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FB Mathematik und Informatik

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Excercise Sheet 3

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Exercise 1. **10 Points**

Weiszfeld's algorithm iterates the operator T defined as

$$p_{j+1} := T(p_j) := \frac{1}{\sum_{i=1}^m \frac{1}{\|p_j - v_i\|_2}} \sum_{i=1}^m \frac{v_i}{\|p_j - v_i\|_2}, \quad j = 1, 2, \ldots$$

to determine the median $p^* = \operatorname{argmin} 1/\mathbb{R}^2/\cdot/\ell_2/\sum$ of a given set of points $V = \{v_i\}_{i=1}^m$ in $\mathbb{R}^2$ w.r.t. Euclidean distances. Consider $V = \{(-1,3),(0,0),(9,9),(10,0)\}$ and compute the median p^* numerically. Try $p_0 = (8,-1)$ and two other starting points of your choice. Plot the resulting iterates.

Exercise 2. **10 Points**

Consider a triangle $\Delta v_1 v_2 p$ in $\mathbb{R}^2$ and let $c = (v_1 + v_2)/2$ be the median of v_1 and v_2. Prove that $\|p - c\|_2 \leq (\|p - v_1\|_2 + \|p - v_2\|_2)/2$.

Exercise 3. **10 Points**

Prove that the Fermat or Torricelli point in a triangle with an obtuse angle ($\geq 2\pi/3$) is the vertex at this obtuse angle.

Exercise 4. **10 Points**

Prove that the median (w.r.t. Euclidean distance) of four points in the plane is

a) the intersection of the two diagonals, if the points form a convex quadrilateral.
b) the point in the triangle, if three points form a triangle containing the fourth.

Hint: Use the Weiszfeld conditions. Choose favorable coordinates in b).

Exercise 5. **Tutorial Session**

Consider the k-median problem $k/N/\cdot/\ell_1/\sum$ w.r.t. Manhattan distances on the 11×11-grid graph N generated by the points $\{0,\ldots,10\}^2$ for a set of points $V = \{(1,8),(2,0),(4,10),(5,4),(7,7),(9,2),(10,5)\}$. Solve the following problems:

a) $1/N/\cdot/\ell_1/\sum$
b) $1/N/R = [0,8] \times [5,5]/\ell_1/\sum$
c) $1/N/R = [0,8] \times [5,6]/\ell_1/\sum$
d) $1/N/R = [0,8] \times [5,6] \cup [8,9] \times [4,7]/\ell_1/\sum$
e) $2/N/\cdot/\ell_1/\sum$
f) $2/N/R = [0,8] \times [5,5]/\ell_1/\sum$
g) $2/N/R = [0,8] \times [5,6]/\ell_1/\sum$
h) $2/N/R = [0,8] \times [5,6] \cup [8,9] \times [4,7]/\ell_1/\sum$

Use integer programming where necessary, and plot all solutions. Which scenarios suffer most from the restrictions?

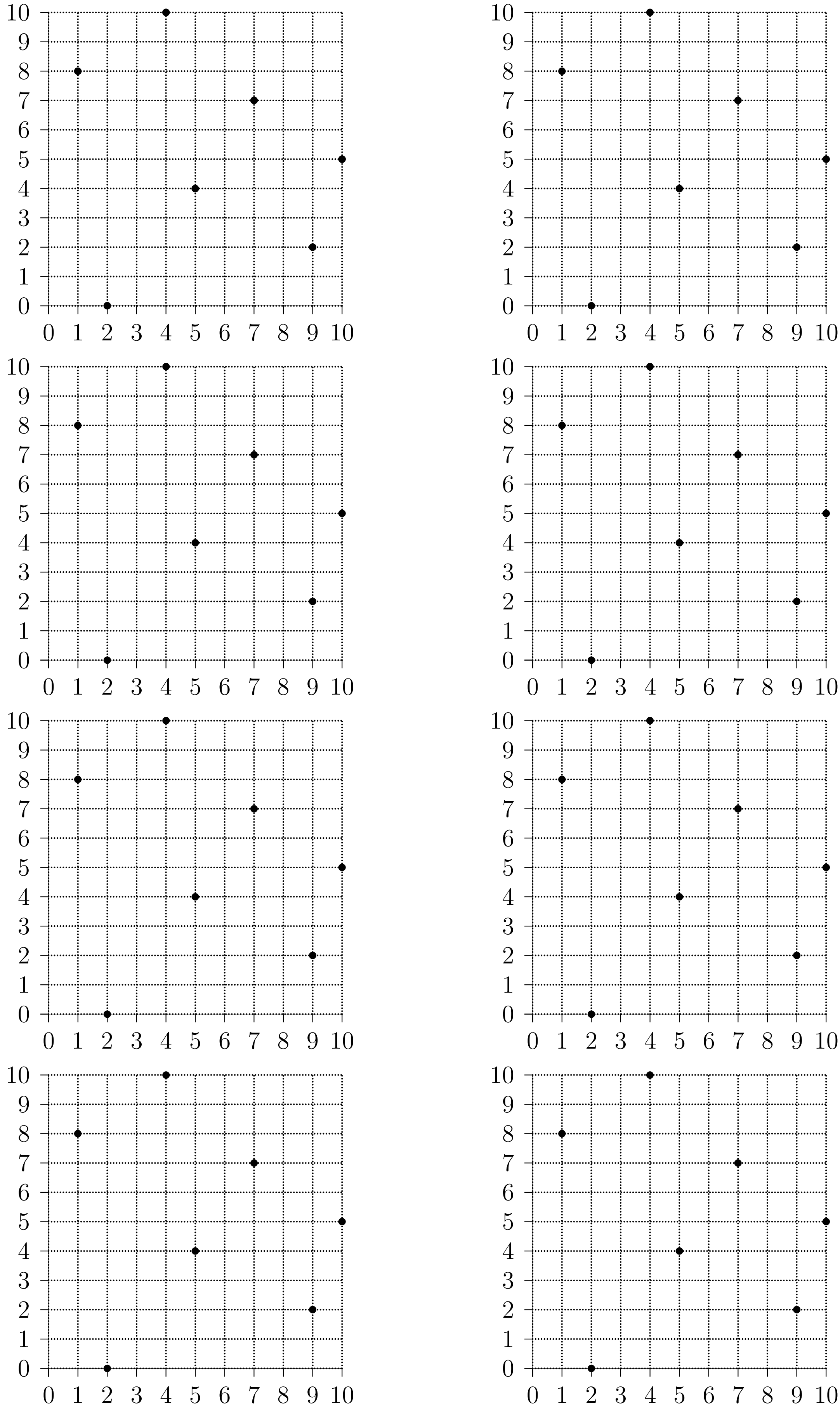

Figure 1: 1-median ℓ_1-problem.