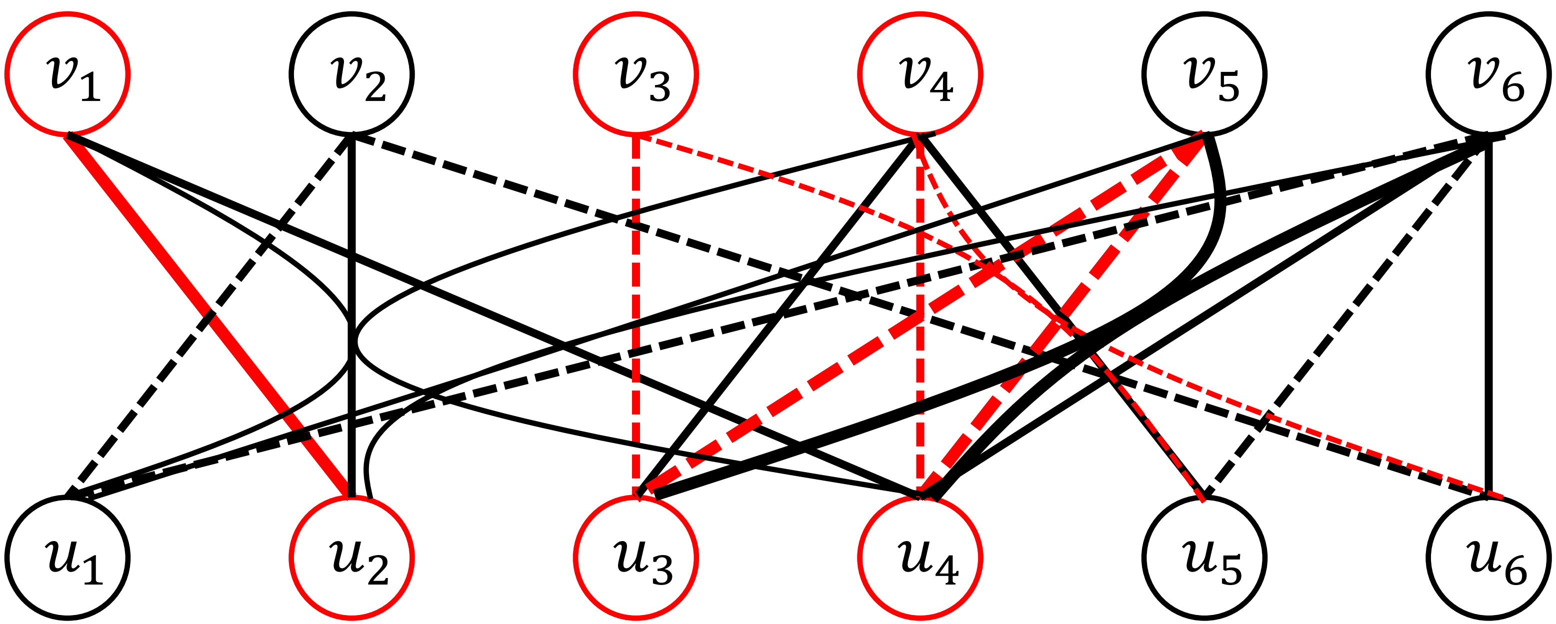
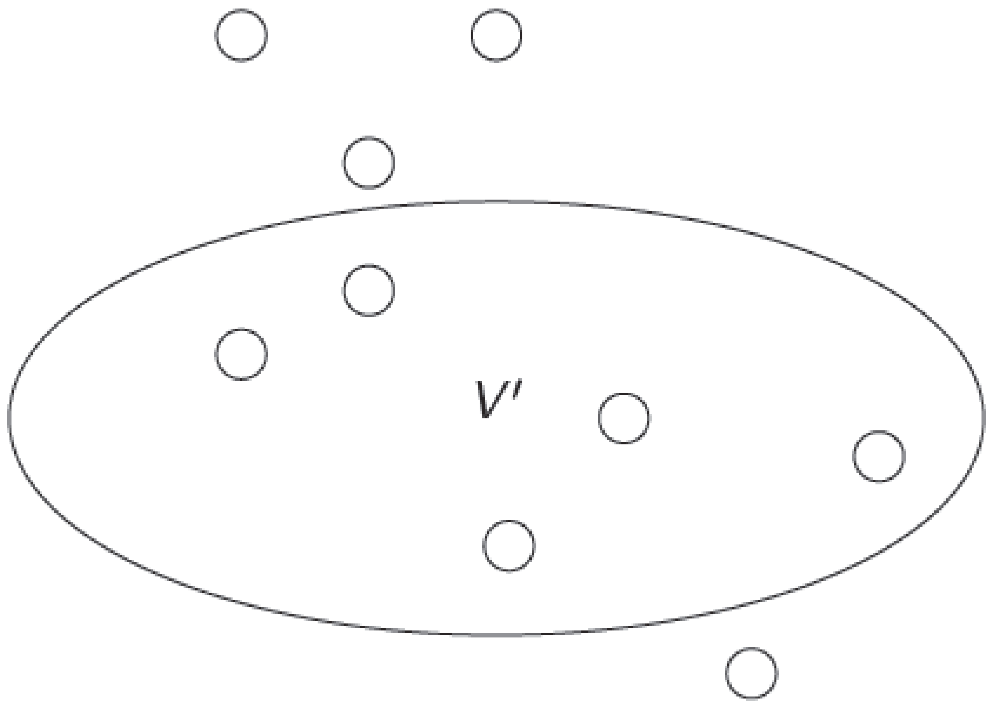


Solution of the LP Relaxation

- Fractional solution, cost = 0.635.
- Consider the 7=2·3+1 cliques associated with the vertices $v_1, v_3, v_4, u_2, u_3, u_4$ and the clique $\{v_5, v_6, u_3, u_4\}, \{v_5, u_3\}, \{v_5, u_4\}$.
- Every red hyperedge is contained in at least two of these cliques.
- We can take at most three of these edges.



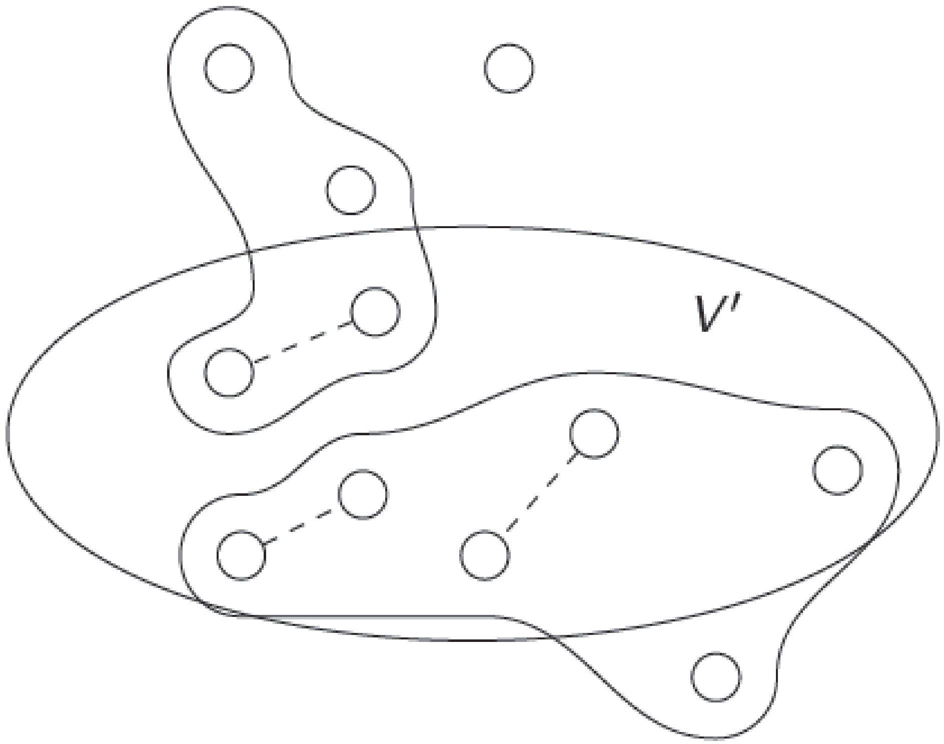
Odd Set Ieqs for the (Perfect) Matching Problem



$$\sum_{e \in E} \left\lfloor \frac{|\{v \in V' : e \in \delta(v)\}|}{2} \right\rfloor x_e \leq \left\lfloor \frac{|V'|}{2} \right\rfloor$$

- Complete description of the matching polytope (together with the degree and non-negativity constraints), Edmonds [1965]

Odd Clique Set Ieqs for General Hypergraphs



$$\sum_{e \in E} \left\lfloor \frac{|\{v \in V' : e \in \delta(v)\}|}{p} \right\rfloor x_e \leq \left\lfloor \frac{|V'|}{p} \right\rfloor \quad \forall V' \subseteq V$$

- Generalize the odd set inequalities for the matching problem
- Related to clique set inequalities by Pêcher & Wagler [2006]

Odd Clique Set Ieqs for General Hypergraphs

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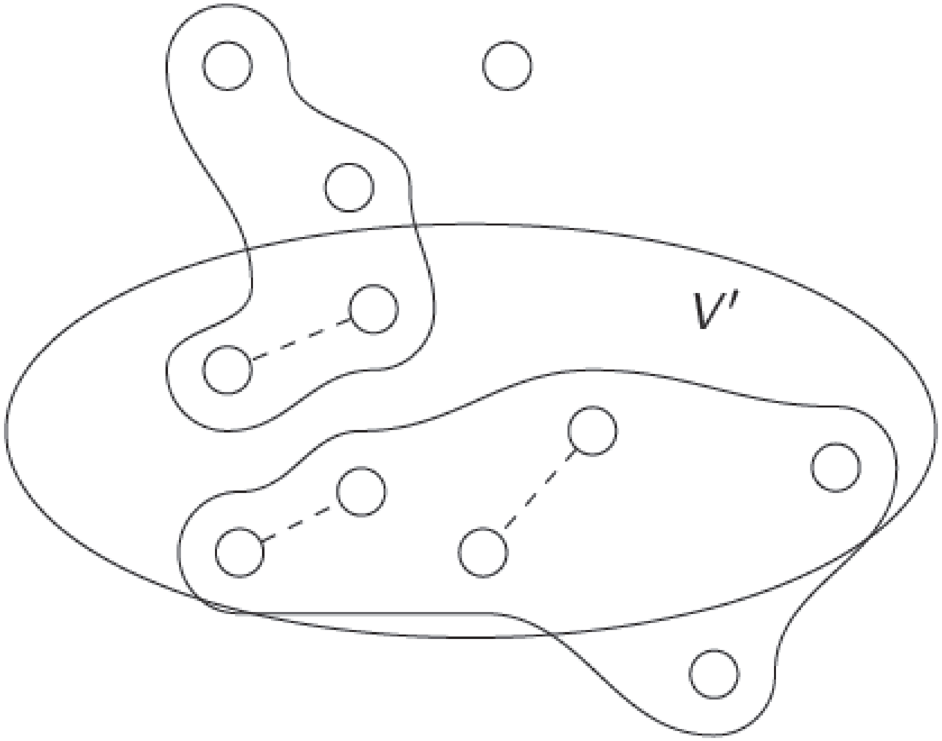
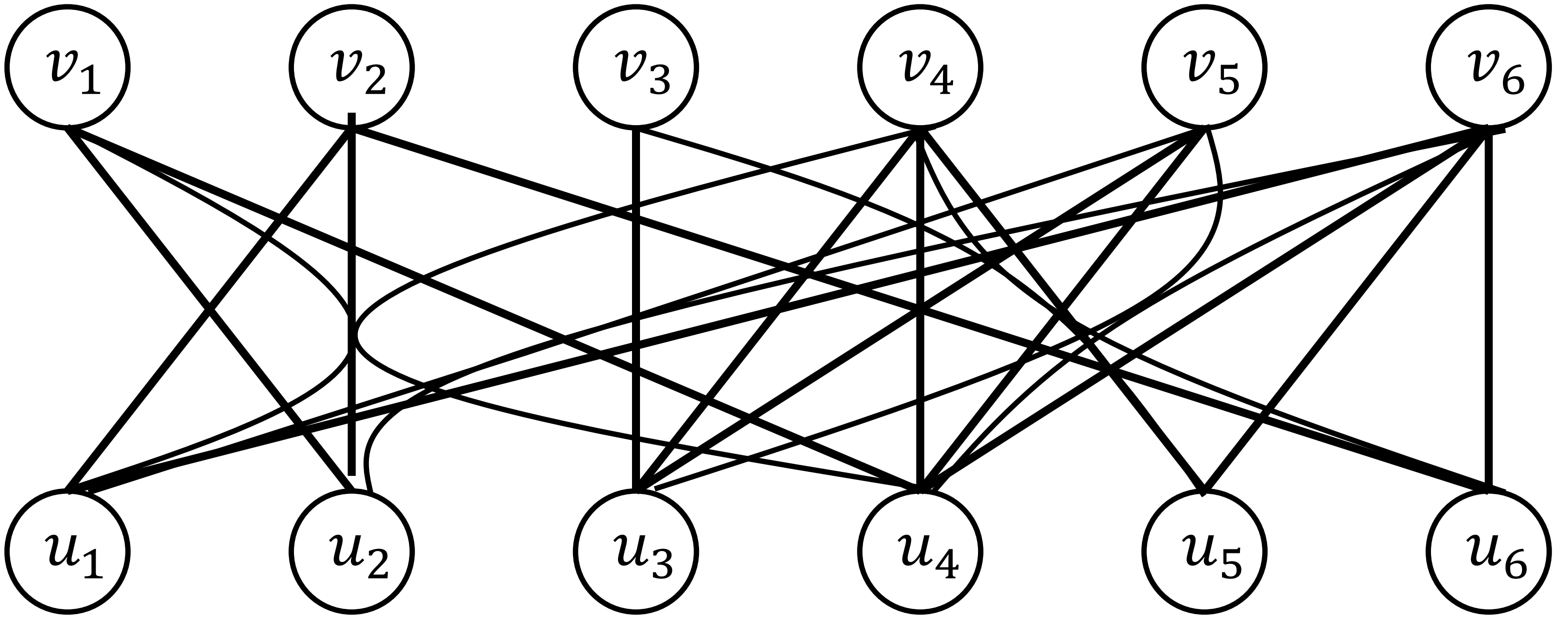


Theorem (B., Heismann [2011])

Let $\mathcal{Q}$ be a set of at least three hyperedge cliques in $G = (V, E)$, $2 \leq p \leq |\mathcal{Q}|$ be an integer number, $r := |\mathcal{Q}| \bmod p$, and $q_e := |\{Q \in \mathcal{Q} : Q \ni e\}|$. Then

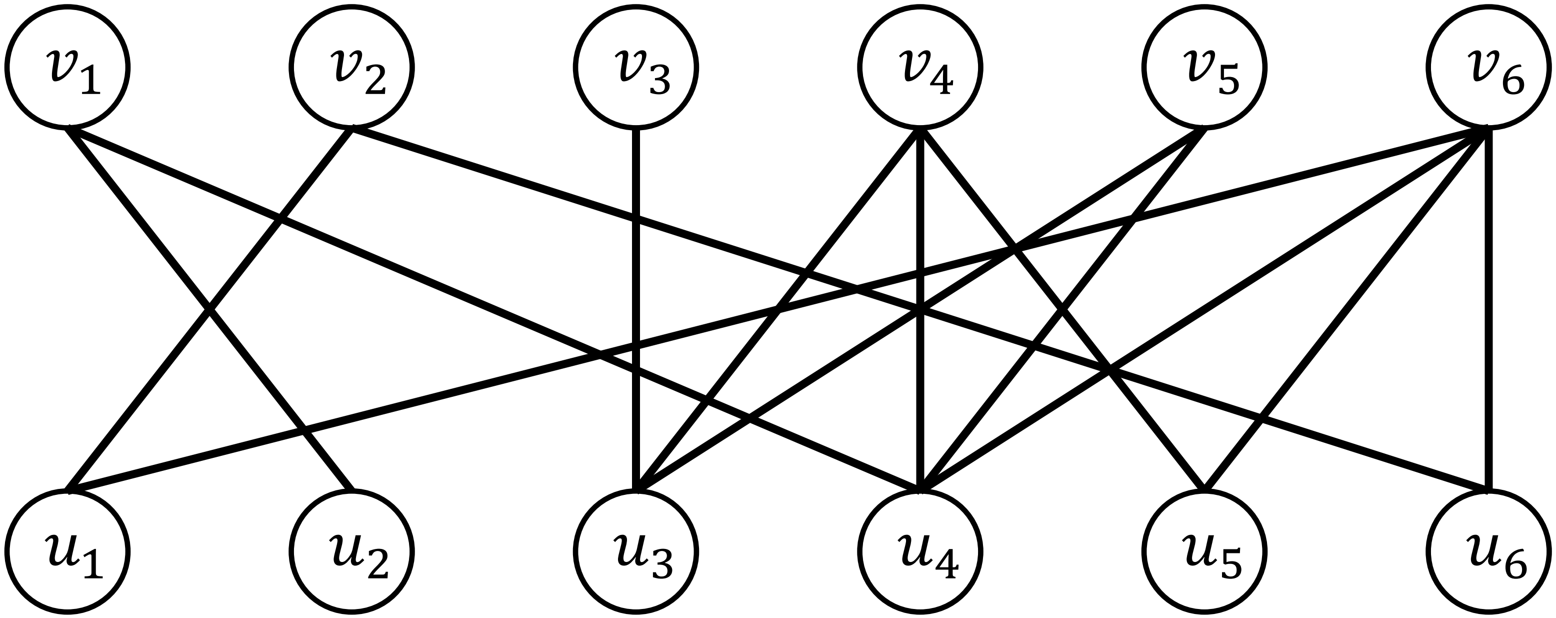
$$\sum_{e \in E} \left(\left\lfloor \frac{q_e}{p} \right\rfloor + \max\left\{0, \frac{q_e \bmod p - r}{p - r}\right\} \right) x_e \leq \left\lfloor \frac{|\mathcal{Q}|}{p} \right\rfloor.$$

A Local Search Heuristic



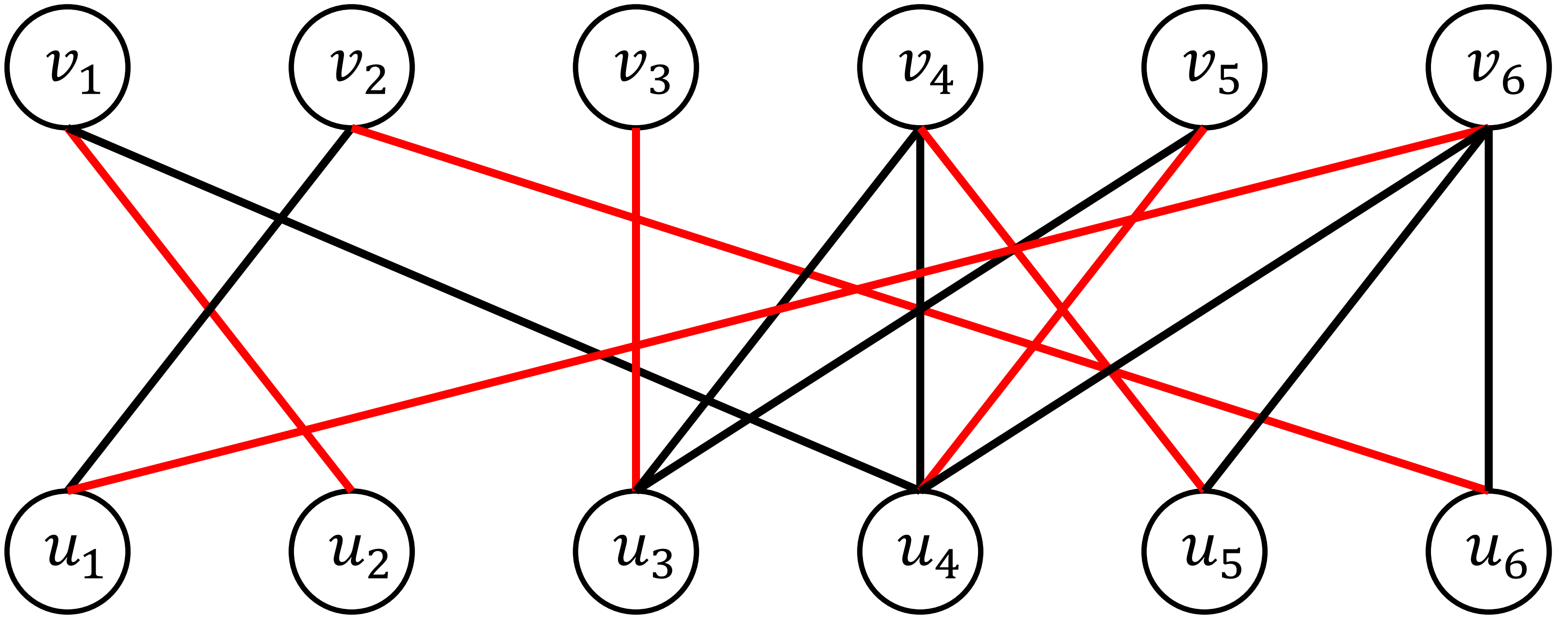
- Reduce to assignment problem by gluing vertices together.

A Local Search Heuristic



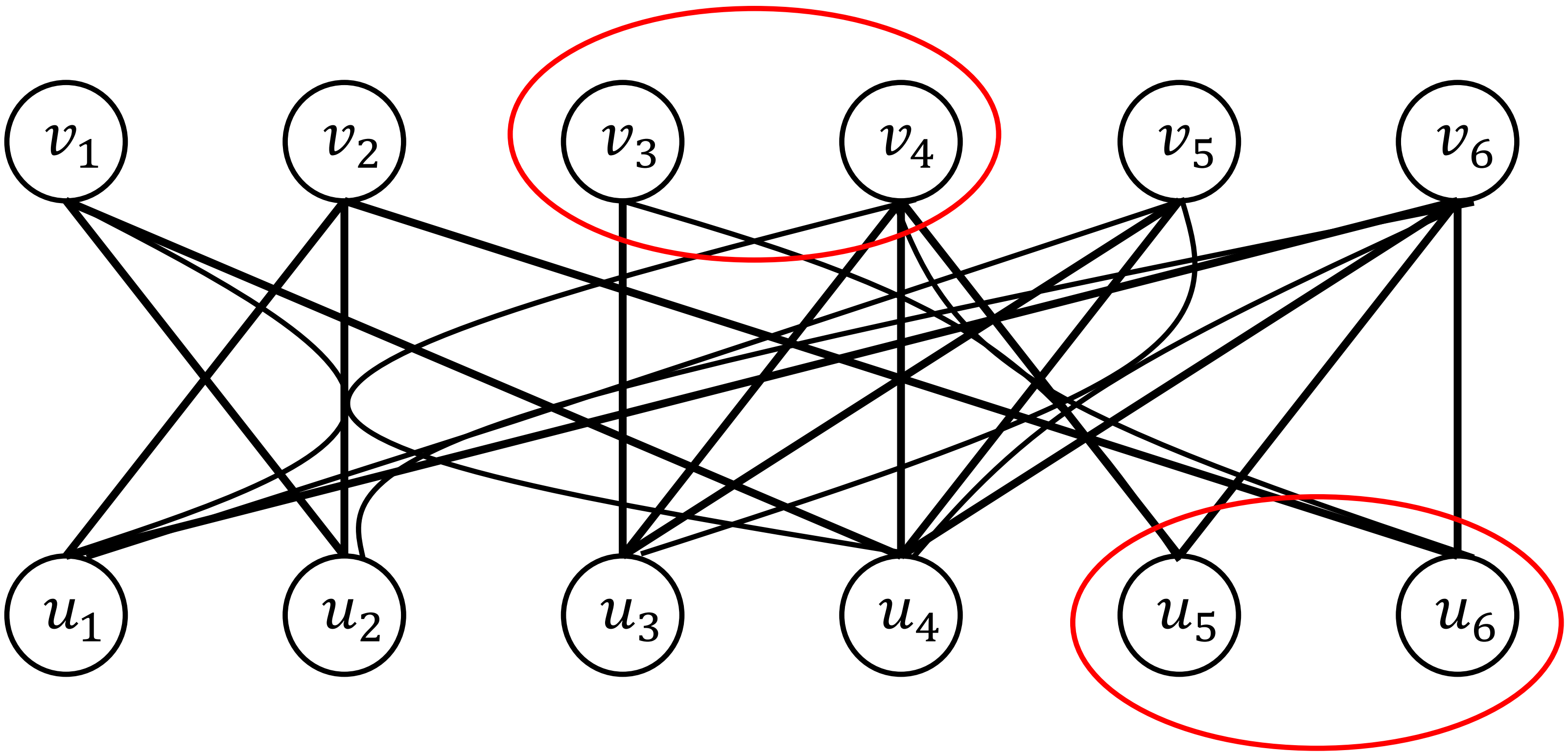
- Reduce to assignment problem by gluing vertices together.

A Local Search Heuristic



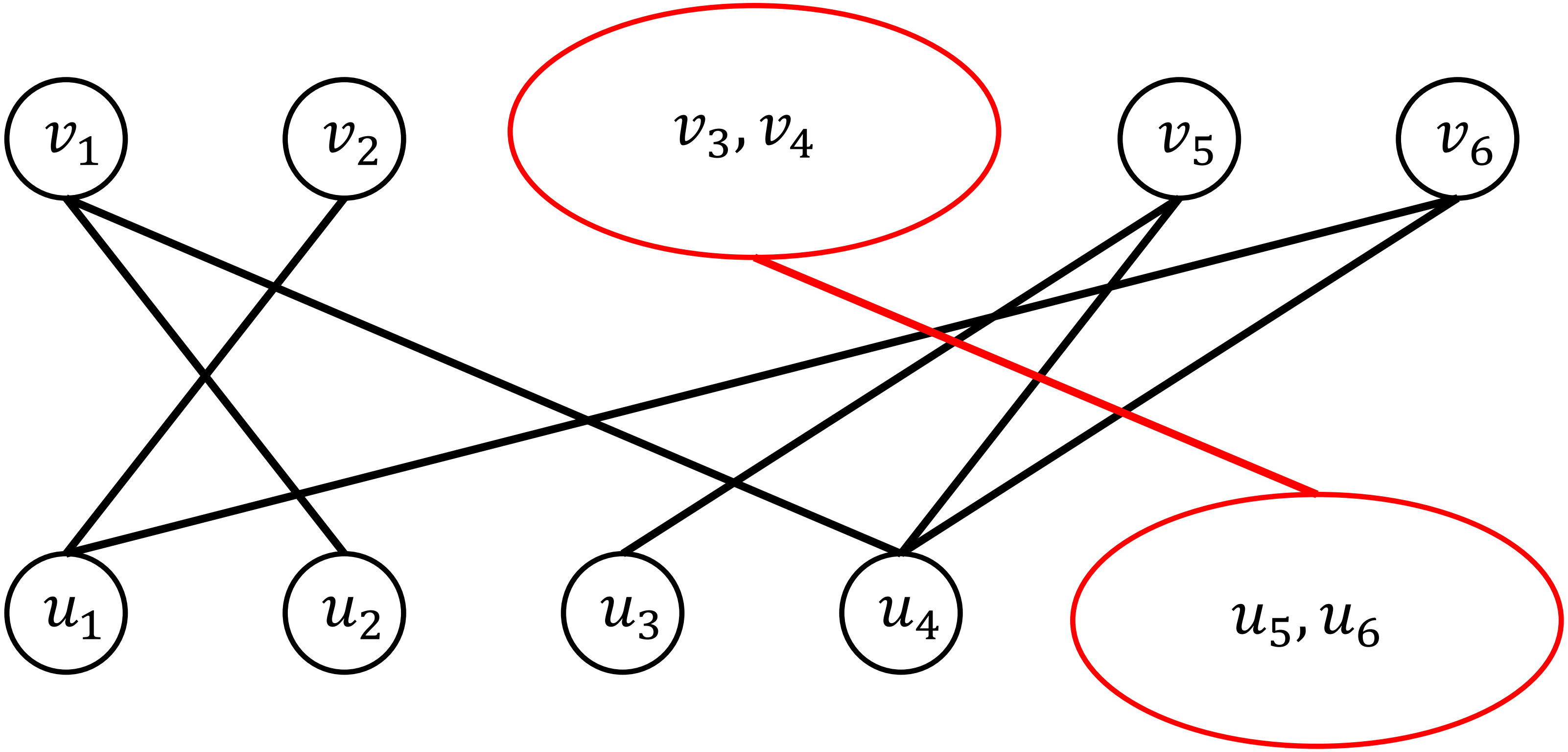
- Reduce to assignment problem by gluing vertices together.
- Cost = 1.46.

A Local Search Heuristic



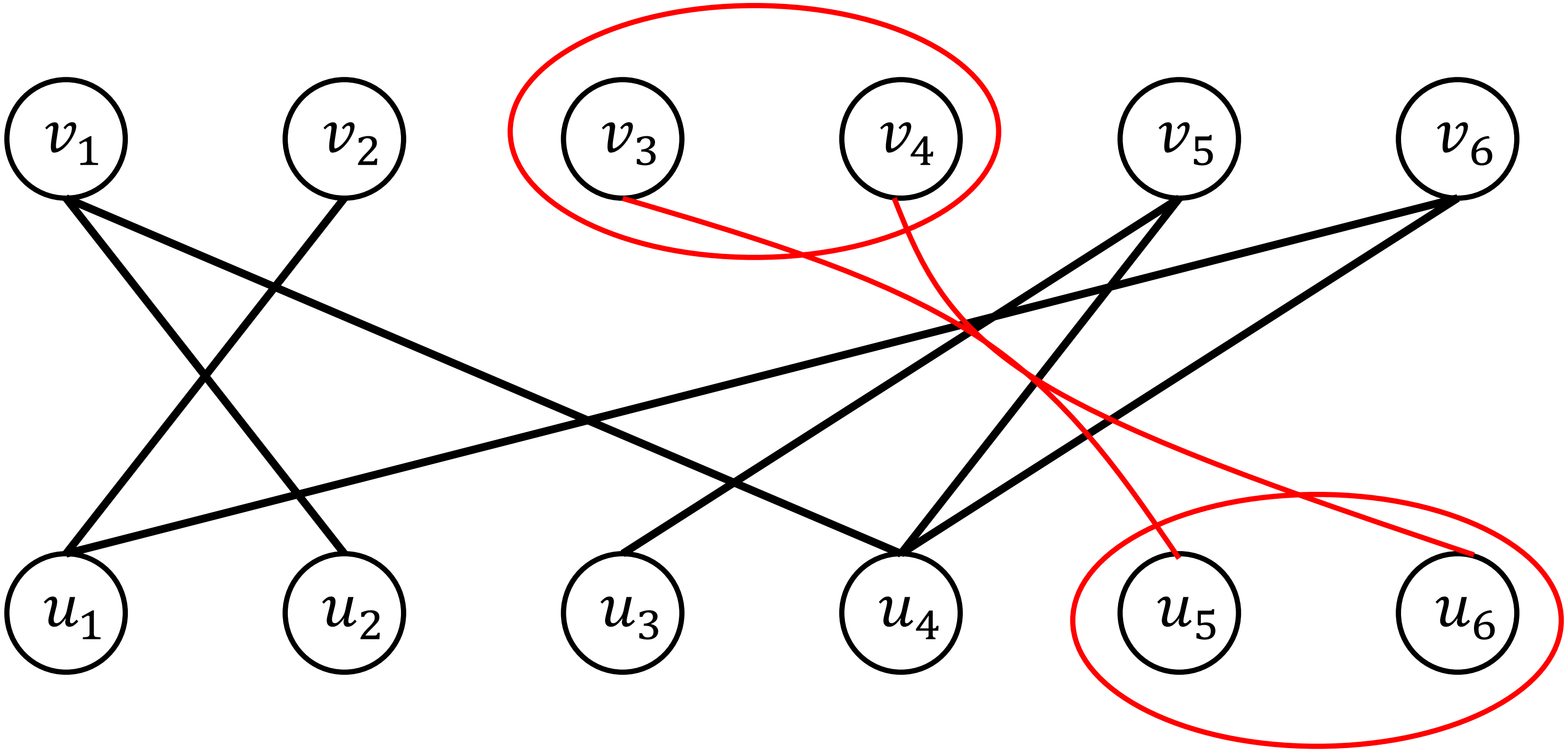
- Reduce to assignment problem by gluing vertices together.

A Local Search Heuristic



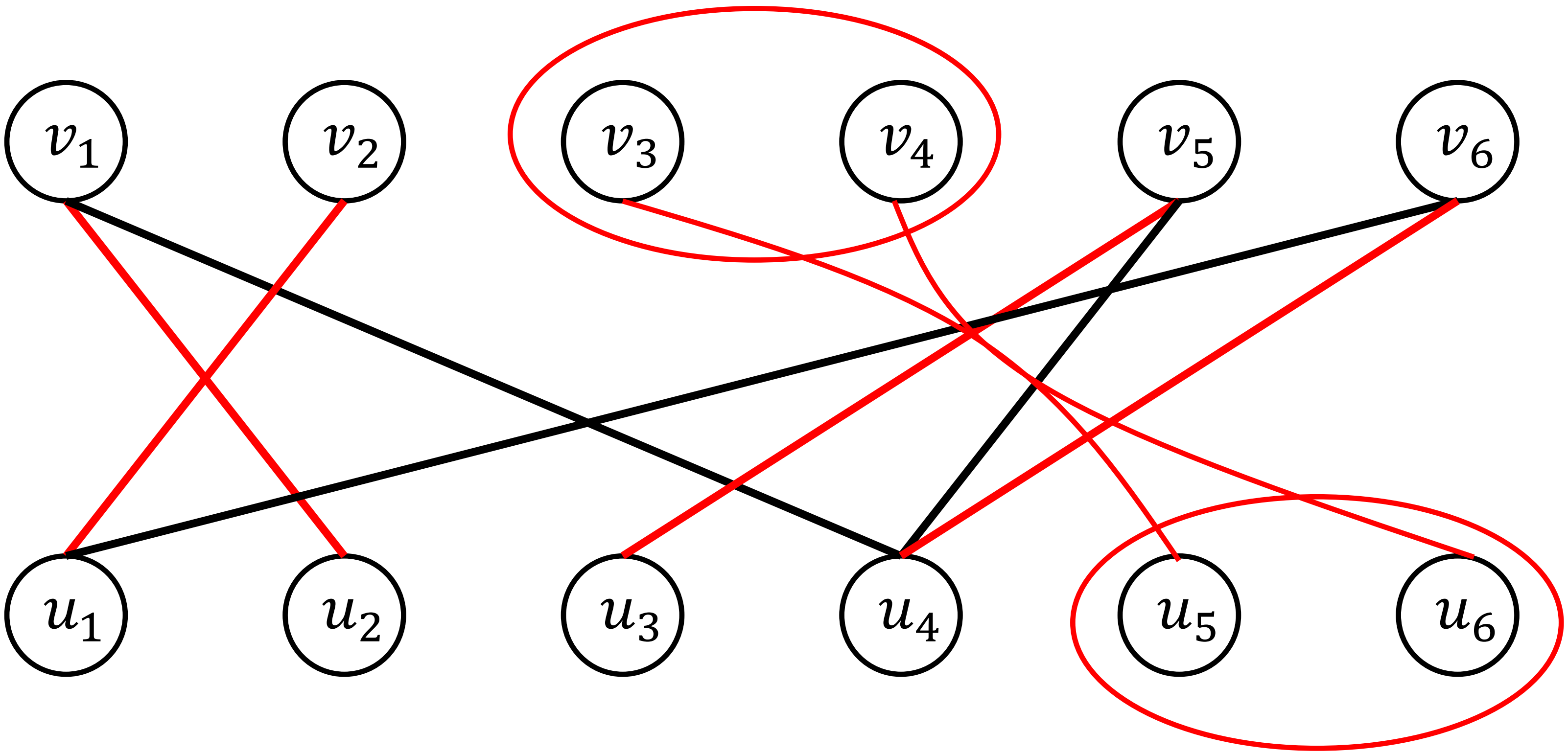
- Reduce to assignment problem by gluing vertices together.

A Local Search Heuristic



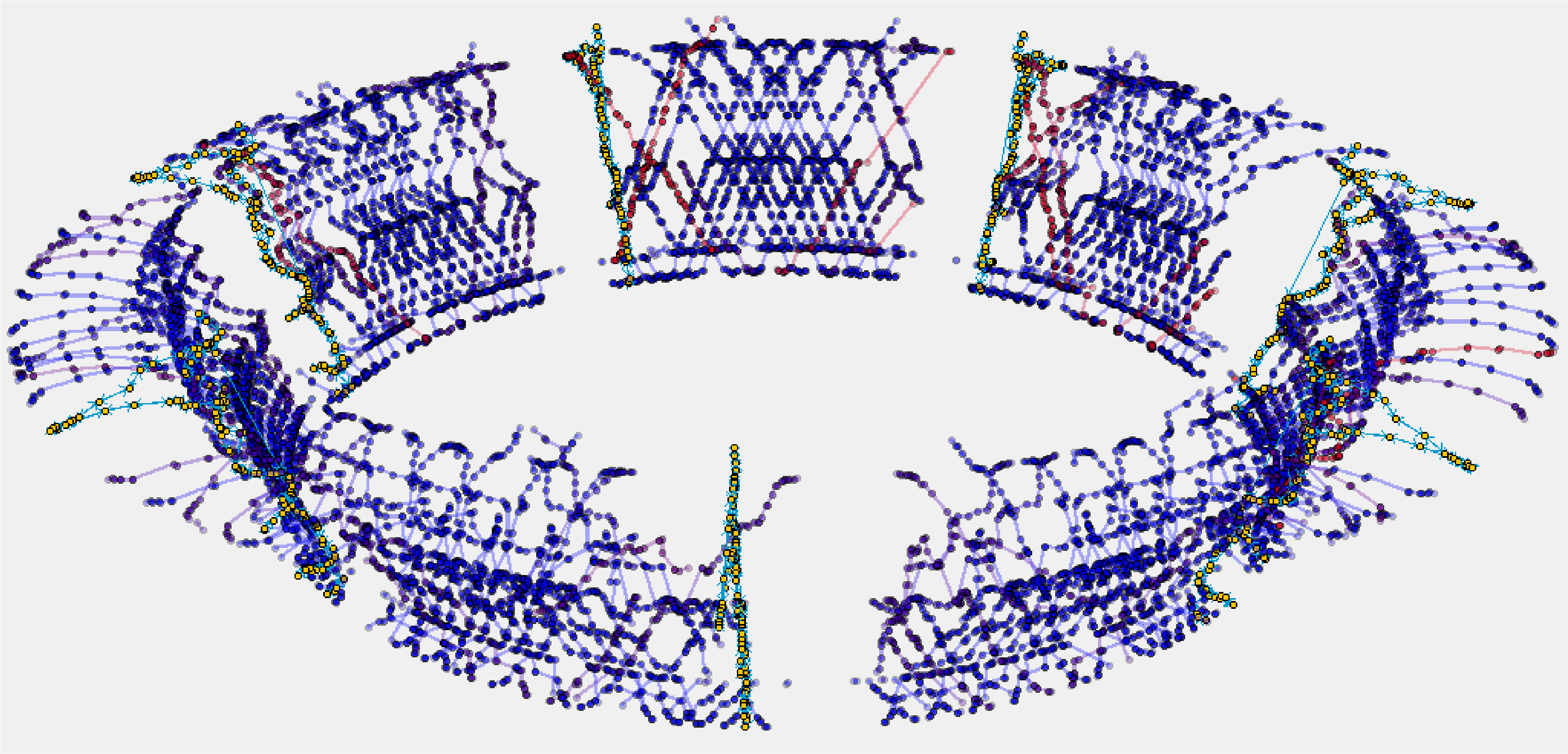
- Reduce to assignment problem by gluing vertices together.

A Local Search Heuristic



- Reduce to assignment problem by gluing vertices together.
- Cost = 0.96.
- Iteratively merge/segregate and solve assignment problems.
- Combine with composite columns method.

Regular Timetable

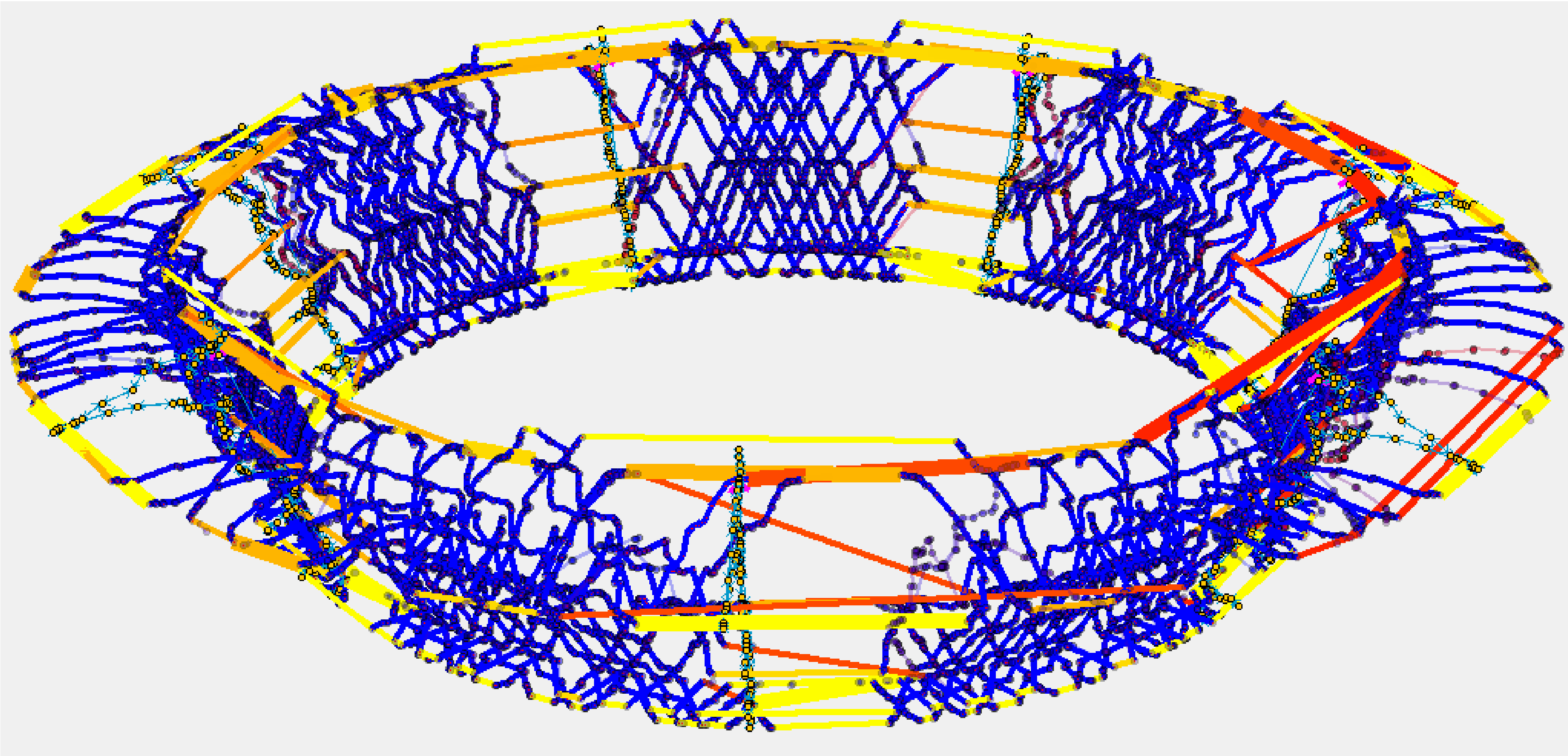


Regularity of the timetable trips:

- Red: Trip done on one day of the week
- Blue: Trip done on all days of the week

Graphics: JavaView, MATHEON F4

Regular Rotations



Regularity of the deadhead trips:

- Yellow: regular
- Red: irregular

Graphics: JavaView, MATHEON F4

Regular Rotations

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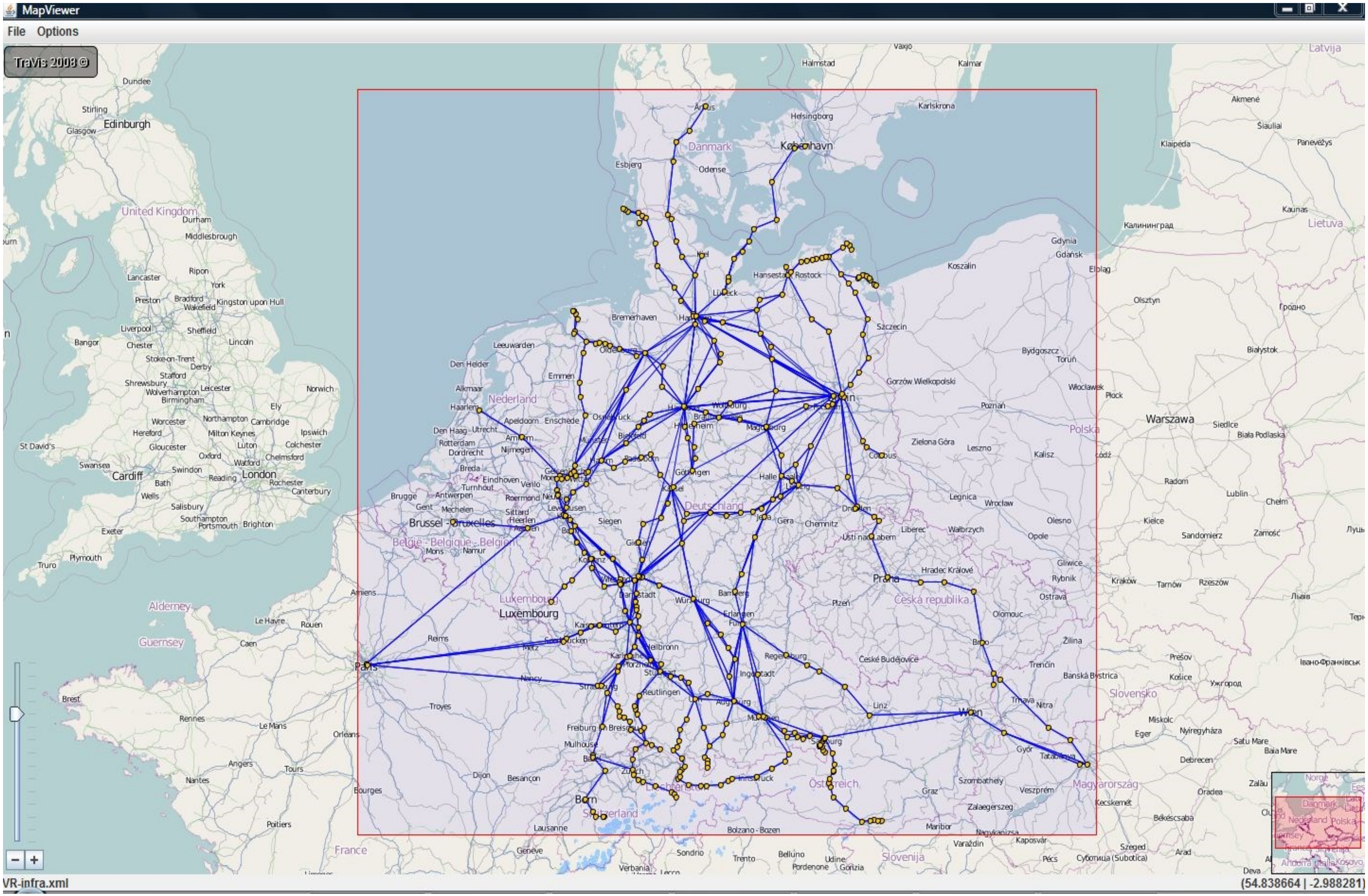
Irregularity in the optimal solution depending on the penalty for irregularity;
61 trains, 75 tasks, 803 stops, 310 trips, 620 vertices.

costs per irreg. turn	costs per irreg. trip	# hyper-edges	# irregular trips	# irregular turns	# vehic-les	time below target cost	dead-head distance cost
0	0	94035	261	310	17	2640	6298.72
1	1	109299	256	130	17	2640	6298.72
10	10	109299	76	105	17	2640	6298.72
100	100	109299	31	121	17	2640	6298.72
1000	1000	109299	25	47	17	2860	6298.13
10000	10000	109299	25	37	17	2860	32087.29

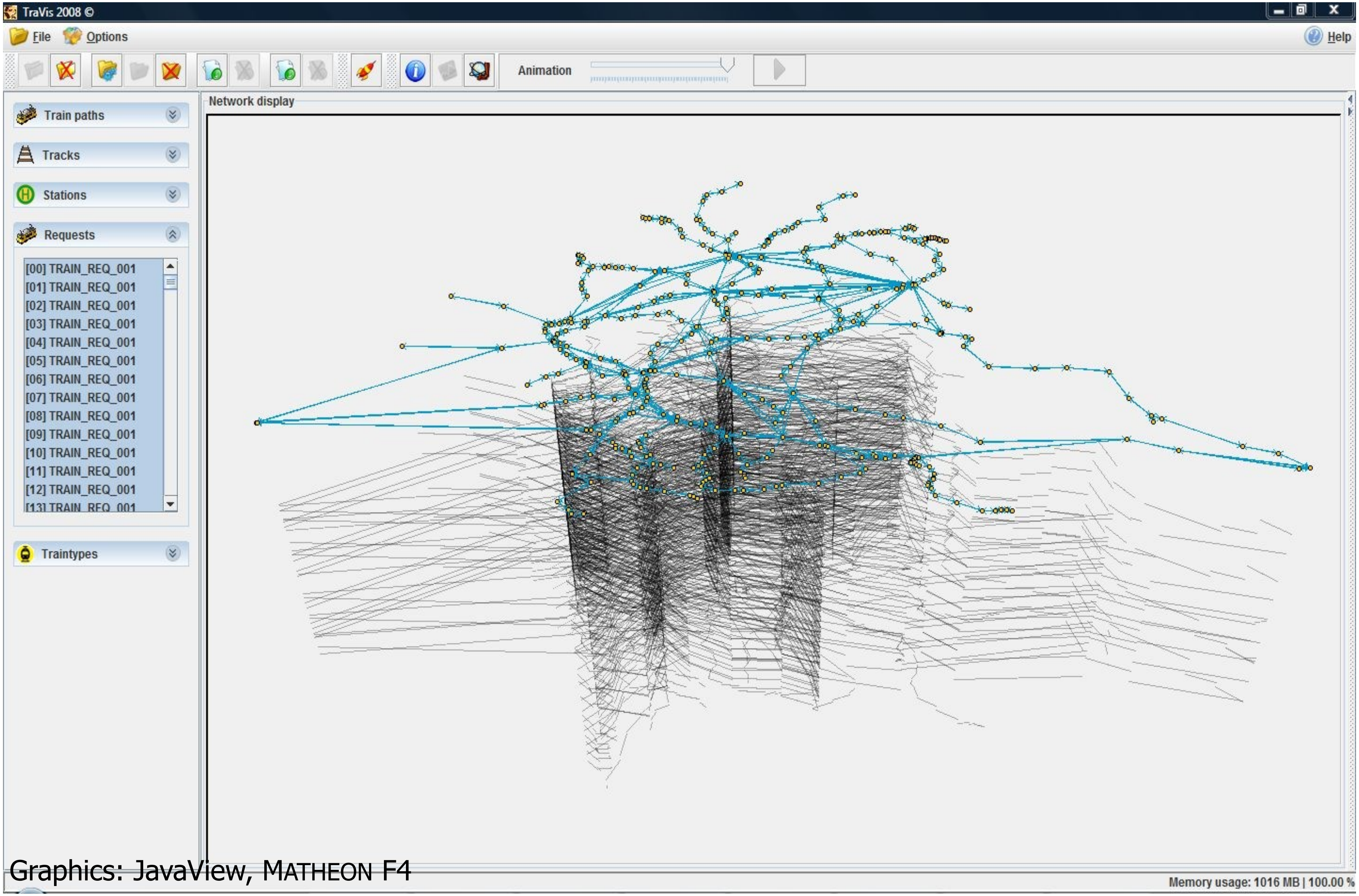
- ▷ Not explicitly optimizing regularity leads to high irregularity
- ▷ Regularity can be enforced by high penalties on arcs
- ▷ High variability

A New Technology: Vehicle Rotation Planning

Starting Point: Trip Network



Timetabled Trips: 1 Day



Graphics: JavaView, MATHEON F4

Timetabled Trips: Standard Week

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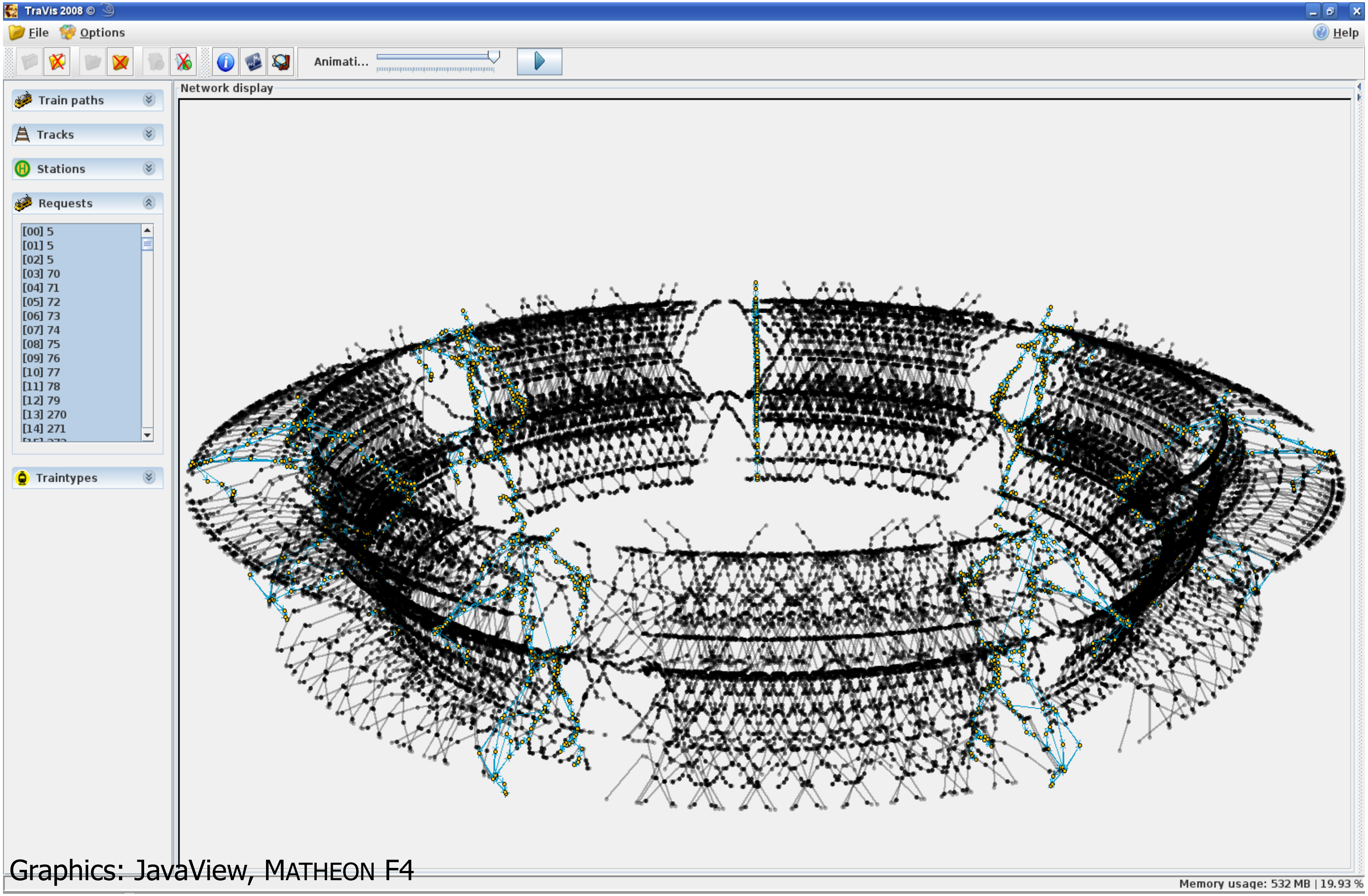
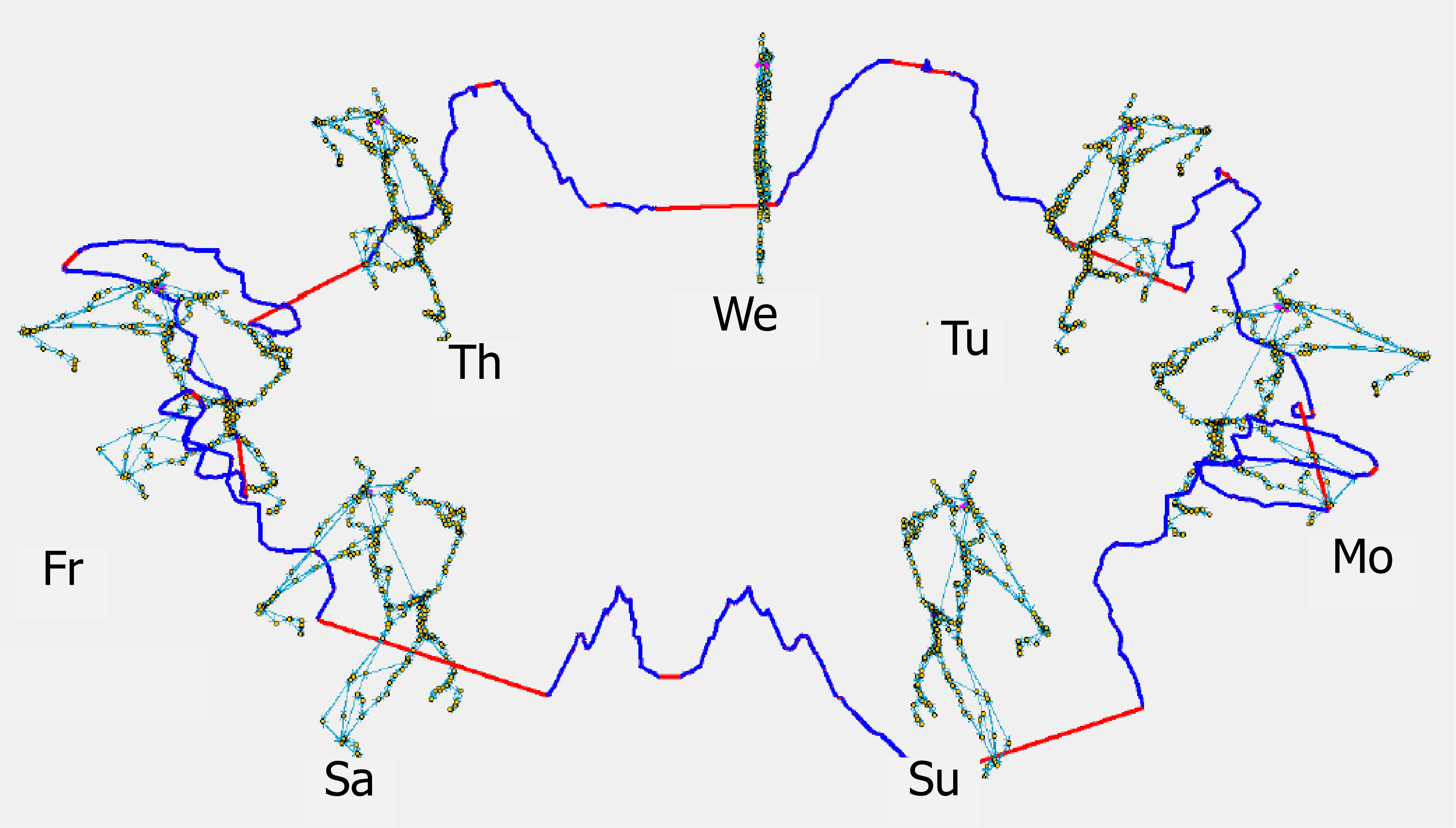


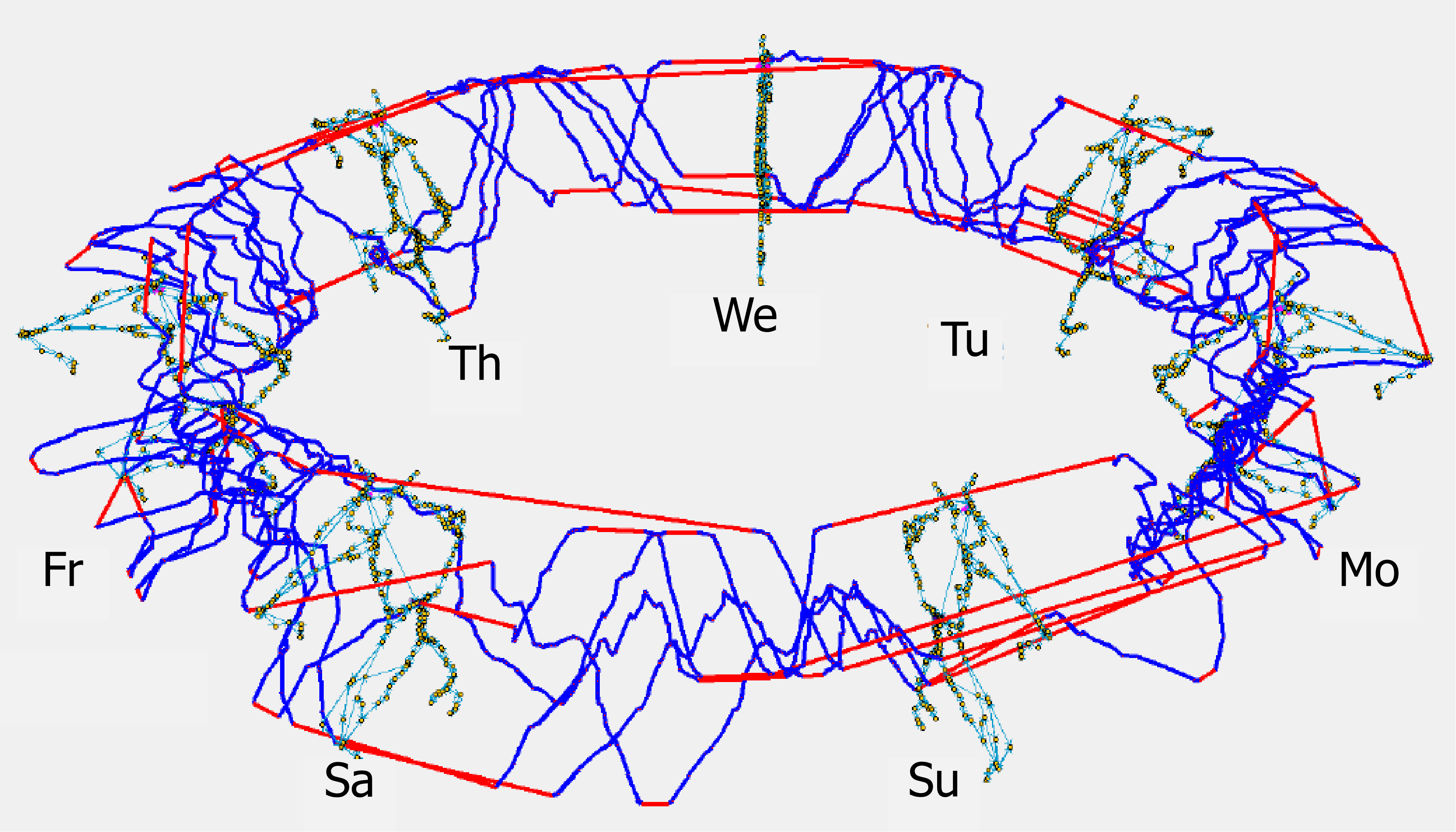
Graphics: JavaView, MATHEON F4

Vehicle Rotation: 1 Week



Graphics: JavaView, MATHEON F4

Vehicle Rotation: 5 Weeks



Graphics: JavaView, MATHEON F4

All Vehicles Rotations: Rotation Plan

(Blue: Timetabled Trips, Red: Deadhead Trips)

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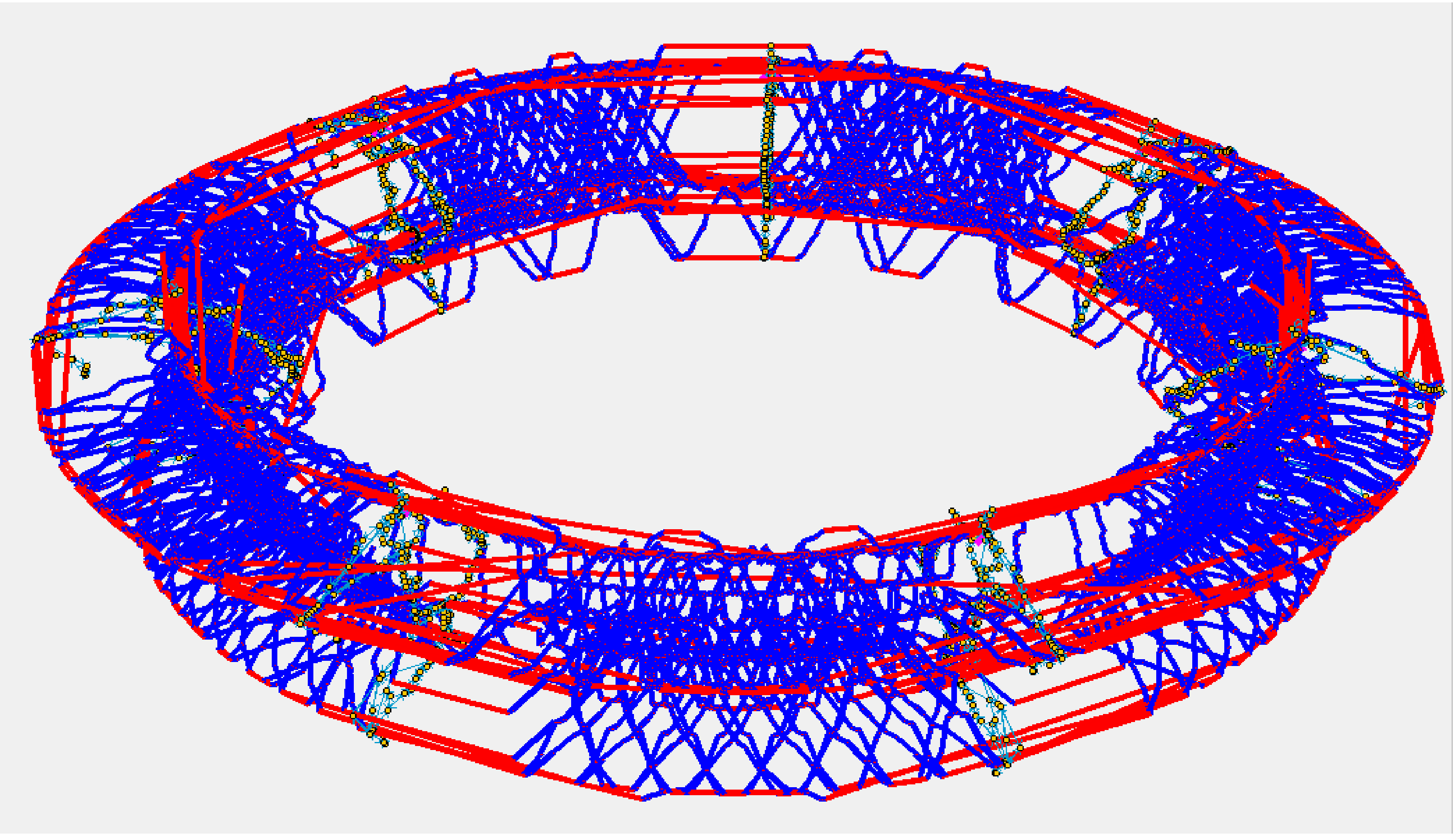

Graphics: JavaView, MATHEON F4

Railway Constraints

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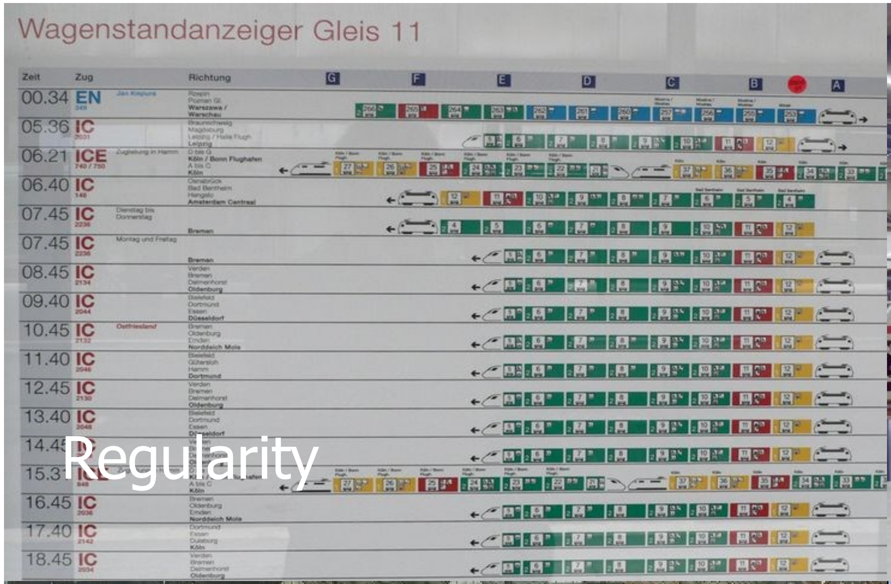
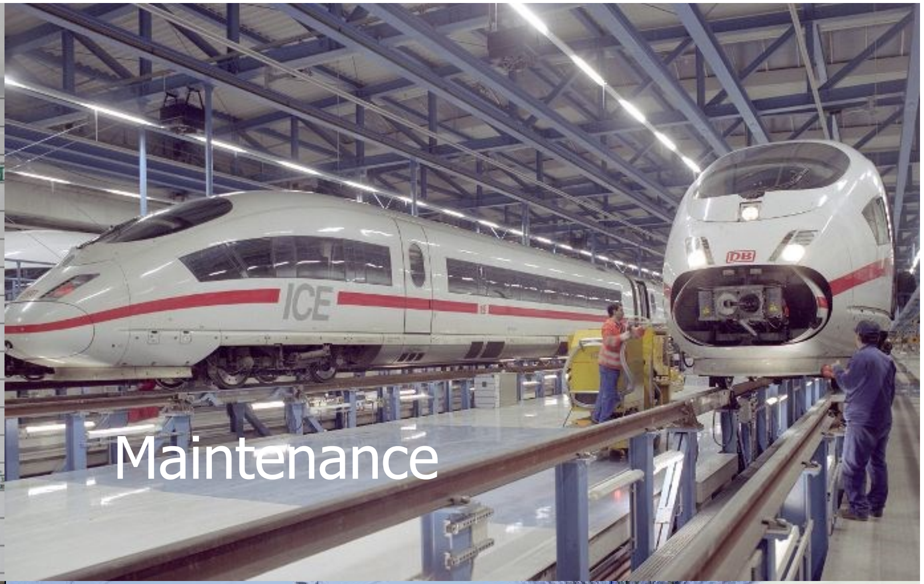
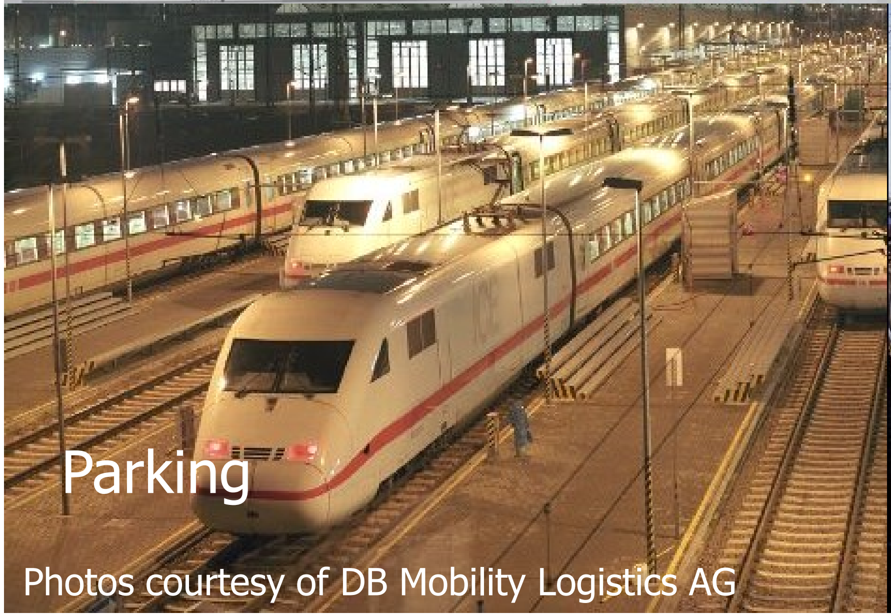
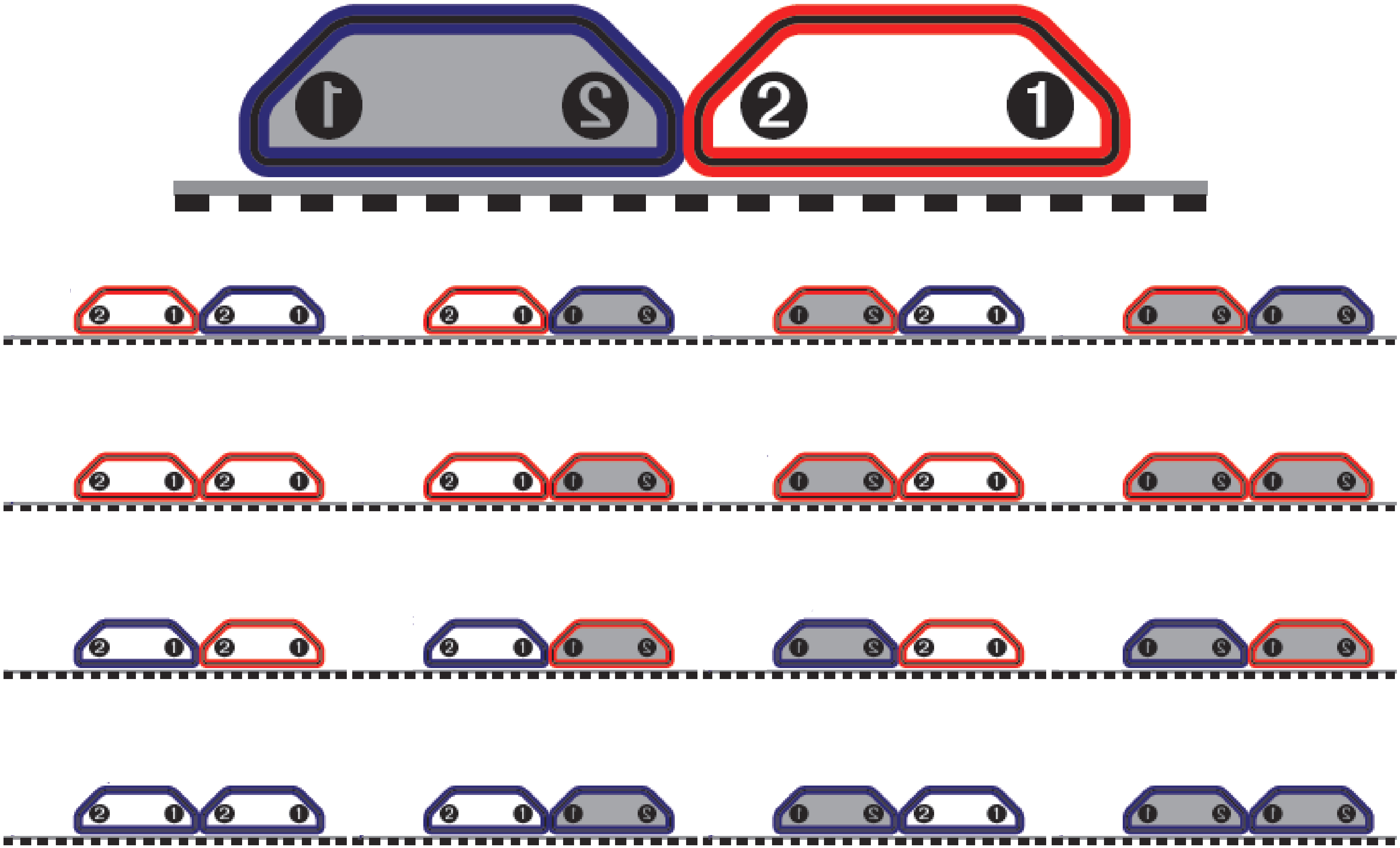




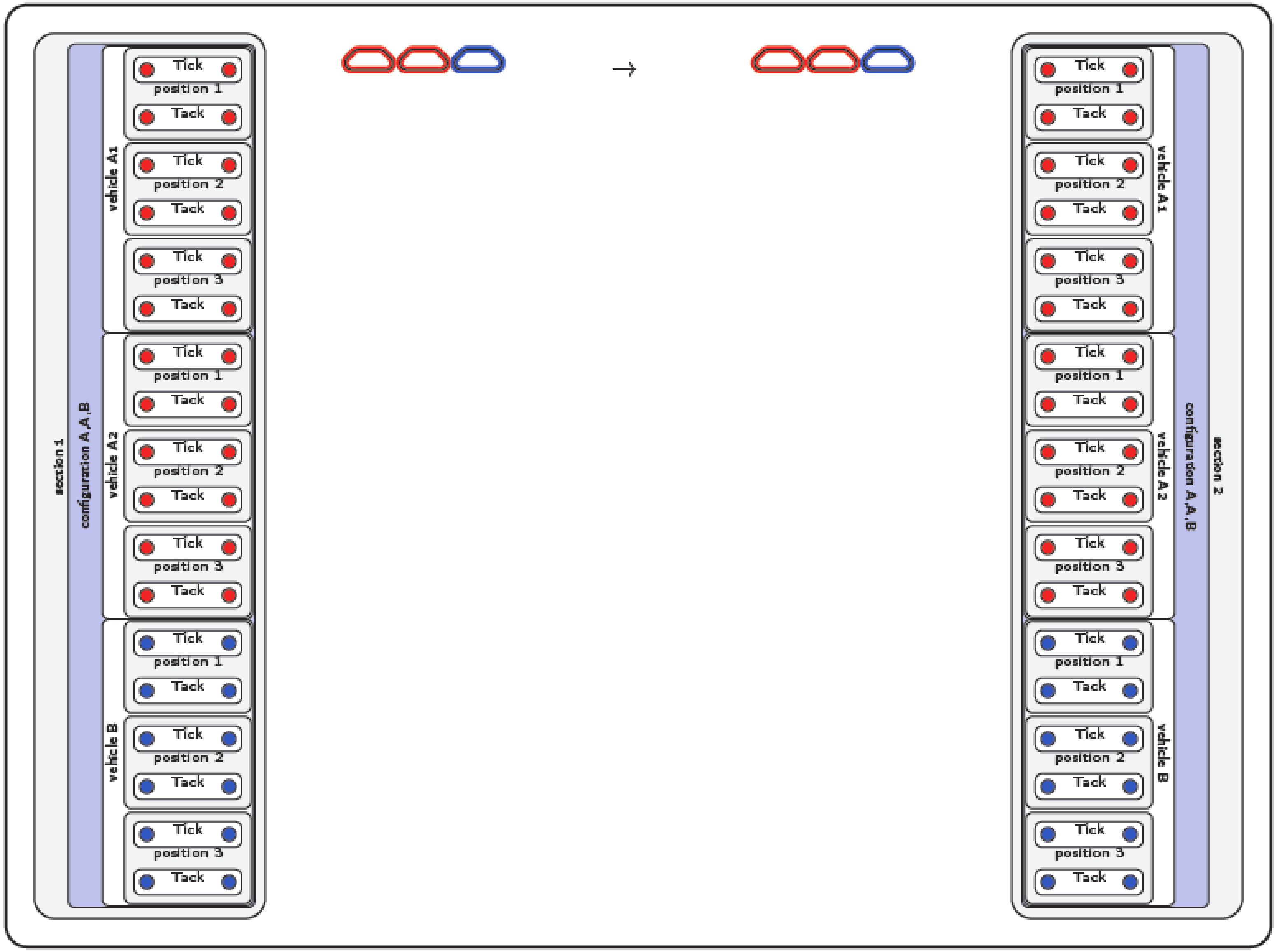


Photos courtesy of DB Mobility Logistics AG

Train Composition: Type, Order, Orientation

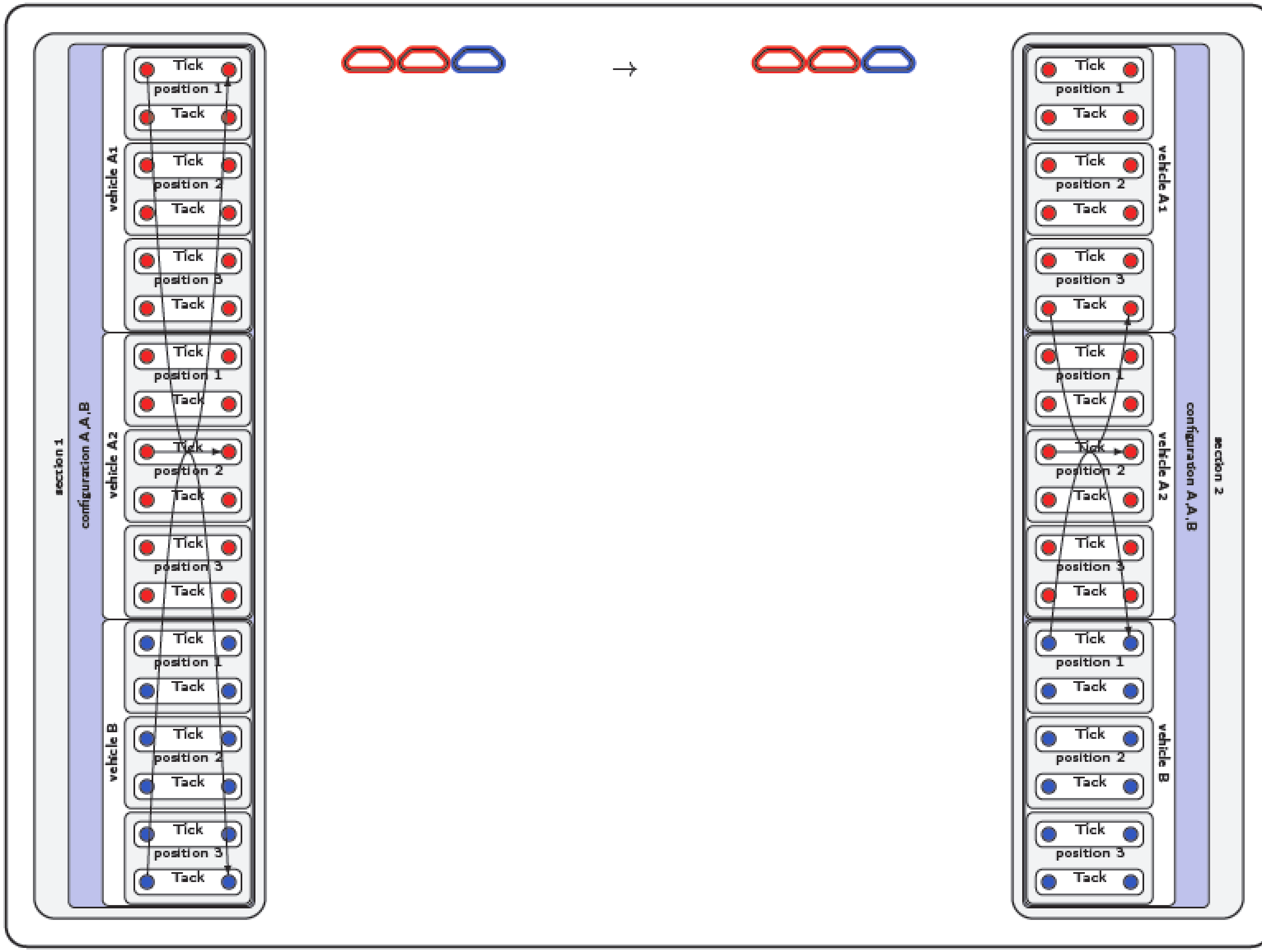


Hypergraph Model

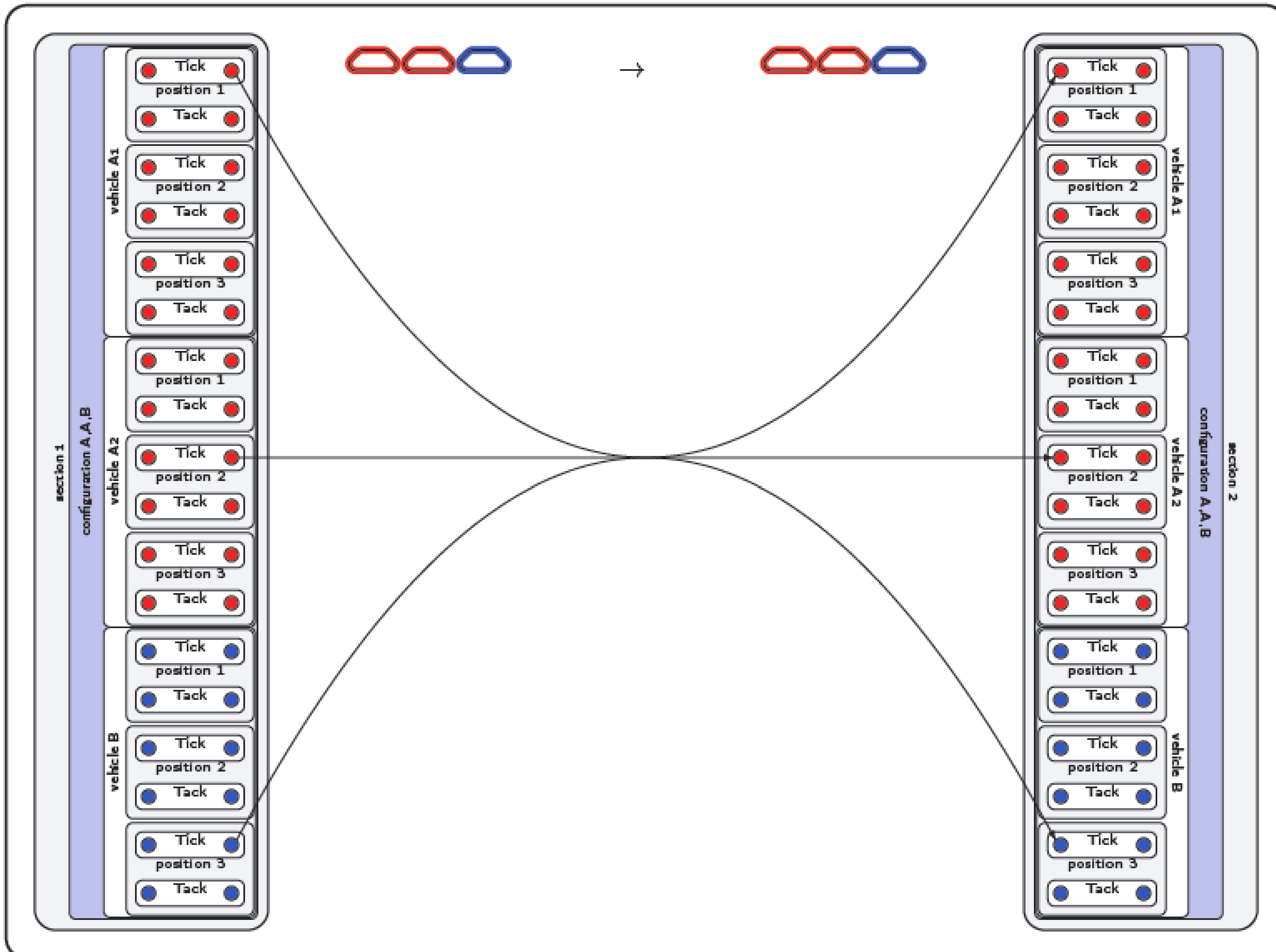


Hypergraph Model

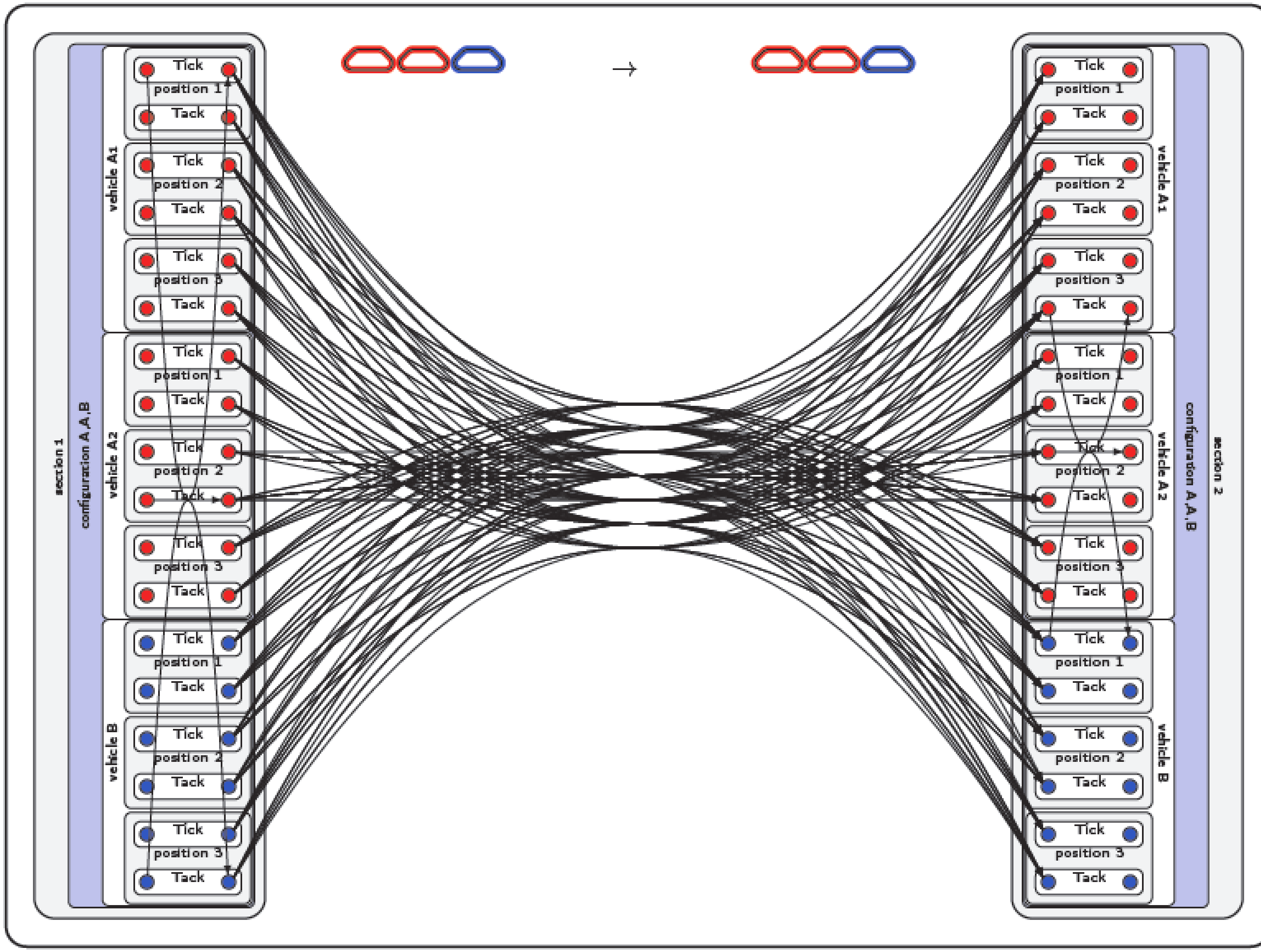
Freie Universität Berlin



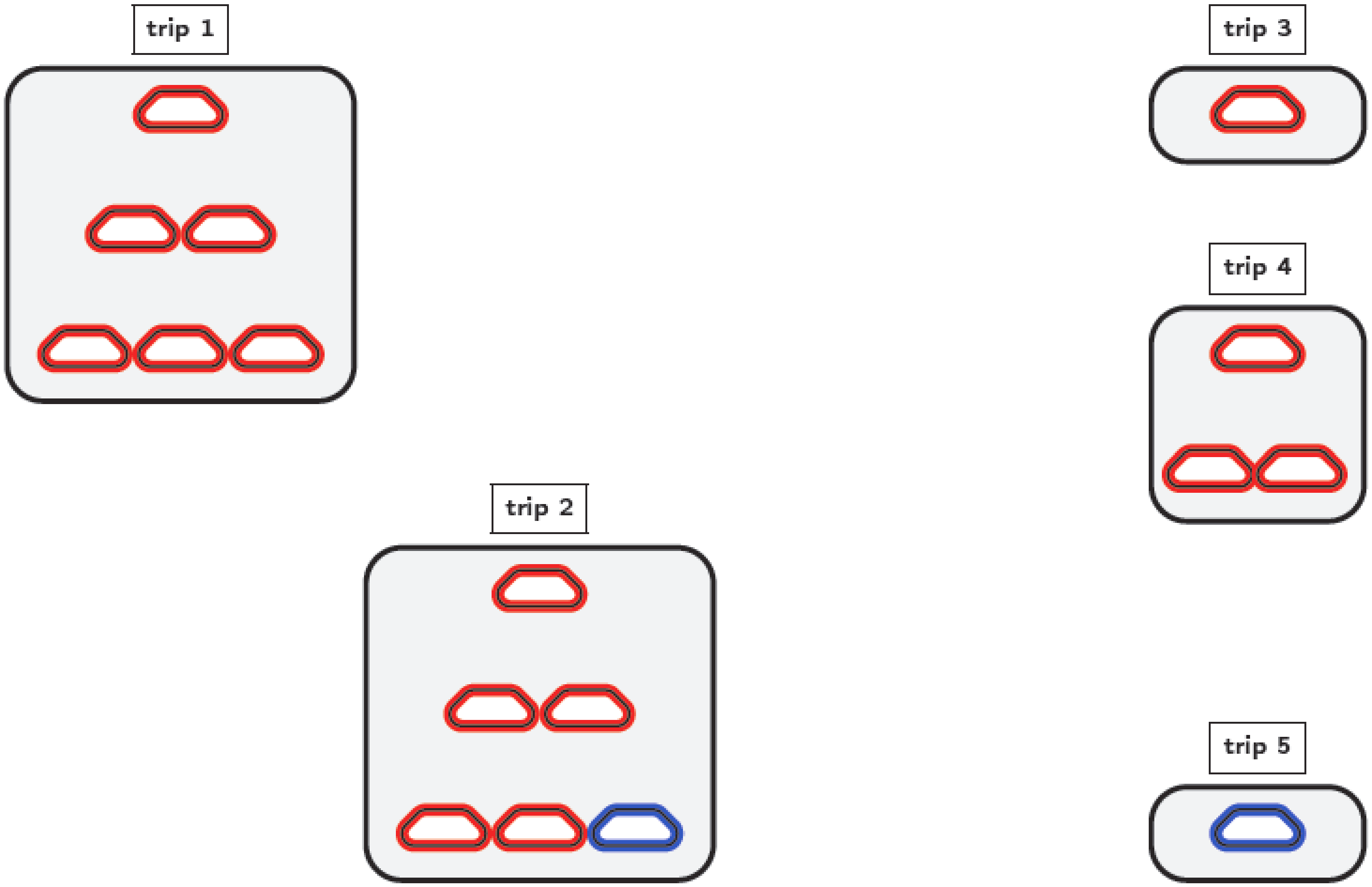
Hypergraph Model



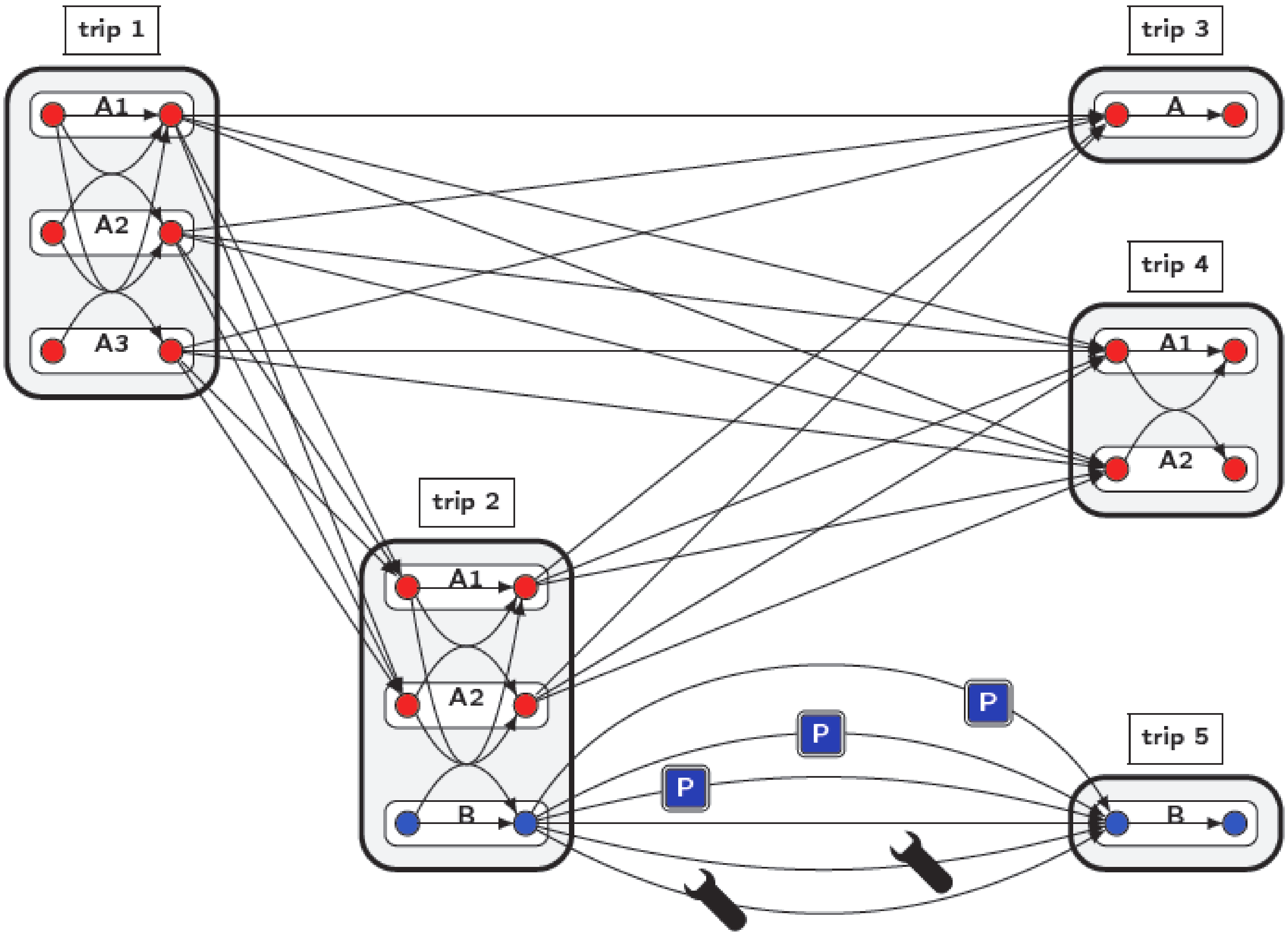
Hypergraph Model



Hypergraph Model



Hypergraph Model



Hyperflow Model

Vehicle Rotation Planning Problem

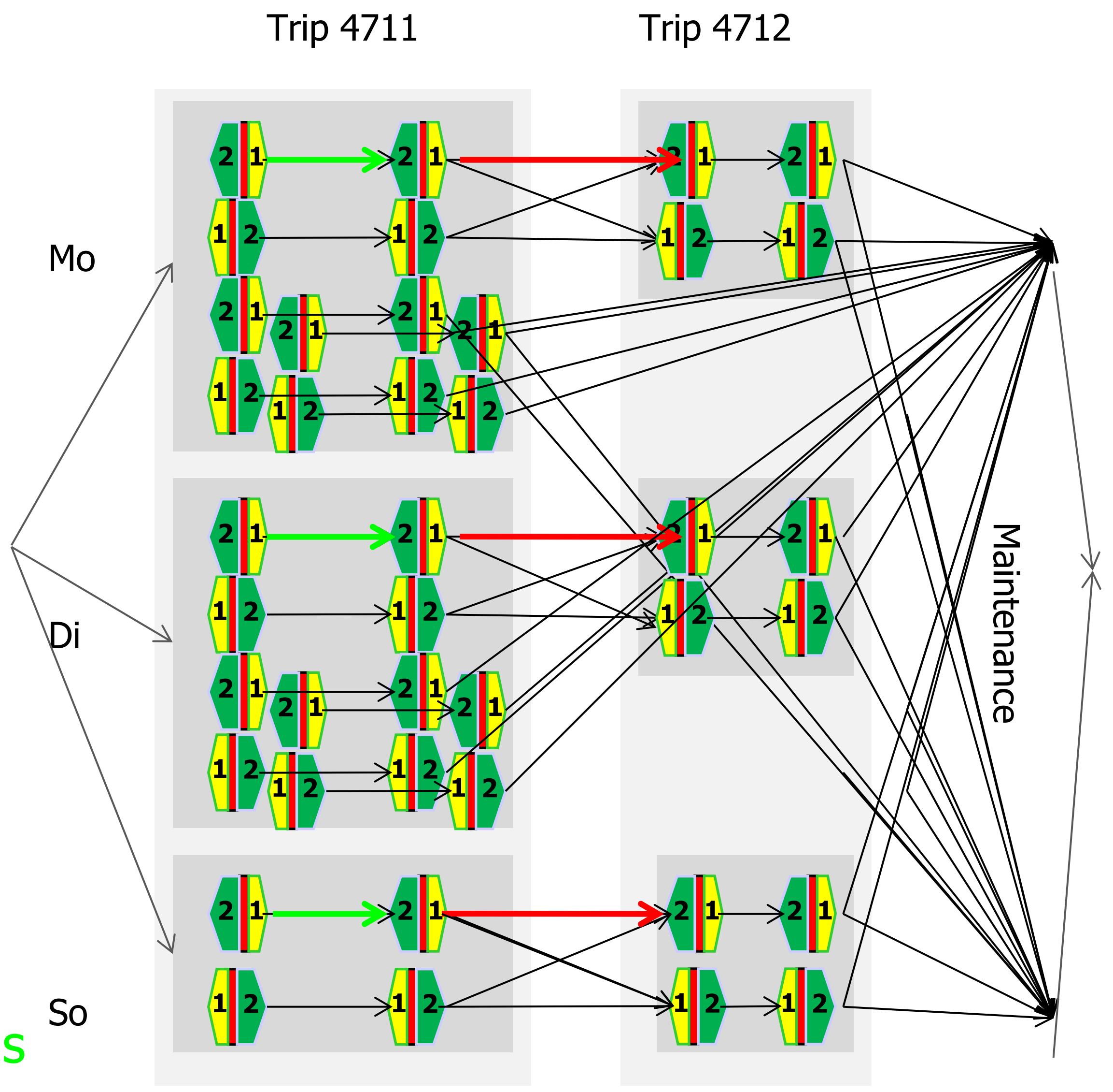
Cover all timetabled trips by rotations such that turns and train composition are regular.

Hypergraph Multi Com-modity Flow Problem

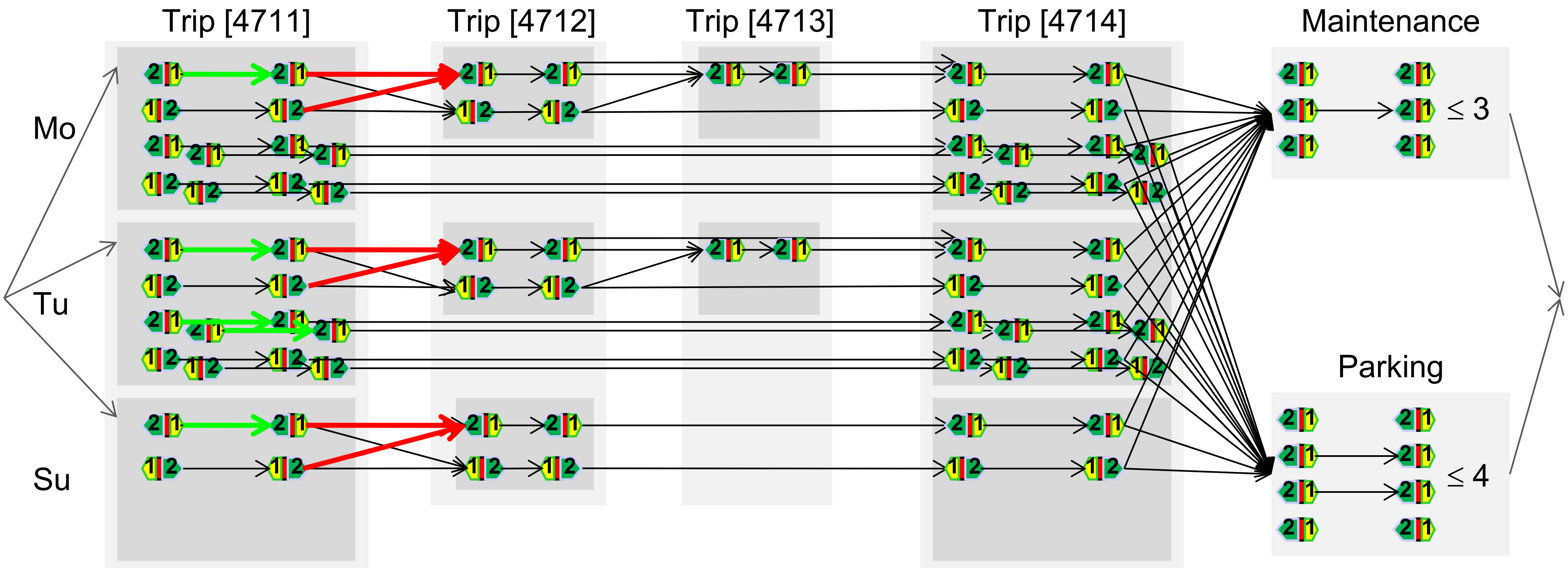
Find a cost minimal hyperflow such that every node configuration is covered by exactly one hyperarc.

Hyperarcs for

- (regular) turns
- (regular) train compositions



Hyperflow Model



- Subarc variables x_a
- Multiarc variables x_m for
  - uniform turns
  - uniform train compositions
- Flow conserv./flow constr.

$$
\begin{aligned}
&\min c^T x \\
x(d,\delta^+(v)) &= x(d,\delta^-(v)) \quad \forall v \in V, d \in D \\
x(\delta^+(t)) &= 1 \quad \forall t \in T \\
x &\in \{0,1\}^{A \cup M}
\end{aligned}
$$

Coarse-to-Fine Method: Setting

- Row and column index sets $I = [m]$, $J = [n]$
- Matrix $A \in \mathbb{R}^{I \times J}$
- Rhs $b \in \mathbb{R}^{I}$
- Objective $\mathrm{c} \in \mathbb{R}^{J}$
- Linear Program and its dual

$$
\begin{aligned}
\min c^T x & \\
Ax &= b \\
x &\geq 0
\end{aligned}
$$

$$
\begin{aligned}
\max\ y^T b & \\
y^T A &= c^T \\
y &\in \mathrm{IR}^I
\end{aligned}
$$

Coarse-to-Fine Method: Notation

- Aggregate/project the rows I of the (LP) by a problem specific coarsening projection $[]: I \to [I]$ (induces an equivalence relation)

- For a column vector $v \in \mathbb{R}^I$ we define the coarsening of v as

  $$[v][i] := (\min\{v_k : k \in I, [k] = [i]\}, \max\{v_k : k \in I, [k] = [i]\}) \cdot \tau(v, i)$$

  where $\tau(v, i) := |\{v_k \neq 0 : [k] = [i]\}|$

- Coarse bimatrix $[A]$, coarse dual vector $[\pi]$

- Coarse objective function $[c] := c$ (no coarsening)

Coarse Reduction

$$\min[c]^T x, [A]x[=][b], x \in \mathbb{R}^{[J]},$$

where

$$[A]x[=][b] :\Leftrightarrow [b]_{[i]2} \leq \sum_{j\in J} [A_{\cdot j}]_{[i]1} x_j,\ \sum_{j\in J} [A_{\cdot j}]_{[i]1} x_j \leq [b]_{[i]2},\ \ \forall [i].$$

Let $P(A,b) := \{Ax = b, x \geq 0\}$ and $P([A],[b]) := \{[A]x[=][b], x \geq 0\}$.

Lemma (B., Reuther, Schlechte, Weider [2015])

$$P(A,b) \subseteq P([A],[b]).$$

The Coarse Reduced Cost

- Multiplication of pairs $(a_1, b_1), (a_2, b_2) \in \mathbb{R}^2$:

$$(a_1, b_1), \cdot (a_2, b_2) \coloneqq \max\{a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2\}$$

- Coarse reduced cost for column j

$$[\overline{c}_j] \coloneqq [c_j] - [\pi]^T \cdot [a_j]$$

Lemma (B., Reuther, Schlechte, Weider [2015])

The coarse reduced cost always underestimates the original reduced cost

$$[\overline{c}_j] \coloneqq [c_j] - [\pi]^T \cdot [a_j] \leq c_j - \pi^T \cdot a_j = \bar{c}_j.$$

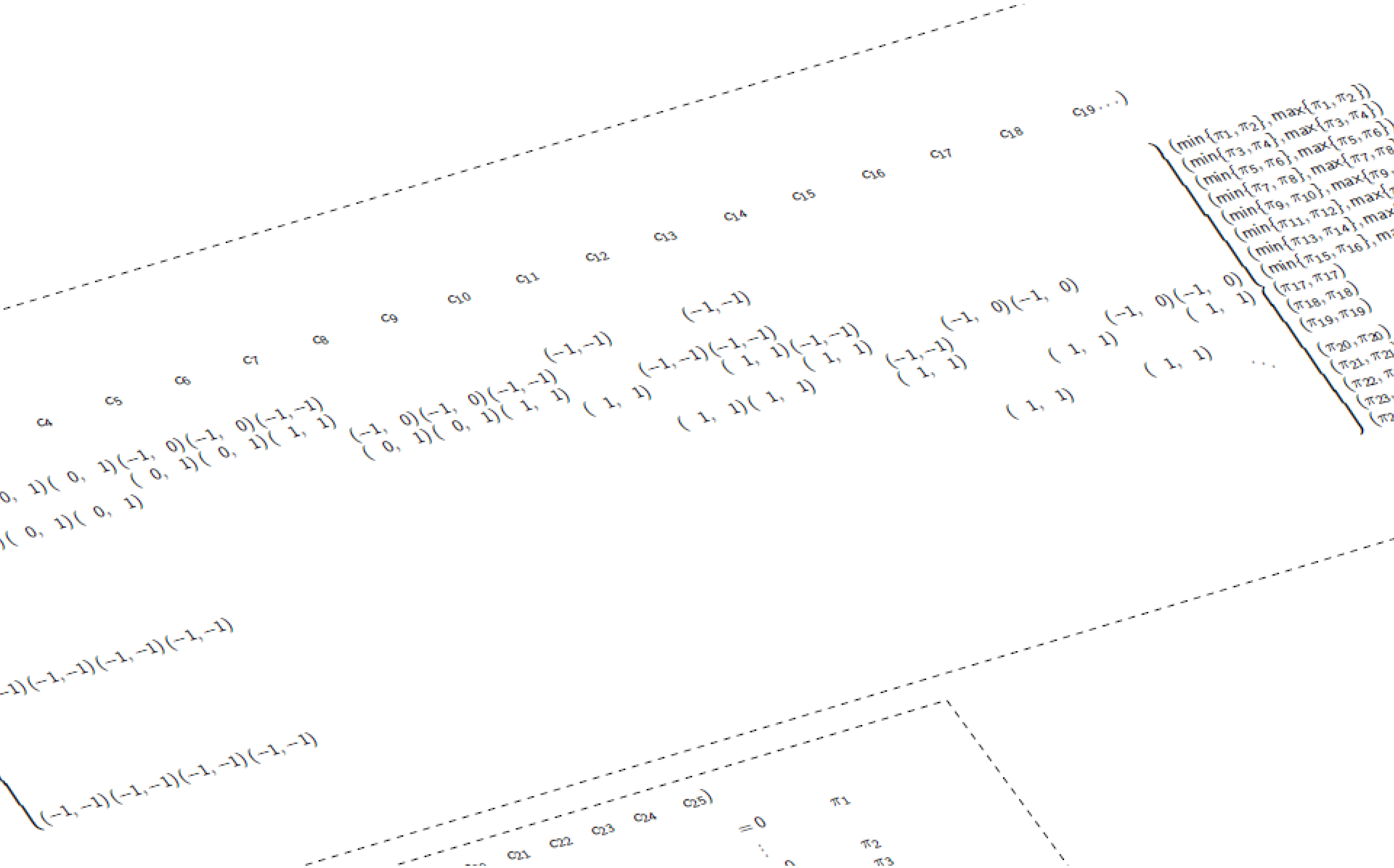
- Use the coarse reduced cost for pricing in the fine model.

Example

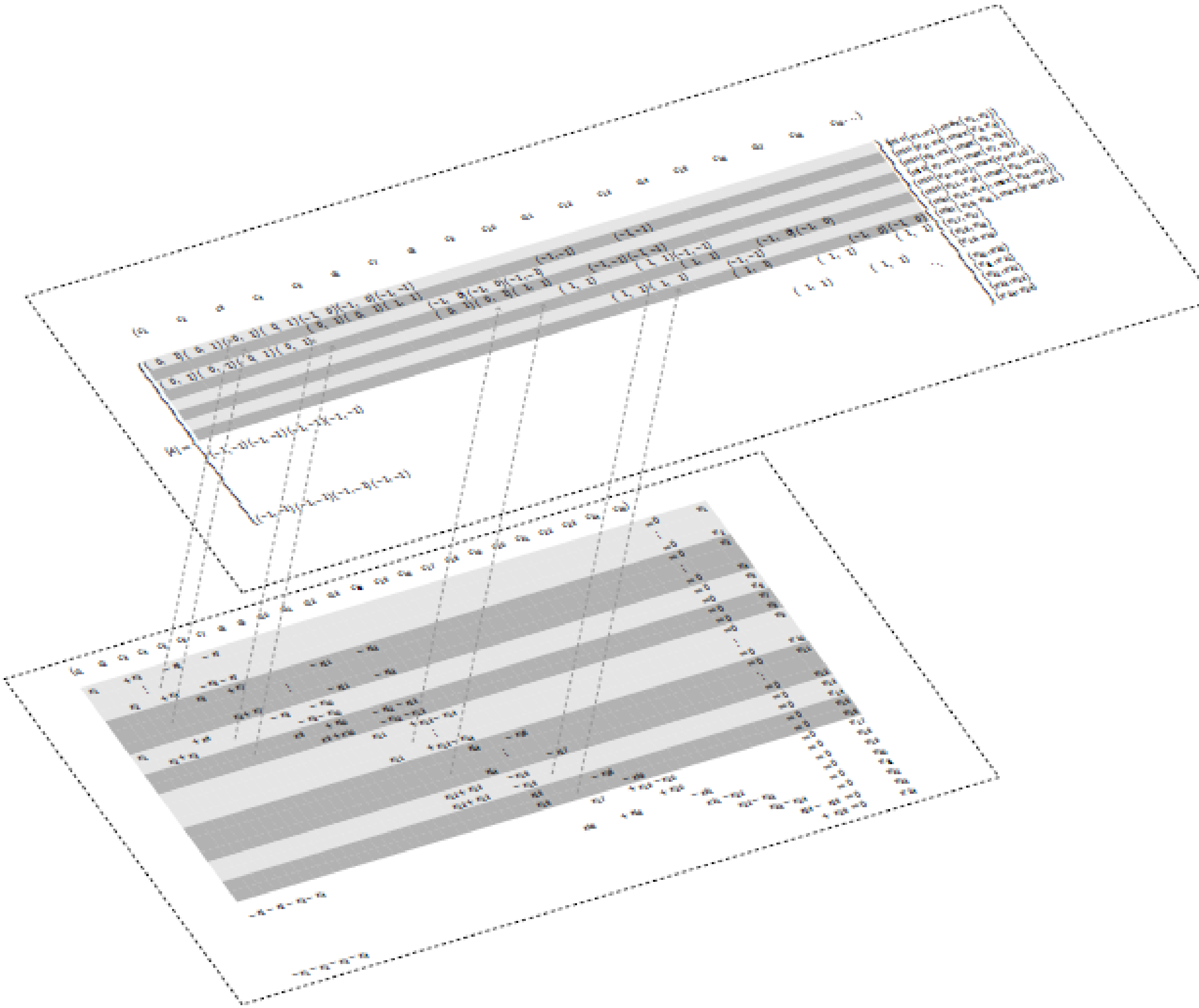
$$
(c_1 \quad c_2 \quad c_3 \quad c_4 \quad c_5 \quad c_6 \quad c_7 \quad c_8 \quad c_9 \quad c_{10} \quad c_{11} \quad c_{12} \quad c_{13} \quad c_{14} \quad c_{15} \quad c_{16} \quad c_{17} \quad c_{18} \quad c_{19} \quad c_{20} \quad c_{21} \quad c_{22} \quad c_{23} \quad c_{24} \quad c_{25})
$$

$$
\begin{array}{lll}
x_1 + x_3 - x_5 - x_7 & = 0 & \pi_1 \\
\vdots & \vdots & \\
x_2 + x_4 - x_6 - x_7 & = 0 & \pi_2 \\
x_5 + x_7 - x_{11} - x_{13} & = 0 & \pi_3 \\
\vdots & \vdots & \\
x_6 + x_7 - x_{11} - x_{13} & = 0 & \pi_4 \\
x_1 + x_4 - x_8 - x_{10} & = 0 & \pi_5 \\
x_2 + x_3 - x_9 - x_{10} & = 0 & \pi_6 \\
x_8 + x_{10} - x_{12} - x_{13} & = 0 & \pi_7 \\
x_9 + x_{10} - x_{12} - x_{13} & = 0 & \pi_8 \\
x_{11} + x_{13} - x_{14} & = 0 & \pi_9 \\
\vdots & \vdots & \\
x_{11} + x_{13} - x_{14} & = 0 & \pi_{10} \\
x_{14} - x_{16} & = 0 & \pi_{11} \\
\vdots & \vdots & \\
x_{14} - x_{17} & = 0 & \pi_{12} \\
x_{12} + x_{13} - x_{15} & = 0 & \pi_{13} \\
x_{12} + x_{13} - x_{15} & = 0 & \pi_{14} \\
x_{15} - x_{18} & = 0 & \pi_{15} \\
x_{15} - x_{19} & = 0 & \pi_{16} \\
x_{17} + x_{19} - x_{20} & = 0 & \pi_{17} \\
-x_1 - x_2 - x_3 - x_4 + x_{20} & = 0 & \pi_{18} \\
x_{16} + x_{18} - x_{21} & = 0 & \pi_{19} \\
x_{21} - x_{22} & = 0 & \pi_{20} \\
x_{22} - x_{23} & = 0 & \pi_{21} \\
x_{23} - x_{24} & = 0 & \pi_{22} \\
x_{24} - x_{25} & = 0 & \pi_{23}
\end{array}
$$

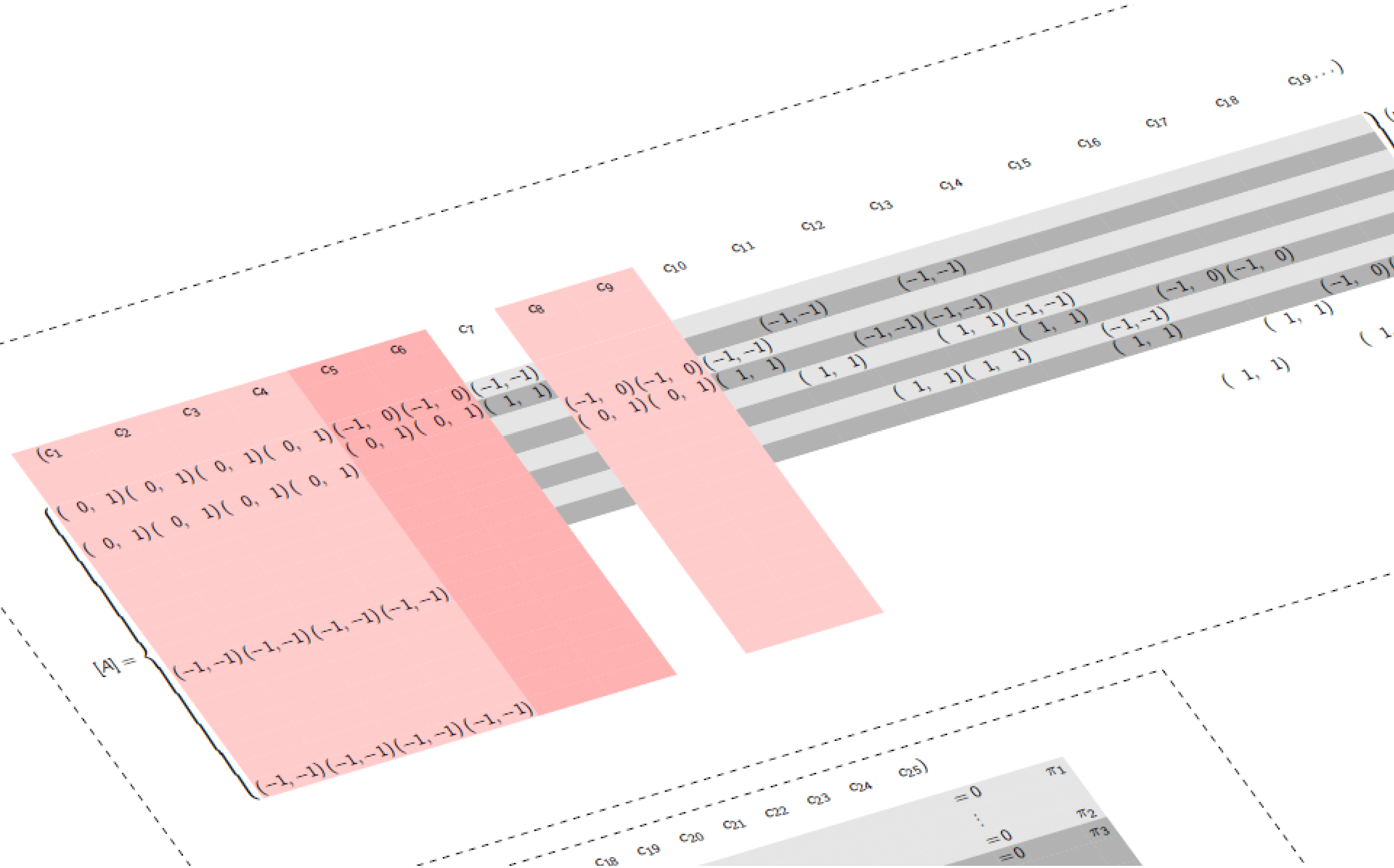
Example


c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14 c15 c16 c17 c18 c19 ...)
c21 c22 c23 c24 c25)
= 0

Example

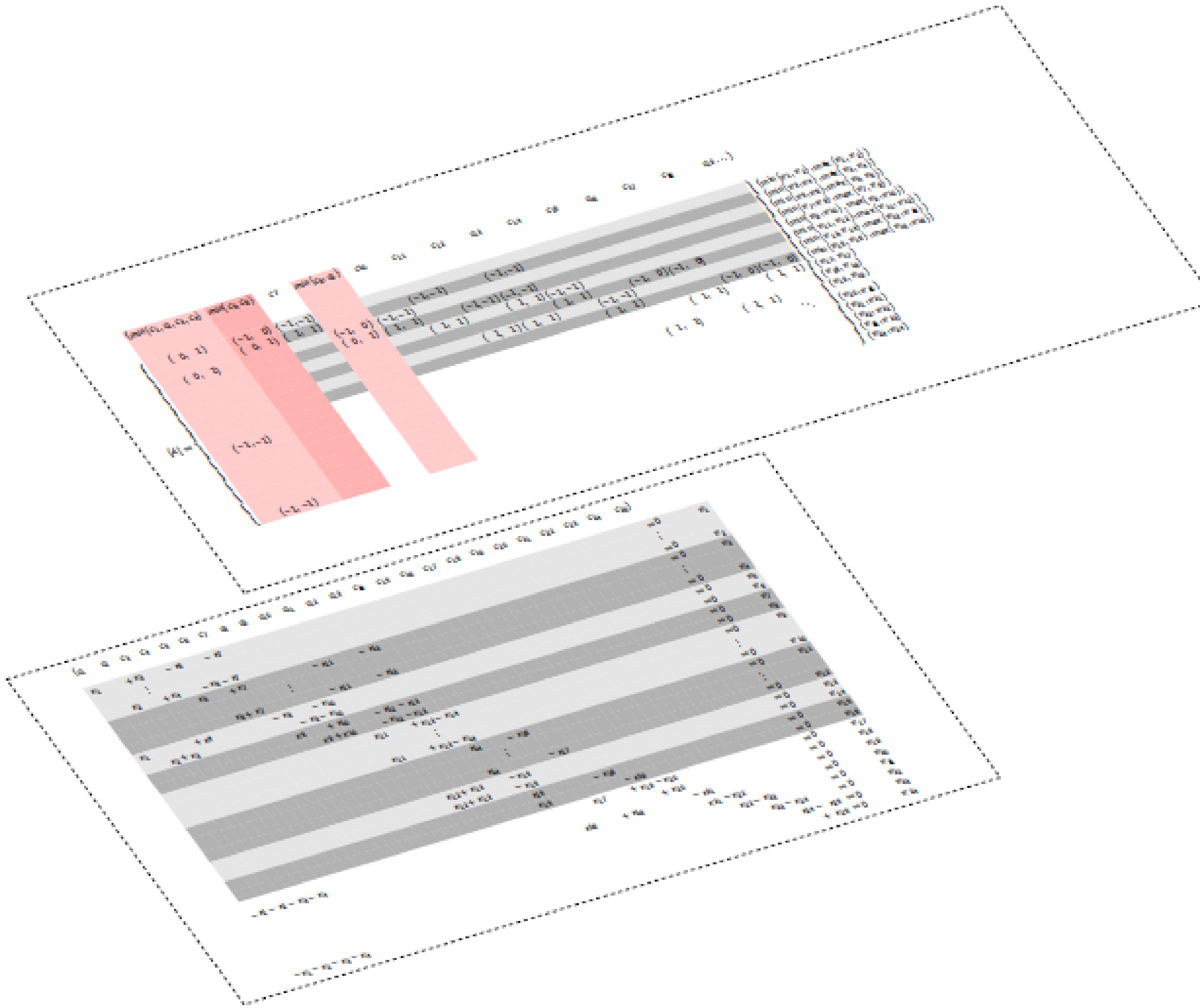


Example

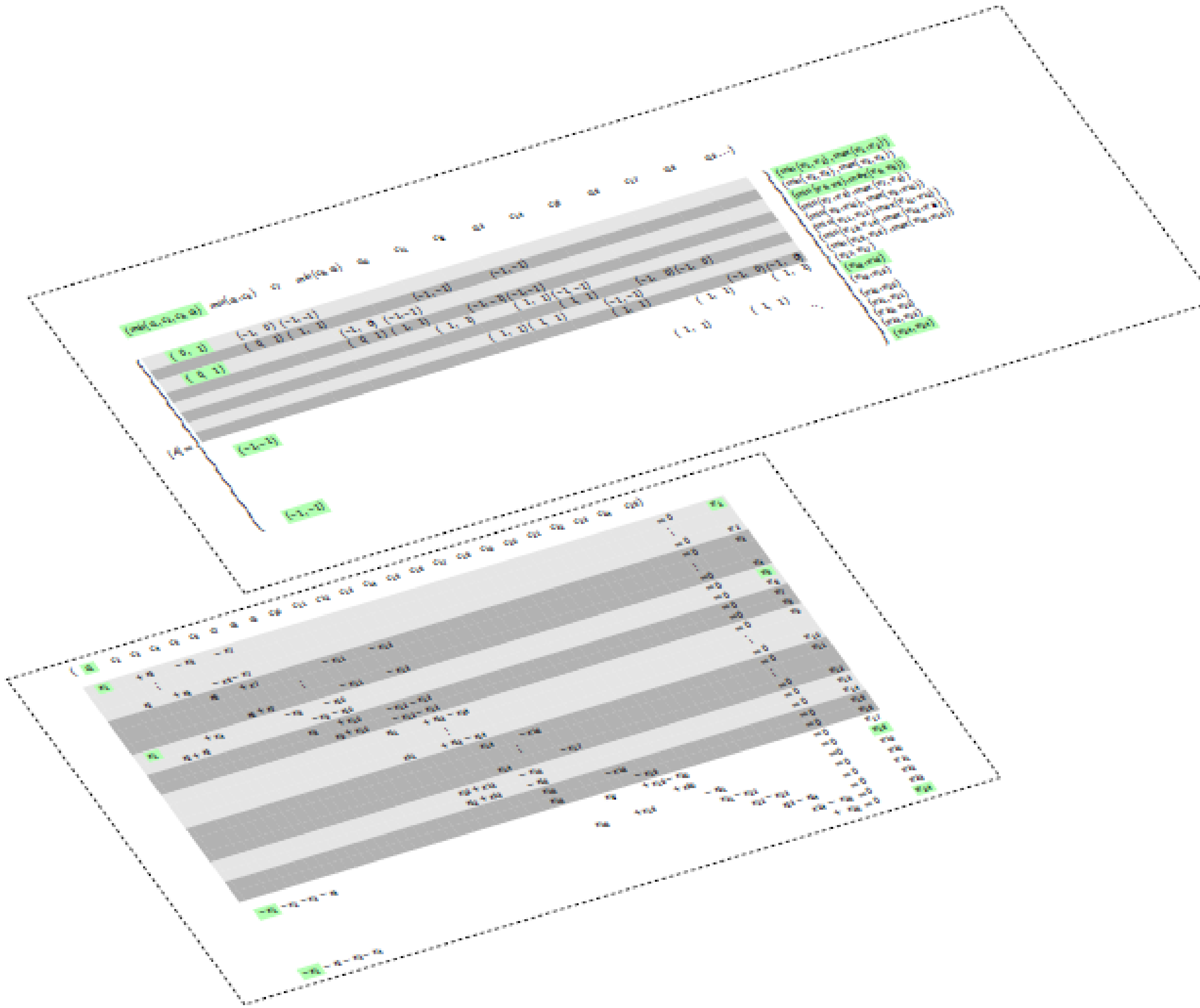


$[A] =$
$(c_1$ c_2 c_3 c_4 c_5 c_6 c_7 c_8 c_9 c_{10} c_{11} c_{12} c_{13} c_{14} c_{15} c_{16} c_{17} c_{18} $c_{19}\ldots)$
c_{18} c_{19} c_{20} c_{21} c_{22} c_{23} c_{24} $c_{25})$
$=0$
$=0$
$=0$
π_1
π_2
π_3

Example



Example



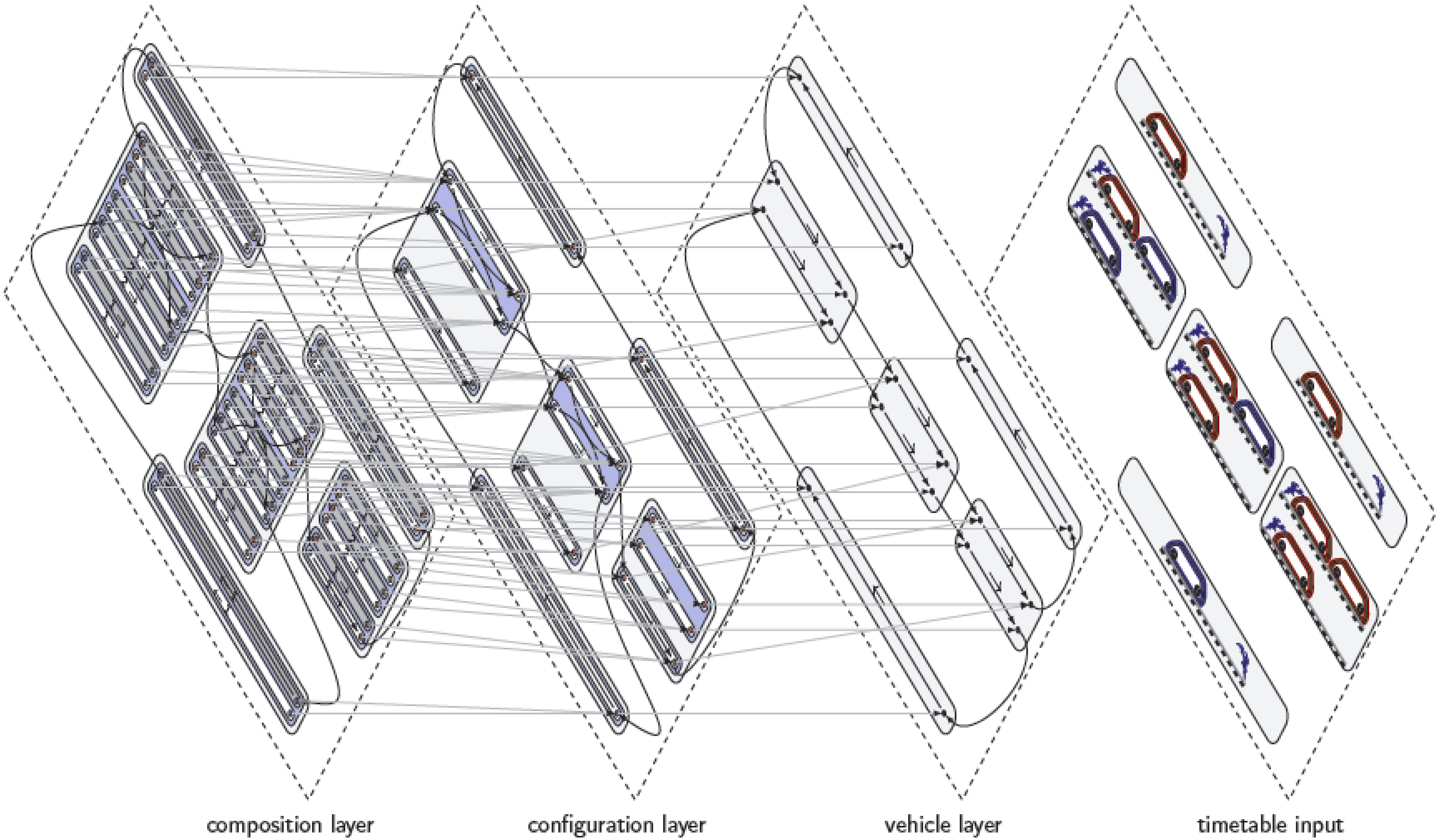
Layers

Problem specific layers:

- composition layer (fine, already seen)
- configuration layer (coarse)
- vehicle layer (very coarse)

The layers are defined in terms of projections of hypergraphs that correspond to the projection of rows of the LP/IP.

Coarse-to-Fine Method



Railway Constraints

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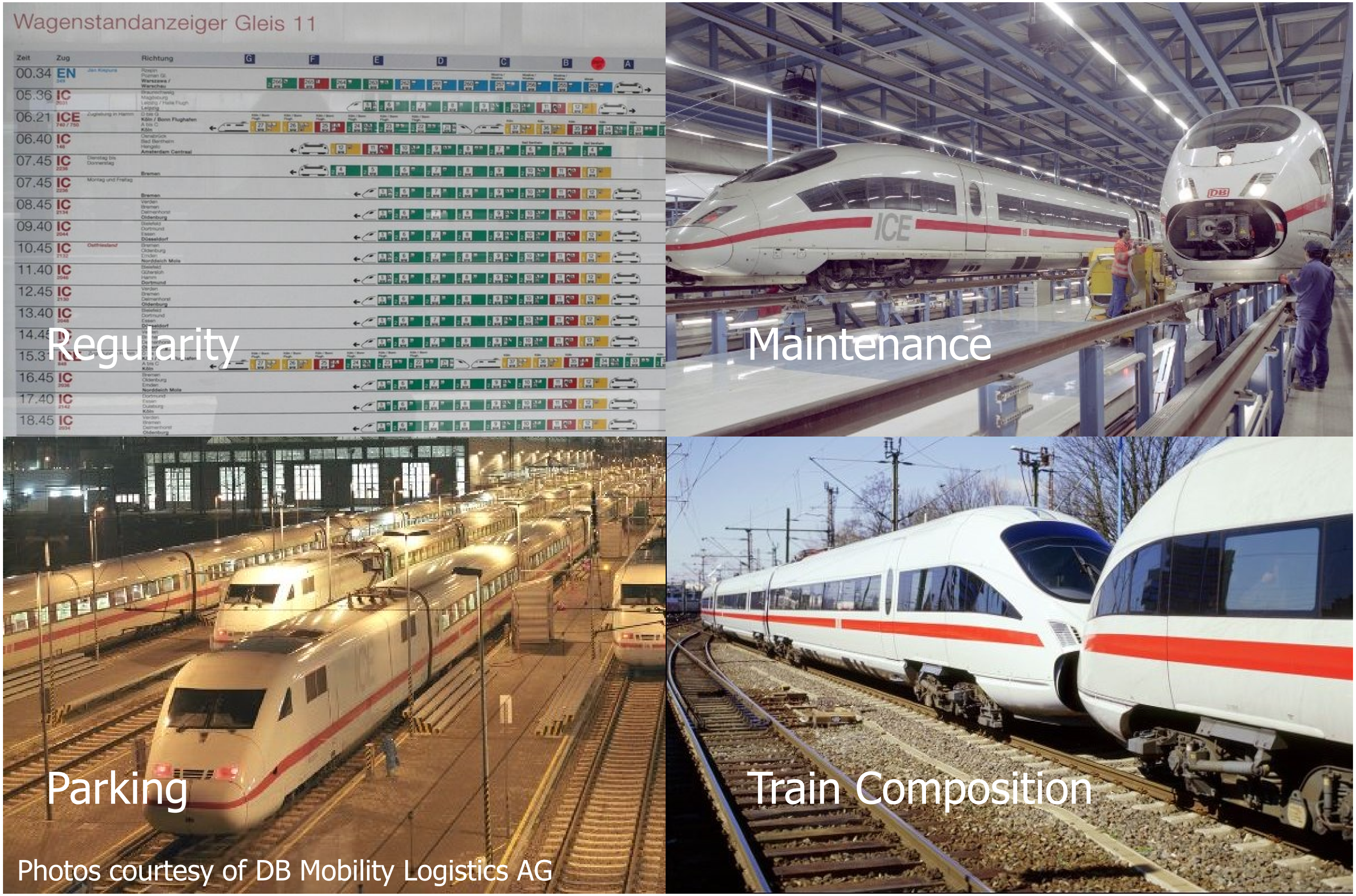

Photos courtesy of DB Mobility Logistics AG

Maintenance: Service Intervals

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Blue: timetabled trips

Green: 4000 km treatment

Dark gray: 8250 km treatment

Yellow: 33000 km treatment

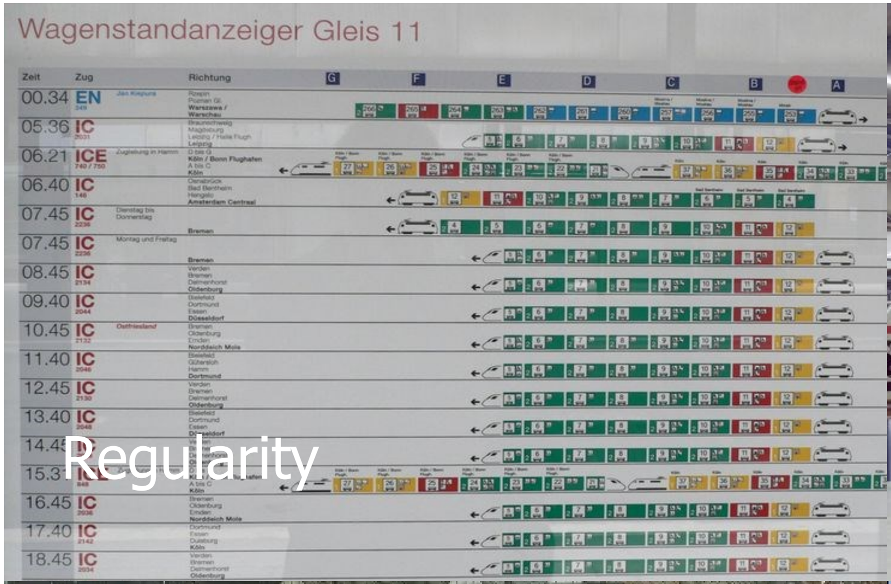
Pink: 66000 km treatment

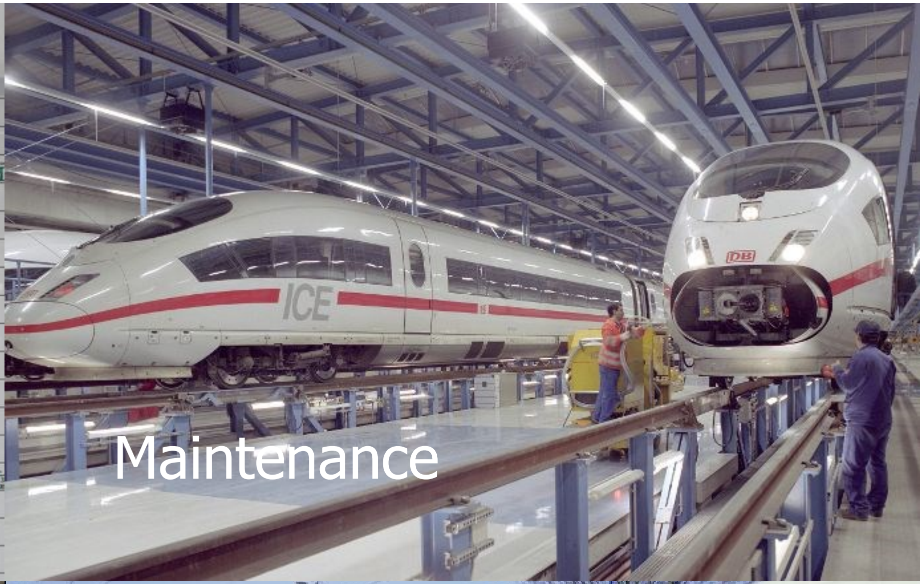
Red: 198000 km treatment

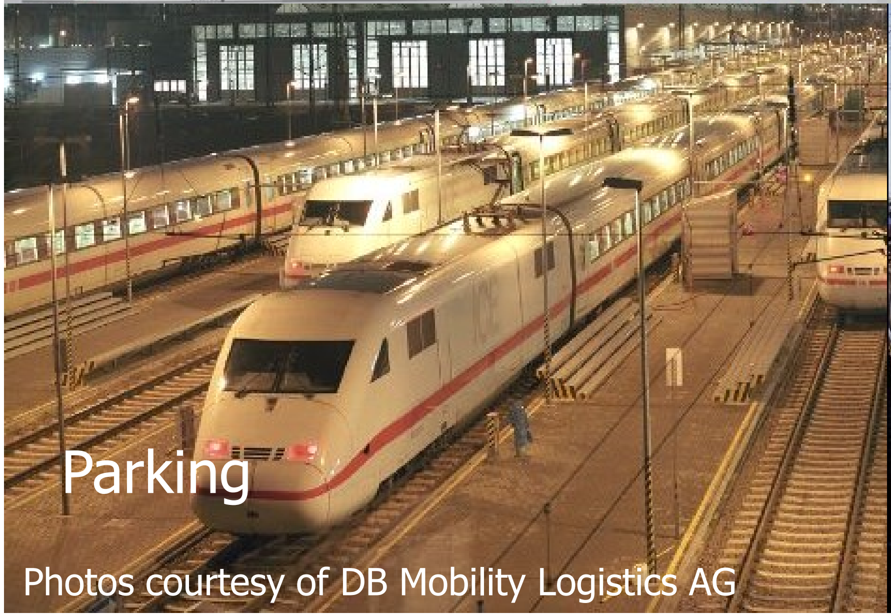
Light gray: 15 days treatment

Turquoise: 30 days treatment

Railway Constraints

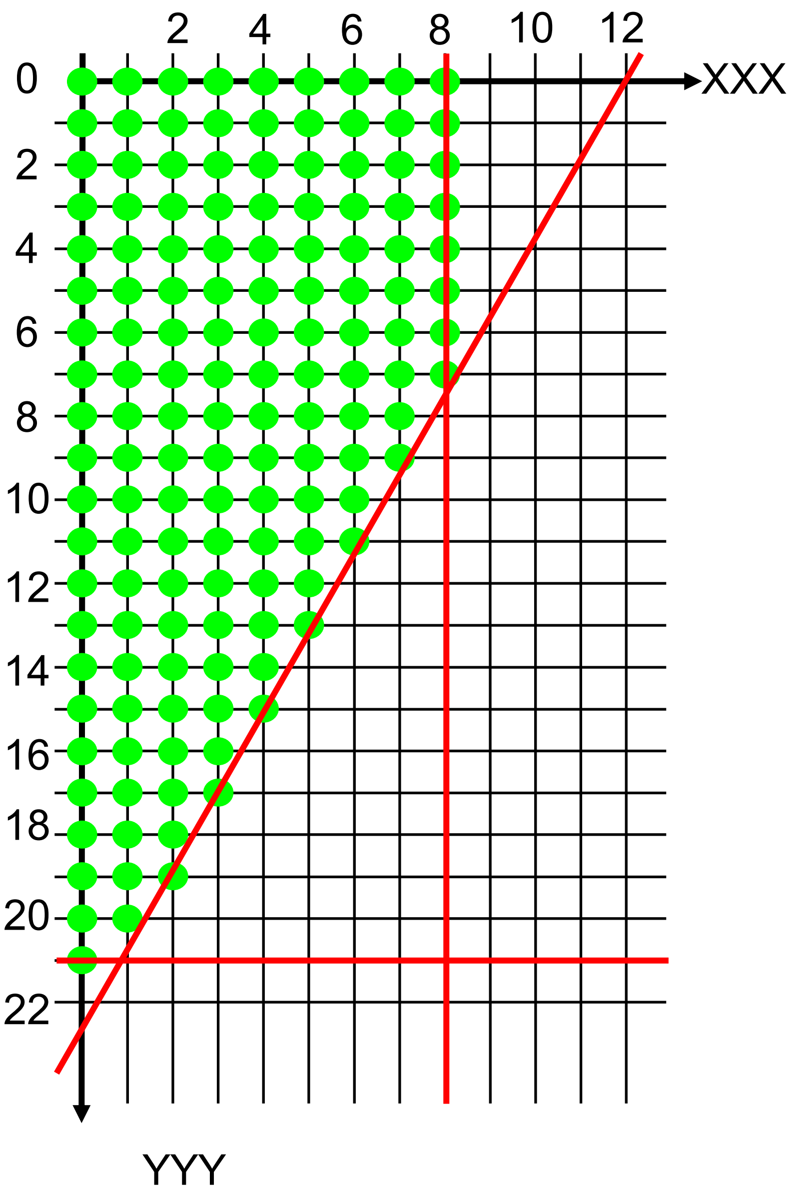






Photos courtesy of DB Mobility Logistics AG

Parking: Keeping It Simple



Siding	Length/m	Feasible Assignments
1	570	XXX, YYY, XXX+YYY, YYY+YYY
2	480	XXX, YYY, YYY+YYY
3	430	XXX, YYY, YYY+YYY
4,5,6	420	XXX, YYY, YYY+YYY
7	410	XXX, YYY, YYY+YYY
8	390	XXX, YYY
9,10,11	240	YYY
12,13,14	210	YYY

Real World Example: Scenario 1

Input	#	Objective	Goal
Timetabled trips	798	Coverage	100%
Connections	171	Rows	Minimum
Maintenance interval • Small: every 12500 km @ 1 depot • Monthly: every 25000 km @ 1 depot • Big: every 50000 km @ 1 depot	3	No of. maintenance services	Minimum
Stations	14		
Depots	7		

Objective	Reference solution	VS-OPT rail
Rows	20 + 300 km deadhead	19 + 300 km deadhead
CPU time (hh:mm)	—:—	00:20

Real World Example: Scenario 2

Input	**#**	**Objective**	**Goal**
Timetabled trips	1292	Trip coverage	100%
Connections	1009	Rows	Minimum
Maintenance intervals • Refuel: every 600 km @ 10 depots • Small: every 15000 km @ 1 depot • Big: every 60000 km @ 1 depot)	3	No of maintenance services	Minimum
Stations	26		
Depots	34		

Objective	**Reference solution**	**VS-OPT rail**
Rows	29 + 5500 km deadhead	26 + 3300 km deadhead
CPU time (hh:mm)	—:—	08:48

Thank you for your attention

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