Freie Universität Berlin
FB Mathematik und Informatik

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Network Design and Operation (WS 2015)

Excercise Sheet 2

Submission: Mo, 02. November 2015, tutorial session

Exercise 1. **5+5 Points**

a) Stable set problem. Show that finding a stable set of size k in a graph is NP-complete by a transformation from the problem to finding a clique of size ℓ.
b) Longest path problem. Show that finding a path of length at least k in a graph is NP-complete.

Exercise 2. **5+5 Points**

Give a polynomial time algorithm for testing a graph for connectedness and analyze its running time.

Exercise 3. 5×2 **Points**

Show the following computing rules for the O-notation.

a) $O(1) + O(1) = O(1)$.
b) $O(\sum_{i=1}^{k} n^i) = O(n^k)$, $k \in \mathbb{N}_0$.
c) $O(n^k) + O(n^\ell) = O(n^{\max\{k,\ell\}})$, $k, \ell \in \mathbb{N}_0$.
d) $O(n^k) \cdot O(n^\ell) = O(n^{k+\ell})$, $k, \ell \in \mathbb{N}_0$.
e) $n! = O(n^n)$.

Exercise 4. **10 Points**

Consider a set of points $V \subseteq \mathbb{R}^2$ in the plane and let $p^* \in \operatorname{argmin} 1/\mathbb{R}^2/\cdot/\ell_2^2/\sum$ be a median w.r.t. squared Euclidean distances. Prove that $p^* \in \operatorname{conv} V$.

Exercise 5. **Tutorial Session**

We will use the graph G in Figure 1 for several exercises; Table 1 lists abbreviations for the station names. Edge data for this graph is contained in the file `edges.dat`, `cap.dat` contains capacity data that we will use in this exercise as costs.

Consider the *even degree problem* (EDP) that asks for a minimum cost set of edges F in G s.t. every node is incident to an even number of F-edges except for the degree one nodes, which must be incident to exactly one F-edge.

a) Model the EDP as an integer program.
b) Write a `zimpl` model for your formulation; you can use the file `deg-skeleton.zpl` as a start.
c) Use `scip` to compute, for all possible even degrees of Utrecht, a min cost solution.
d) Draw all these solutions (a separate drawing for each solution).
e) Compile a table with the optimal objective values.

Exercise 6. **Tutorial Session**

Consider the 1-median problem $1/\mathbb{R}^2/\cdot/\ell_1/\sum$ w.r.t. Manhattan distances for a set of points $V \subseteq \mathbb{Z}^2$ with integer coordinates. Use Figure 2 to construct an instance of this problem with at least 6 different points s.t.

1. the set of medians is a line segment.
2. the set of medians is a single point.

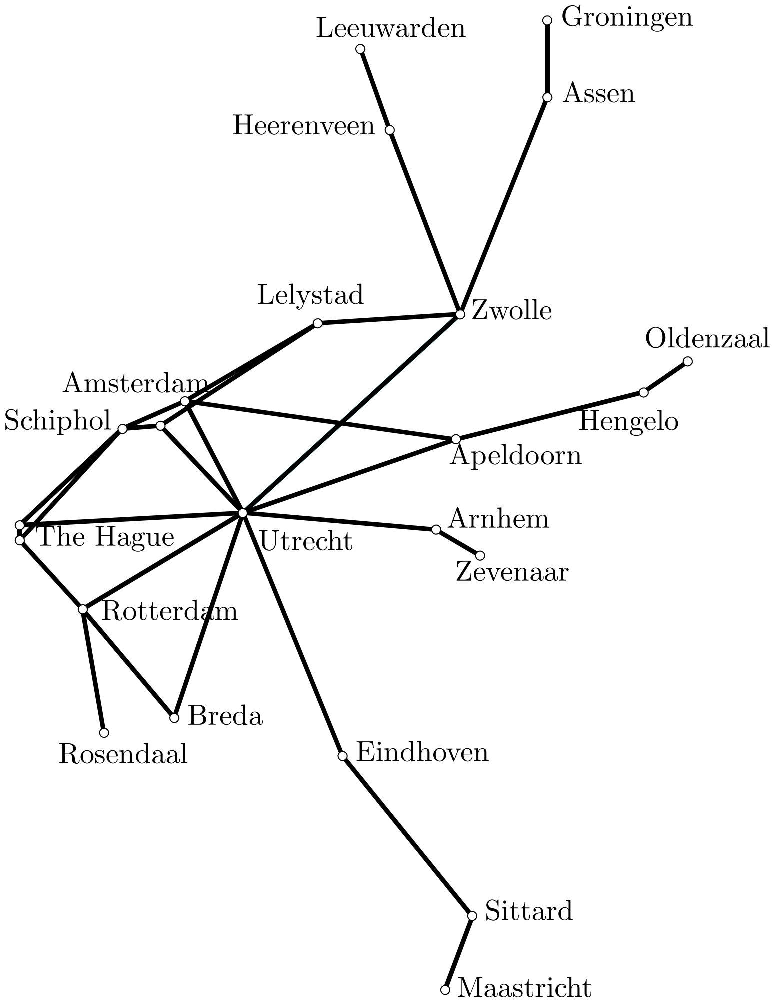

Figure 1: The Dutch high-speed railway network.

Ah	Arnhem
Apd	Apeldoorn
Asd	Amsterdam CS
Asdz	Amsterdam Zuid WTC
Asn	Assen
Bd	Breda
Ehv	Eindhoven
Gn	Groningen
Gv	Den Haag HS
Gvc	Den Haag CS
Hgl	Hengelo
Hr	Heerenveen
Lls	Lelystad Centrum
Lw	Leeuwarden
Mt	Maastricht
Odzg	Oldenzaal Grens
Rsdg	Rosendaal Grens
Rtd	Rotterdam CS
Shl	Schiphol
Std	Sittard
Ut	Utrecht CS
Zl	Zwolle
Zvg	Zevenaar Grens

Table 1: Station names and abbreviations in the Dutch high-speed railway network.

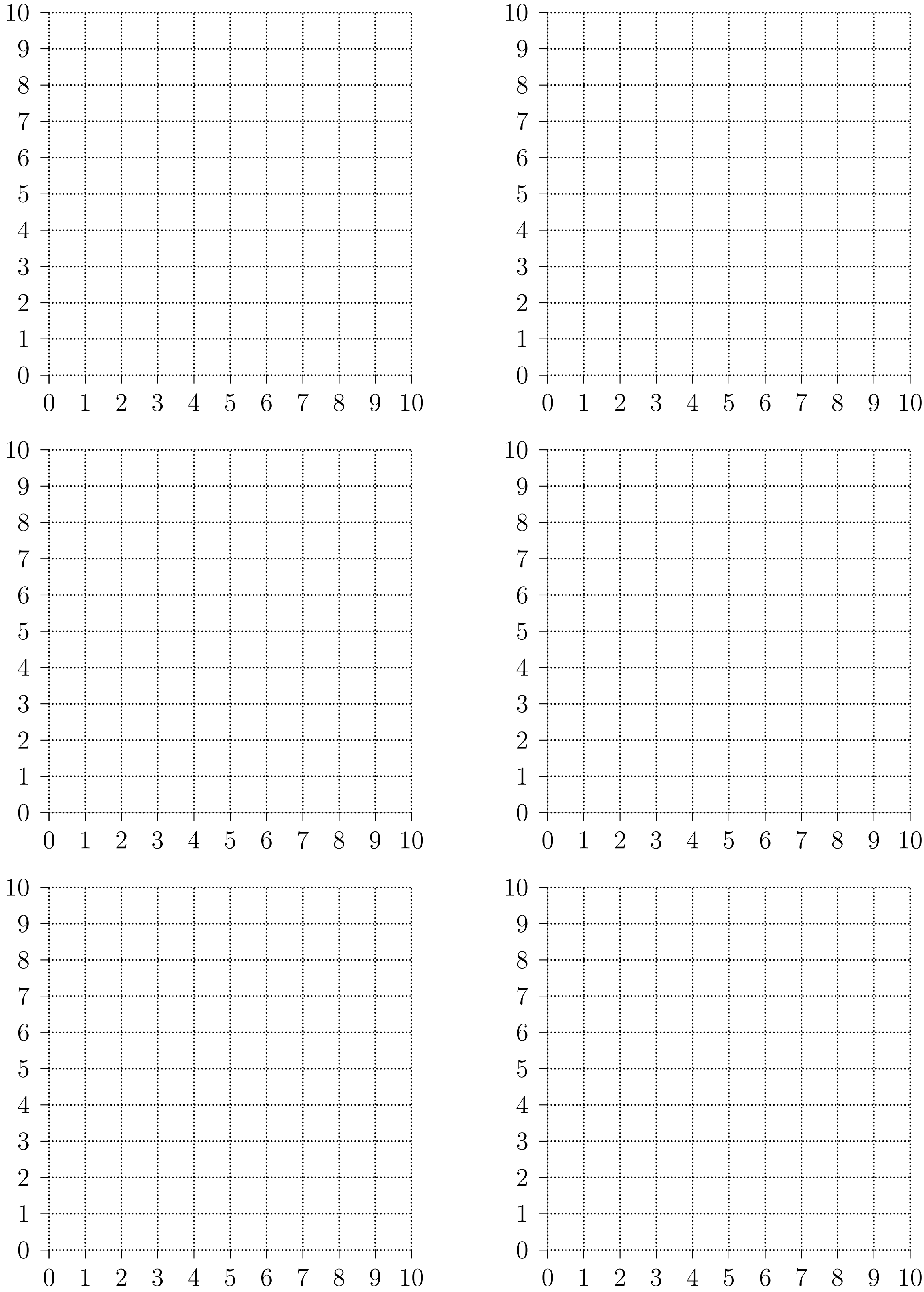

Figure 2: 1-median ℓ_1-problem.