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FB Mathematik und Informatik

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Network Design and Operation (WS 2015)

Excercise Sheet 1

Submission: Mo, 26. October 2015, tutorial session

Exercise 1. **8+2 Points**

Show that connectedness induces an equivalence relation on the nodes of an undirected graph. What are the equivalence classes?

Exercise 2. **10 Points**

Show that an edge is a bridge of an undirected graph if and only if it is not contained in a cycle.

Exercise 3. **10 Points**

Show that an undirected graph is bipartite if and only if it doesn't contain an odd cycle (with an odd number of nodes = number of edges).

Exercise 4. **10 Points**

Show that a tree with maximum degree $\Delta(G) := \max_{v \in V} \delta(v)$ has at least $\Delta(G)$ leaves (nodes of degree 1).

Exercise 5. **Tutorial Session**

The *assignment problem* involves a complete bipartite graph $G = (U, V, E)$ with the same number of nodes $|U| = |V|$ on both sides and a matrix of edge weights $c \in \mathbb{Q}^E = \mathbb{Q}^{U \times V}$. An *assignment* is a set of edges $M \subseteq E$ such that each node u is contained in exactly one edge uv that matches or assigns it to v. The assignment problem is to find an assignment M of minimum cost $c(M)$.

Solve the assignment problem given by the matrix in Table 1.

8	7	9	9
5	2	7	8
6	1	4	9
2	3	2	6

Table 1: Assignment problem

Exercise 6. **Tutorial Session**

Solve the problem in exercise 5 by integer programming using the programs `zimpl` and `scip`.

a) Download `zimpl` and `scip` from `zibopt.zib.de` and have a look at the example section in the `zimpl` manual.
b) Formulate the assignment problem as an integer linear program (IP).
c) Translate your IP model into a `zimpl` model.
d) Solve the model using `scip`.
e) Construct the LP relaxation of the model.
f) Solve the LP relaxation using `scip`.
g) Prove that the solution of the LP relaxation is always integer.