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FB Mathematik und Informatik

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Network Design and Operation (WS 2015)

Excercise Sheet 6

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Exercise 1. **8+2 Points**

Let $w^1 \geq \cdots \geq w^s > w^{s+1} = 0$ be integers and denote by $H(n) := \sum_{i=1}^{n} \frac{1}{i}$ the n-th harmonic number, $n \in \mathbb{N}$. Prove that

$$\sum_{i=1}^{s}(w^i - w^{i+1})/w^i \leq \sum_{i=1}^{s}\left(H(w^i) - H(w^{i+1})\right) \leq H(w^1).$$

Exercise 2. **4+4+2 Points**

Consider a *discrete stop location problem on a line*, i.e., in a network $N = (S, E)$ of collinear nodes. Prove:

a) The covering matrix A_r^{cov} associated with a discrete stop location problem on a line has the *consecutive ones property*, i.e., the columns and rows of A_r^{cov} can be permuted in such a way that $a_{ri} = 1 = a_{rk}$ implies $a_{rj} = 1$, $i \leq j \leq k$, for all $r \in V$, $i, k \in S$.
b) A 0/1 matrix with the consecutive ones property is totally unimodular.
c) The set covering problem associated with a discrete stop location problem on a line can be solved by linear programming.

Exercise 3. **10 Points**

Consider the ***continuous*** *stop location problem* in a network $N = (S, E)$ with demand points V and covering radius r:

$$\text{(CSL)} \qquad p = |U|/N/\operatorname{cov}_r(U) = V/\ell_2/p.$$

Prove: If (CSL) is feasible, the optimum is finite.

Exercise 4. **2+6+2 Points**

Consider the *continuous stop location problem* in a network $N = (S, E)$ with demand points V and covering radius r

$$\text{(CSL)} \qquad p = |U|/N/\operatorname{cov}_r(U) = V/\ell_2/p$$

and denote

$$S_r := S \cup \{x \in N : \exists v \in V : \|x - v\|_2 = r\}.$$

Prove that (CSL) can be reduced to (DSL):

a) S_r is finite.
b) Consider an edge $e = uv \in E$ and let $S_r^e := S_r \cap e = \{s_1, s_2, \dots, s_k\}$. Order $\{s_1, \dots, s_k\}$ as $u = s_1 <_e \dots <_e s_n = v$ with respect to the natural order along the edge $e = uv$, starting from u. Let s be a point on uv between s_i and s_{i+1}, i.e., $s_i <_e s <_e s_{i+1}$. Then $\mathrm{cov}_r(s_i) \supseteq \mathrm{cov}_r(s)$ and $\mathrm{cov}_r(s_{i+1}) \supseteq \mathrm{cov}_r(s)$.
c) (CSL) has an optimal solution $S^* \subseteq S_r$.

Exercise 5. **Tutorial Session**

Consider the bus stop location problem on the left of Fig. **??** with unit costs, potential stops $S = \{s_1, \dots, s_7\}$, demand points $V = \{v_1, \dots, v_5\}$ and covering radius r with associated covering matrix

$$A_r^{\mathrm{cov}} = \begin{array}{c} \\ \\ \\ \\ \\ \end{array} \begin{array}{ccccccc|c} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & p_1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & p_2 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & p_3 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & p_4 \\ 1 & 1 & 1 & 1 & 0 & 1 & 0 & p_5 \end{array}$$

(cf. Schöbel [2003]). Solve this problem using set covering techniques, illustrating your reasoning in the figure.

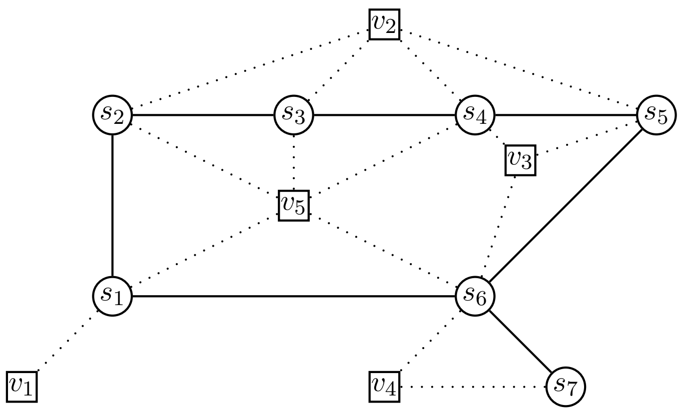

Figure 1: Bus stop location problem.

Exercise 6. **Tutorial Session**

Peter wants to buy a new computer but has little money. To save costs, he decides to purchase individual parts that he plans to assemble himself. In a tedious internet search, he identifies five suppliers that sell the parts that he wants at the following prices (in Euros):

supplier	1	2	3	4	5
main board	100	120	105	120	110
CPU	190	200	205	190	210
RAM	65	50	45	50	65
hard disk	105	80	85	100	105
frame	45	45	50	40	40

All suppliers charge (in Peter's opinion) ridiculously high delivery costs independent of the size of the order, namely, suppliers 1, 2, and 3 charge 20 Euros, supplier 4 charges 25 Euros, and supplier 5 charges 10 Euros. Clenching his teeth, Peter recalls his math class. After all, a computer is also a kind of infrastructure. At least, he can ... yeah, what can he do?

a) Develop an optimization model for Peter's computer acquisition problem.
b) What kind of model is this?
c) Solve the model. What is the optimal solution?
d) Solve the linear programming relaxation of the model. What happens?
e) Fix the suppliers and resolve the linear programming relaxation of the model. What happens now?
f) Peter is seldom at home, and his neighbors don't want to store more than one parcel for him. What should he do?