TECHNISCHE UNIVERSITÄT BERLIN
INSTITUT FÜR MATHEMATIK

Wintersemester 11/12

Computational Integer Programming

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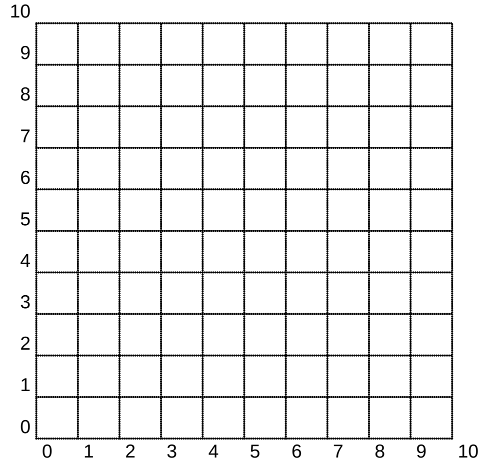
Exercise sheet 10

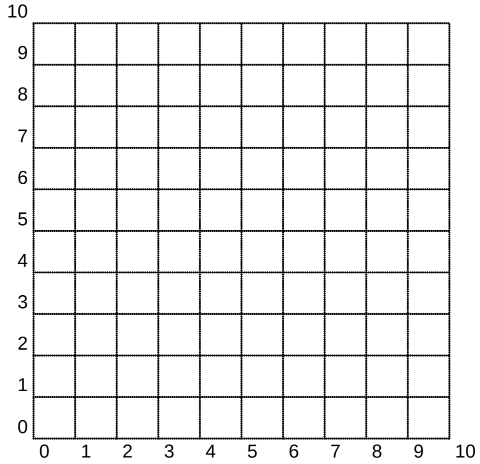
Deadline: Thu, 12 Jan. 2012, by email to `borndoerfer@zib.de`

Check your proofs using the program GEODUAL by Mike Jünger, Michael Schulz, and Steffen Zychowicz at `http://www.informatik.uni-koeln.de/lsjuenger/research/geodual`.

Exercise 1. **(Tutorial session)**

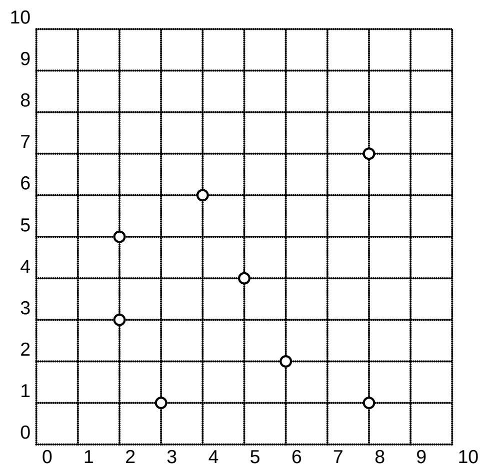
Construct an example of a 2D Euclidean perfect matching problem in the plane such that no disc packing proves optimality.

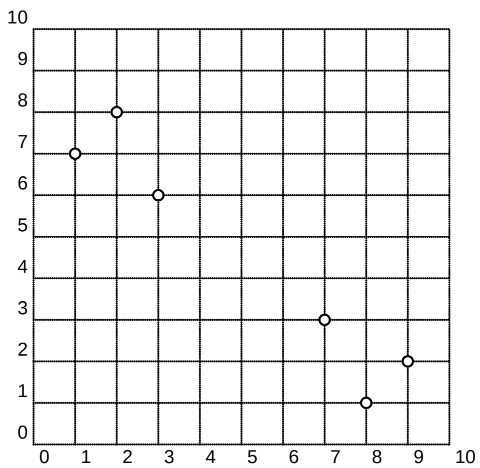




Exercise 2. **(Tutorial session)**

Solve the following instances of the 2D Euclidean perfect matching problem:



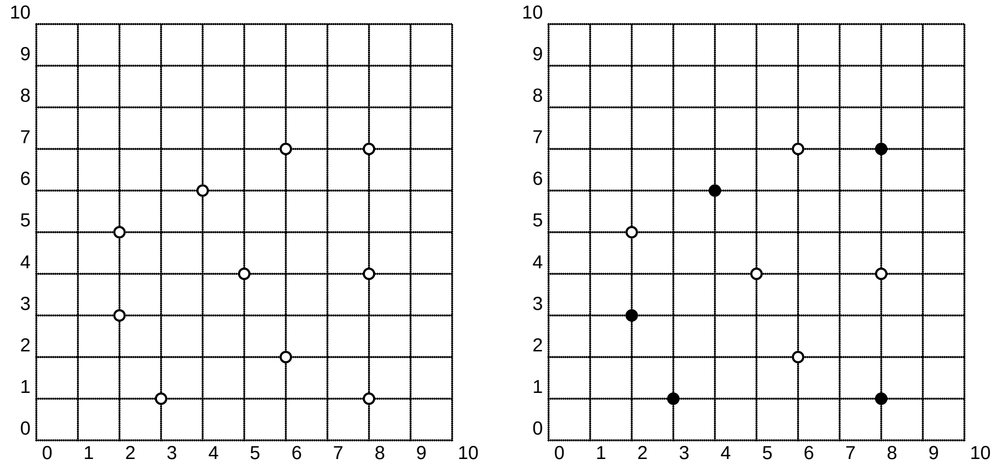


Exercise 3. **(Tutorial session)**

The node covering problem looks for a minimum weight collection of edges in a graph such that each node is covered by at least one edge, in a 2D Euclidean node covering problem the nodes correspond to points in the plane and the weights to Euclidean distances. Prove that an optimal solution of a 2D Euclidean node covering problem for an even number of nodes is a perfect matching.

Exercise 4. **(Tutorial session)**

Construct 2-approximate solutions of the following 2D Euclidean spanning tree (left) and Steiner tree problems (right, terminals are black):



Exercise 5. **10 points**

Call a Steiner node of degree three or more in a Steiner tree a *branching node*. Prove that a Steiner tree Y for a terminal set R contains at most $|R|-2$ branching nodes.

Exercise 6. **10 points**

Consider two branching nodes v_0 and v_k in a minimum cost Steiner tree that are connected via a path $p = (v_0, v_1, \ldots, v_k)$ such that all inner nodes v_i, $i = 1, \ldots, k-1$, have degree two and none of them is a terminal. Prove that p is a shortest path.

Exercise 7. **10 points**

There is a one-to-one correspondence between minimum cost Steiner trees on terminals T and branching nodes S in a graph G and minimum spanning trees in a complete graph $\tilde{G}$ on nodes $T \cup S$ whose edge weights correspond to shortest path distances in G.

Exercise 8. **10 points**

Using the results of the previous three exercises, construct an algorithm for the undirected Steiner tree problem that has pseudo-polynomial running time in the number of terminals. **Hint:** Enumerate all sets of branching nodes.