Freie Universität Berlin
FB Mathematik und Informatik

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Network Design and Operation (WS 2015)

Excercise Sheet 4

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Exercise 1. **10 Points**

Prove that the smallest enclosing circle of n points in the plane is uniquely determined by 3 points.

Exercise 2. **5+5 Points**

Solving the 1-center network location problem $1/N/\cdot/\mathrm{sp}/\max$ requires the solution of minimization problems

$$(\mathrm{P}_{uv}) \qquad \min \max_{w \in V} \min_{\lambda \in [0,1]} \{\mathrm{sp}(u,w) + \lambda c_{uv}, \mathrm{sp}(v,w) + (1-\lambda) c_{uv}\}$$

for all undirected edges $uv \in E$.

a) The functions

$$\lambda \mapsto \max_{w \in V} \min_{\lambda \in [0,1]} \{\mathrm{sp}(u,w) + \lambda c_{uv}, \mathrm{sp}(v,w) + (1-\lambda) c_{uv}\}$$

are continuous and piecewise affine with at most $2|V|$ pieces.

b) $(\mathrm{P})_{uv}$ can be solved in linear time.

Exercise 3. **5+3+2 Points**

Consider a modification of the `select` median finding algorithm that subdivides m given numbers into groups of k elements, where k can be different from 5.

a) Ignoring integrality issues, derive a recursion for the run time.
b) What is the ratio in the resulting geometric series?
c) What is special about $k = 5$?

Exercise 4. **10 Points**

Consider real number $a_1, \ldots, a_m$ and positive weights $w_1, \ldots, s_m$; let $W := \sum_{i=1}^m w_i$. The *weighted median* of $\{\{a_i\}\}$ w.r.t. w_i is

$$w\text{–med}\,\{\{a_i\}\} := [a_k : \sum_{a_i < a_k} w_i < W/2, \sum_{a_i > a_k} w_i \leq W/2; a_\ell : \sum_{a_i < a_k} w_i \leq W/2, \sum_{a_i > a_\ell} w_i < W/2].$$

Prove that the median of $\{\{a_i\}\}$ is the weighted median of $\{\{a_i\}\}$ w.r.t. weights $w \equiv \mathbb{1}/m$.

Exercise 5. **Tutorial Session**

Consider the 6-node graph $N = (V, E)$ in Fig. 1 with distances d_{ij} and demands w_i as drawn next to the edges and nodes.

a) Solve the warehouse location problem $1/V/\cdot/\mathrm{sp}/\sum w_i$.
b) Solve the warehouse location problem $2/V/\cdot/\mathrm{sp}/\sum w_i$ by fixing the solution of a) and adding a second warehouse in a best possible way.
c) Develop an IP formulation for $2/V/\cdot/\mathrm{sp}/\sum w_i$.
d) Solve your formulation from c).
e) Did b) produce the optimum?
f) Solve the network center problem $1/V/\cdot/\mathrm{sp}/\max$.
g) Solve the network center problem $2/V/\cdot/\mathrm{sp}/\max$ by fixing the solution of f) and adding a second center in a best possible way.
h) Develop an IP formulation for $2/V/\cdot/\mathrm{sp}/\max$.
i) Solve your formulation from h).
j) Did g) produce the optimum?

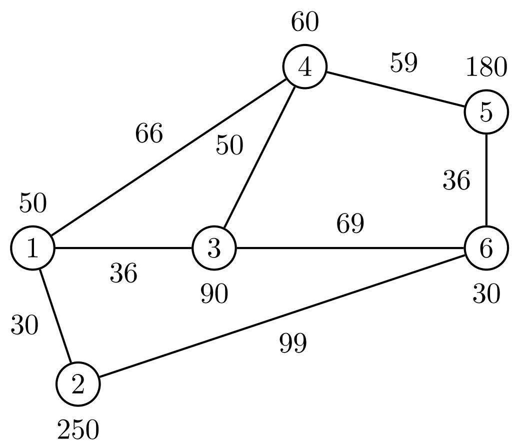

Figure 1: Warehouse location/network center problem.