

Network Design and Operation (WS 2015)

Excercise Sheet 11

Submission: Mo, 25. January 2016, tutorial session

Exercise 1.

4 Points

Show through an example that the set of extreme points of a non-empty compact set does not need to be closed.

Exercise 2.

8 + 5 Points

Let $A \in \mathbb{R}^{m \times n}$ be a matrix with linearly independent columns and let C be a non-empty convex subset of \mathbb{R}^n .

- a) Show that a vector $x \in C$ is an extreme point of C if and only if Ax is an extreme point of the image AC ;
- b) Show through an example that if the columns of A are linearly dependent, then Ax can be an extreme point of AC for some non-extreme point x of C .

Exercise 3.

7+7+7 Points

Prove the following statements:

1. Every tree made up of more than one node has at least one leaf.
2. An undirected graph is a tree if and only if it is connected and possesses $|N| - 1$ edges.
3. Given any two distinct nodes i, j of a tree, there exists a unique path from i to j .
4. If we add a single edge to a tree, the resulting graph contains exactly one cycle (if we do not distinguish between cycles defined over the same set of nodes).