

Vehicle Rotation Planning and Hyperassignments

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2015 Workshop on Combinatorial Optimization with Applications in Transportation and Logistics

Beijing, 30.07.2015

A Set Partitioning Example

What is the minimum cost of covering the letters A–L with a subset of the following sets s.t. no two sets intersect?

{A, B, D, G, H, J},	-100	{D, K},	0.81
{A, H},	0.24	{E, F, G, H},	0.71
{A, J},	0.43	{E, F, I, J},	0.62
{B, G},	0.13	{E, I},	0.04
{B, L},	0.02	{E, J},	0.06
{C, D, K, L},	0.02	{F, G},	0.14
{C, I},	0.19	{F, J},	0.53
{D, I},	0.05	{F, K},	0.08
{D, J},	0.11	{F, L}.	0.04



A Set Partitioning Example



{A, B, D, G, H, J}, {A, H}, {A, J}, {B, G}, {B, L}, {C, D, K, L}, {C, I}, {D, I}, {D, J}, {D, K}, {E, F, G, H}, {E, F, I, J}, {E, I}, {E, J}, {F, G}, {F, J}, {F;K}, {F; L}.



A Set Partitioning Example

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{A, B, D, G, H, J}, {A, H}, {A, J}, {B, G}, {B, L}, {C, D, K, L}, {C, I}, {D, I}, {D, J}, {D, K}, {E, F, G, H}, {E, F, I, J}, {E, I}, {E, J}, {F, G}, {F, J}, {F;K}, {F; L}. All hyperedges that do not intersect {A, B, D, G, H, J}:



Hall's Theorem implies that there is no solution of cost < 0.

Overview

- The Hypergraph Assignment Problem
- Random Hyperassignments
- Complexity Results
- Partitioned Hypergraphs
- Polyhedral Results
- A Local Search Heuristic
- Regular ICE Rotations
- Vehicle Rotation Planning
- Coarse-to-Fine Method

Bipartite Hypergraphs

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A hypergraph G is called bipartite if

- its vertex set can be written as the disjoint union of two vertex sets
 U and V with the same size |U| = |V|, and
- every hyperedge e ∈ E has the same number |e ∩ U| = |e ∩ V| of vertices in U and V.

We then represent G as a triple G = (U, V, E).



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A hyperassignment is a subset H of E such that there is exactly one incident hyperedge for every vertex.





Definition (Hyperassignment Problem)

Input: A bipartite hypergraph G = (U, V, E) with edge costs $c_e \in \mathbb{R}$. Output: A minimum cost hyperassignment H^* in G, i.e., a hyperassignment H^* s.t.

 $c(H^*) = \min\{c(H), H \text{ is a hyperassignment in } G\}$

or the statement that no hyperassignment exists.

$$\min \begin{array}{c} c^T x \\ x(\delta^+(v)) = 1 \quad \forall v \in U \cup V \\ x(\delta^-(v)) = 1 \quad \forall v \in U \cup V \\ x \in \{0,1\}^E \end{array}$$

The HAP is a special type of set partitioning problem.

Motivation: ICE Connections





Timetabled Trips: 1 Day

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Timetabled Trips: Standard Week

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Graphics: JavaView, MATHEON F4

Vehicle Rotation (5 Weeks)





Graphics: JavaView, MATHEON F4

Rotation Plan: Follow-on Trip Assignment

(Blue: Timetabled Trips, Red: Deadhead Trips)





Graphics: JavaView, MATHEON F4



Timetable Regularity

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Rotation Regularity





Modeling Regularity via Hyperedges





Hyperassignment Solution

ICE 136 ICE 757 (Thu) (Tue) ICE 757 ICE 757 (Mon) (Sun) ICE 51 ICE 9 (Mon) (Mon) ICE 9 (Tue) ICE 51 ICE 9 (Tue) (Wen)



Bipartite Hypergraph Model







Theorem (Mézard, Parisi [1985], Aldous [1992, 2001])

The expected optimal objective value E of the assignment problem for random instances on complete bipartite graphs and uniformly i. i. d. edge costs in [0,1] or exponentially i. i. d. edge costs with mean 1 converges to

$$E = \frac{\pi^2}{6} = 1.6449 \dots$$

if the number of vertices tends to infinity.

Random Hypergraph Assignment Problems

(1 000 runs with exponentially i. i. d. edge costs with mean 1)

ean 1) _{Fi}



G _{2,n}	E	σ (E)	# e: e >2	σ(#e: e >2)
10	1.019	0.206	5.3	2.0
20	1.039	0.141	10.4	2.8
30	1.049	0.117	15.3	3.4
40	1.045	0.097	20.5	3.9
50	1.054	0.085	25.4	4.3
60	1.050	0.080	30.6	4.7
70	1.053	0.079	35.6	5.1
80	1.054	0.069	40.6	5.4
90	1.053	0.066	45.9	5.8
100	1.057	0.063	50.6	6.3
110	1.054	0.060	56.1	6.4
120	1.052	0.056	61.1	6.7
130	1.054	0.053	66.3	6.9
140	1.053	0.051	71.3	7.1
150	1.051	0.050	76.2	7.5
160	1.054	0.048	81.2	7.6



Theorem (Mézard, Parisi [1985], Aldous [1992, 2001])

The expected optimal objective value E of the assignment problem for random instances on complete bipartite graphs and uniformly i. i. d. edge costs in [0,1] or exponentially i. i. d. edge costs with mean 1 converges to

$$E = \frac{\pi^2}{6} = 1.6449 \dots$$

if the number of vertices tends to infinity.

Theorem (B., Heismann [2014])

The expected optimal objective value E of the hyperassignment problem for random instances on complete bipartite hypergraphs $H_{2,n}$ with exactly n proper hyperedges and exponentially i. i. d. edge costs with mean 1 converges to

0.3718 < E < 1.8310

as the number of vertices tends to infinity.

Relations to Hyperflow Problems

- Cambini, Gallo, Scutellà [1992]
 Minimum cost flows on hypergraphs; solves only LP relaxation
- Jeroslow, Martin, Rarding, Wang [1992]
 Gainfree Leontief substitution flows; does not hold for HAP

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Theorem (B., Heismann [2011], Heismann [2014])

- 1. The HAP is NP-hard and APX-hard, even for bipartite hypergraphs with maximum hyperedge size 4.
- 2. The set packing/covering relaxations of the HAP are NP-hard, even for bipartite hypergraphs with maximum hyperedge size 6.
- 3. The LP/IP gap can be arbitrarily large.
- 4. The determinants of basis matrices can be arbitrarily large.



Solution of the LP Relaxation

Fractional solution, cost = 0.615.





Solution of the LP Relaxation

- Fractional solution, cost = 0.615.
- The red hyperedge clique inequality separates this solution.
- Cliques can be separated efficiently by exploiting a "partitioning structure".





Partitioned Hypergraphs

• A bipartite hypergraph G = (U, V, E) is called partitioned with maximum part size $d \in \mathbb{N}$ if there exist pairwise disjoint $\leq d$ -element sets U_1, \ldots, U_p and V_1, \ldots, V_q called the parts of G s.t. every hyperedge intersects exactly one part in U and one part in V.



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Theorem (B., Heismann [2012])

Every HAP in a bipartite hypergraph G = (U, V, E) can be polynomially transformed into a HAP in a partitioned hypergraph with $d = 0.5 \max_{e \in E} |e|$.





Theorem (B., Heismann [2011])

- Every (hyperedge) clique in a partitioned hypergraph is a subset of the incident hyperedges $\delta(P)$ of some part *P*.
- The (hyperedge) conflict graph contains no holes of any size and no antiholes of size < 7.</p>





Theorem (B., Heismann [2011])

There exists an extended formulation with $O(|U|^{d+1})$ variables that implies all clique inequalities.

min $c^T x$ $\begin{aligned} x(\delta^+(v)) &= 1 \quad \forall v \in U \cup V \\ x(\delta^-(v)) &= 1 \quad \forall v \in U \cup V \\ x &\in \{0,1\}^A \end{aligned}$





configuration 1



configuration 3

configuration 2

Further Polyhedral Results

- Let G_{2,n} be the complete partitioned directed hypergraph on 2n nodes with n parts of size 2.
- Let $P(G_{2,n})$ be the HAP polytope associated with $G_{2,n}$.
- $P(G_{2,6})$ is completely described by 14 049 facets.



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- Every facet of $P(G_{2,6})$ can be described in many different ways, in particular, in the form

$$\sum_{e \in E_1} x_e - \sum_{e \in E_{-1}} x_e \le 1.$$



Example of two facets with coefficients -1 and +1 only

- Red: coefficient -1
- ▷ Black: coefficient 1

Hyperedges are drawn as connections between the surroundings of the corresponding vertices.



Further Polyhedral Results

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$$\sum_{e \in E_1} x_e - \sum_{e \in E_{-1}} x_e \le 1.$$

- The 14 049 facets fall into 30 symmetry classes, generated by swapping U and V, permuting parts inside U and V, permuting vertices inside of parts, resulting in 4 608 vertex and hyperedge permutations.
- 16 symmetry classes are understood, including nonnegativity constraints, clique inequalities, and odd clique set inequalities.
- Facet classification done by HUHFA program available at <u>http://comopt.ifi.uni-heidelberg.de/people/hildenbrandt/HUHFA/</u>.

Solution of the LP Relaxation

- Fractional solution, cost = 0.635.
- Consider the 7=2·3+1 cliques associated with the vertices $v_1, v_3, v_4, u_2, u_3, u_4$ and the clique $\{v_5, v_6, u_3, u_4\}, \{v_5, u_3\}, \{v_5, u_4\}$.


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- Every red hyperedge is contained in at least two of these cliques.
- We can take at most three of these edges.



Odd Set Ieqs for the (Perfect) Matching Problem Berlin



 Complete description of the matching polytope (together with the degree and non-negativity constraints), Edmonds [1965]

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Odd Clique Set Ieqs for General Hypergraphs



$$\sum_{e \in E} \left\lfloor \frac{|\{v \in V' : e \in \delta(v)\}|}{p} \right\rfloor x_e \le \left\lfloor \frac{|V'|}{p} \right\rfloor \quad \forall V' \subseteq V$$

- Generalize the odd set inequalities for the matching problem
- Related to clique set inequalities by Pêcher & Wagler [2006]

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Odd Clique Set Ieqs for General Hypergraphs



Theorem (B., Heismann [2011])

Let Q be a set of at least three hyperedge cliques in $G = (V, E), 2 \le p \le |Q|$ be an integer number, $r := |Q| \mod p$, and $q_e := |\{Q \in Q : Q \ni e\}|$. Then $\sum_{e \in E} \left(\left| \frac{q_e}{p} \right| + \max\left\{ 0, \frac{q_e \mod p - r}{p - r} \right\} \right) x_e \le \left| \frac{|Q|}{p} \right|.$

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- Reduce to assignment problem by gluing vertices together.
- Cost = 1.46.

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- Reduce to assignment problem by gluing vertices together.
- Cost = 0.96.
- Iteratively merge/segregate and solve assignment problems.
- Combine with composite columns method.

Regular Timetable

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Regularity of the timetable trips:

- Red: Trip done on one day of the week
- Blue: Trip done on all days of the week

Graphics: JavaView, MATHEON F4

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Regular Rotations





Regularity of the deadhead trips:

- Yellow: regular
- Red: irregular

Graphics: JavaView, MATHEON F4

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Irregularity in the optimal solution depending on the penalty for irregularity; 61 trains, 75 tasks, 803 stops, 310 trips, 620 vertices.

costs per irreg. turn	costs per irreg. trip	# hyper- edges	# irregular trips	# irregular turns	# vehic- les	time below target cost	dead- head distance cost
0	0	94035	261	310	17	2640	6298.72
1	1	109299	256	130	17	2640	6298.72
10	10	109299	76	105	17	2640	6298.72
100	100	109299	31	121	17	2640	6298.72
1000	1000	109299	25	47	17	2860	6298.13
10000	10000	109299	25	37	17	2860	32087.29

> Not explicitly optimizing regularity leads to high irregularity

- Regularity can be enforced by high penalties on arcs
- High variability

A New Technology: Vehicle Rotation Planning





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Starting Point: Trip Network



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Timetabled Trips: 1 Day

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Timetabled Trips: Standard Week







Graphics: JavaView, MATHEON F4

Vehicle Rotation: 5 Weeks





Graphics: JavaView, MATHEON F4

All Vehicles Rotations: Rotation Plan

(Blue: Timetabled Trips, Red: Deadhead Trips)





Graphics: JavaView, MATHEON F4

Railway Constraints

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Photos courtesy of DB Mobility Logistics AG

Train Composition

Maintenance

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Train Composition: Type, Order, Orientation

















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trip 1



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Hyperflow Model

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Vehicle Rotation Planning Problem

Cover all timetabled trips by rotations such that turns and train composition are regular.

Hypergraph Multi Commodity Flow Problem

Find a cost minimal hyperflow such that every node configuration is covered by exactly one hyperarc.

Hyperarcs for

- (regular) turns
- (regular) train compositions



Hyperflow Model

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- \triangleright Subarc variables X_a
- \triangleright Multiarc variables x_m for
 - uniform turns
 - uniform train compositions
- ▷ Flow conserv./flow constr.

 $\min c^{T} x$ $x(d, \delta^{+}(v)) = x(d, \delta^{-}(v)) \quad \forall v \in V, d \in D$ $x(\delta^{+}(t)) = 1 \qquad \forall t \in T$ $x \quad \in \{0,1\}^{A \cup M}$

Coarse-to-Fine Method: Setting

- Row and column index sets I = [m], J = [n]
- Matrix $A \in \mathbb{R}^{I \times J}$
- Rhs $b \in \mathbb{R}^{I}$
- Objective $c \in \mathbb{R}^J$
- Linear Program

 $\min c^T x$ Ax = b $x \ge 0$

and its dual

 $\max y^{T} b$ $y^{T} A = c^{T}$ $y \in \mathbf{IR}^{I}$

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- Aggregate/project the rows *I* of the (LP) by a problem specific coarsening projection []: *I* → [*I*] (induces an equivalence relation)
- For a column vector $v \in \mathbb{R}^{I}$ we define the coarsening of v as $[v][i] \coloneqq (\min\{v_k: k \in I, [k] = [i]\}, \max\{v_k: k \in I, [k] = [i]\}) \cdot \tau(v, i)$ where $\tau(v, i) \coloneqq |\{v_k \neq 0: [k] = [i]\}|$
- Coarse bimatrix [A], coarse dual vector [π]
- Coarse objective function $[c] \coloneqq c$ (no coarsening)

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$$\min[c]^T x, [A] x [=] [b], x \in \mathbb{R}^{[J]},$$

where

$$[A]x[=][b]: \Leftrightarrow [b]_{[i]2} \le \sum_{j \in J} [A_{\cdot j}]_{[i]1} x_j, \ \sum_{j \in J} [A_{\cdot j}]_{[i]1} x_j \le [b]_{[i]2}, \ \forall [i].$$

Let $P(A, b) \coloneqq \{Ax = b, x \ge 0\}$ and $P([A], [b]) \coloneqq \{[A]x[=][b], x \ge 0\}$.

Lemma (B., Reuther, Schlechte, Weider [2015])

$$P(A,b) \subseteq P([A],[b]).$$

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The Coarse Reduced Cost

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• Multiplication of pairs $(a_1, b_1), (a_2, b_2) \in \mathbb{R}^2$:

 $(a_1, b_1), (a_2, b_2) \coloneqq \max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$

Coarse reduced cost for column j

$$\overline{[c_j]} \coloneqq [c_j] - [\pi]^T \cdot [a_j]$$

Lemma (B., Reuther, Schlechte, Weider [2015])

The coarse reduced cost always underestimates the original reduced cost

$$\overline{[c_j]} \coloneqq [c_j] - [\pi]^T \cdot [a_j] \le c_j - \pi^T \cdot a_j = \overline{c_j}.$$

• Use the coarse reduced cost for pricing in the fine model.

Example



(c1	c ₂ c ₃	C4 (CБ	<i>c</i> 6	C7	<i>c</i> 8	C9	C10	c ₁₁	c ₁₂	C13	C14	C15	C16	C17	C18	C19	C20	c ₂₁	C22	C23	C24	c ₂₅)	
<i>x</i> ₁	$+ x_{3}$	— ;	хs	_	- X7																		=0	π_1
	:																						:	
	x ₂ +	×4	_	- x ₆ -	- X7																		=0	π_2
		,	×5	+	- x7			-	$-x_{11}$		- x ₁₃												=0	π_3
							1																	
				x ₆ +	- X7			-	- x ₁₁		- x ₁₃												=0	π_4
x_1	+	x4			-	- x ₈	-	- x ₁₀															=0	π_5
	$x_2 + x_3$					-	- xg -	- x ₁₀															=0	π_6
						<i>x</i> 8	+	- x ₁₀		$-x_{12}$	$-x_{13}$												=0	π_7
							Xg -	- x ₁₀		$-x_{12}$	$-x_{13}$												=0	π_8
									<i>x</i> ₁₁		$+x_{13}$	$-x_{14}$											=0	π_9
											- 1												:	
									<i>x</i> 11		$+x_{13}$	$-x_{14}$											=0	π_{10}
												x ₁₄		- x ₁₆									=0	π_{11}
													:										:	
												x ₁₄			- x ₁₇								=0	π_{12}
										x ₁₂	$+x_{13}$		$-x_{15}$										=0	<i>π</i> 13
										x ₁₂	$+x_{13}$		$-x_{15}$										=0	π_{14}
													x ₁₅		-	- x ₁₈							=0	π_{15}
													x15			-	- x ₁₉						=0	π_{16}
															x ₁₇	-	+ x ₁₉ ·	$-x_{20}$					=0	π_{17}
$-x_{1}$	$-x_2 - x_3 - x_3$	×4																$+x_{20}$					=0	π_{18}
														<i>x</i> 16	-	+ x ₁₈		-	$-x_{21}$				=0	π_{19}
																			x ₂₁ -	- x ₂₂			=0	π_{20}
																				x ₂₂ -	- x ₂₃		=0	π_{21}
																					x23 ·	- x ₂₄	=0	π_{22}
																						×24 -	$-x_{25} = 0$	π_{23}



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Example





Example





Layers



Problem specific layers:

- composition layer (fine, already seen)
- configuration layer (coarse)
- vehicle layer (very coarse)

The layers are defined in terms of projections of hypergraphs that correspond to the projection of rows of the LP/IP.





Railway Constraints

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Zeit	Zug		Richtung	G	E	E	D	C	B	B
00.34	EN	Jan Kiepuns	Roman Poemen GE Warszawe /					Barra' Mada	-	
05.36	IC	1	Warschau Eksundchweig Magesturp	66						<u>⊜</u> →
06.21	ICE	Zugleburg in Hamm	Leoping / Huma Hugh Leoping D bes G	Star / Bank	Marillan Marilland	New Line Law	M N		日20 日	
00.21	740/750		A bis C Köln	+						
06.40	140		Cenabricos Biad Bertheim Hongelo Amateriam Castrani		-		- 2 ¹¹ . B	and T and the first sector	ter terberten a	al brefute
07.45	IC	Dienstag bis Donnenstag								
07.45	IC	Montag und Fraitag	Bremen	1	+()					
09.45	2236		Bremen Venden			+ <u>C</u> 18 11	- H - H	* . R ** . R		
00.40	2334		Bremen Detreenhorst Oldenburg			+ @			* BR*	
09.40	IC		Delefality Oortmand Essen							20 -
10.45	IC	Dathiesland	Bramen Oldanburg Tordes	0.000						
11.40	IC		Northfaich Mola Ebeeleich Gütereich				24 24			
10 45	2046		Harren Dortmund Vanden			+ (10 10	·PI ·PI		24 1 1 1	
12.45	2130		Bremen Deimenhorst Oldenburg			+ (1 1	·M" ·R	· Plat · Pl	a. 1920	
13.40	IC		Bielefeld Dortmund Essen			+	Nº N	21 1		
14.45	D							N 9 EAN IN 10 E		
15.31					tile/here tile/here Fuge Rage	Annual An		Circles Circle	AND Revenues of	
16 45	10		A bis C Köls Bromen	+ / = 23		· [8] 24 2 (8) ** 1				
10.45	21036		Oldenburg Einden Norddeich Mote			+ @	· 🕅 " 🛛 🕅		17 M 20 M	
17.40	IC		Essen Dueterg			+ @	A. H	22 2	·· · ··	R ()
18.45	IC		Verden Bramen Detherhorst			+ - 1 1		285 28	·	



Photos courtesy of DB Mobility Logistics AG

Train Composition

Maintenance

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Maintenance: Service Intervals



Blue: timetabled trips Green: 4000 km treatment Dark gray: 8250 km treatment Yellow: 33000 km treatment Pink: 66000 km treatment Red: 198000 km treatment Light gray: 15 days treatment Turquoise: 30 days treatment

Freie Universität

Berlin

Railway Constraints

Freie Universität



Zeit	Zug		Richtung	G	E	130	151	63		-	
00.34	EN	Jan Kingtura	Roman GL Poeman GL					Martin Martin			A
05.36	IC		Warschau Braunschweig Magbiourg							響-	<u> </u>
06.21	ICE	Zugleising in Harris	Leipzig D ber 0 Köln / Bonn Flughaten	-	Alles / Barris Frage. Robert Regist.	Hard Barry State				HE	<u></u>
06.40	1C		A bis C Köln Osnatzisck Eliad Benthelm	+ <u></u>					Mar Barlantas	Barl Instant	
07.45	IC	Denstag bis Domentag	Horgelo Amaterdam Centraal		+ <u>()</u>			·	· [8].	- PI -	
07.45	IC	Montag und Freitag	Bremen	-	+ (<u> </u>				No 19 20		
08.45	IC		Bremen Verden Premen			+ <u>C</u> II II	· H* · P		** E *	2	<u></u>
09.40	IC IC		Delinerkost Oldenburg Dielefuly Outpresed			+ C E E			2" MC"	12	<u> </u>
10.45	IC	Osthiesland	Essen Düsseldorf Dramen Observis es			+ C 1 1 1			2* E C?	3	<u> </u>
11 40	in IC		Ernden Nordsteich Mole Beiefeld			+ C 16 19			2* P 2*	1	\simeq
12 45	IC		Verden Dortmund			+ C II I	· PI · PI	* . P	** P (**	12	
13.40	IC		Delmenhorat Oldenburg Elisiefeid			• C 1 1	·M. ·H		** PI X *	2	<u> </u>
14 45	2040		Essen Dimseldorf			+ <u>A</u>	M. N		2" E.C."	1	
15.31	36				Mitchen Mitchen						
16.45	IC	- 3-	A bis C Rôle Brenen	+ <u>/</u> =]]=		· (1936) · (1977) ·				* - 四	
17.40	2008		Ender Nordseich Mate			+ <u>~ 1</u> 1					<u></u>
18.45	IC		Duatorg Köln Verder			+ <u>~</u> II II					
10.40	2004		Detherboret			+ C 11 11			- Misa		



Photos courtesy of DB Mobility Logistics AG

Train Composition

Maintenance

Parking: Keeping It Simple

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Feasible Assignments

XXX, YYY, XXX+YYY,

XXX, YYY, YYY+YYY

XXX, YYY, YYY+YYY

XXX, YYY, YYY+YYY

XXX, YYY, YYY+YYY

YYY+YYY

XXX, YYY

YYY

YYY



Vehicle Rotation	Planning and	l Hyperassignments	CO∈TL	2015



Input	#	Objective	Goal
Timetabled trips	798	Coverage	100%
Connections	171	Rows	Minimum
 Maintenance interval Small: every 12500 km @ 1 depot Monthly: every 25000 km @ 1 depot Big: every 50000 km @ 1 depot 	3	No of. maintenance services	Minimum
Stations	14		
Depots	7		

Objective	Reference solution	VS-OPT rail
Rows	20 + 300 km deadhead	19 + 300 km deadhead
CPU time (hh:mm)	:	00:20



Input	#	Objective	Goal
Timetabled trips	1292	Trip coverage	100%
Connections	1009	Rows	Minimum
 Maintenance intervals Refuel: every 600 km @ 10 depots Small: every 15000 km @ 1 depot Big: every 60000 km @ 1 depot) 	3	No of maintenance services	Minimum
Stations	26		
Depots	34		

Objective	Reference solution	VS-OPT rail
Rows	29 + 5500 km deadhead	26 + 3300 km deadhead
CPU time (hh:mm)	:	08:48

Thank you for your attention



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