Stochastic and diffusive processes: Topics for exam

Dr. Luca Donati

Zuse Institute Berlin
Modeling and Simulation of Complex Processes
Computational Molecular Design

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Rules

- The exam will be oral and will generally last about 40 minutes.
- The course program is subdivided in 8 topics, listed into the pdf.
- Students will prepare a 30 minutes blackboard presentation for each topic.
- Students can bring notes.
- Students can bring a laptop or printed figures to show examples.
- At the beginning of the exam, one topic will be randomly chosen for each student.
- Students will present the topic without interruptions (unless serious errors emerge during the presentation).
- Questions about the chosen topic, or other topics, will be addressed in the last 5-10 minutes.
- The date of the exam will be decided by doodle.

- Describe the experiment conducted by Robert Brown and the main results.
- Describe the key points of Einstein's theory for Brownian motion (diffusion equation and the Stokes-Einstein relation).
- Describe the key points of Langevin's theory for Brownian motion.

References

[1] Lecture notes 1.
[2] Gardiner W., Handbook of Stochastic Methods 1994, chapter 1.
[3] Brown R. 1828.
[4] Einstein A. 1905.
[5] Langevin P. 1908.

- Describe Markovian processes and the Chapman-Kolmogorov equation
- Describe the differential CKE (dCKE), the Master equation and the Kramers-Moyal expansion.
- Why the Kramers-Moyal expansion can be truncated at the second order? (Pawula theorem).

References

[1] Lecture notes 3.
[2] Gardiner W., Handbook of Stochastic Methods 1994, chapter 3.
[3] Pawula R.,1967.

- Describe the master equation.
- Describe the method of moments generating functions and apply to the pure birth process.
- Describe Gillespie's algorithm.

References

[1] Lecture notes 3, 4b.
[2] Gillespie D.T., Markov Processes 1992, Chapter 5.
[3] Shorack G.R., Probability for statisticians 2000, Chapter 7.

4 Generalized Langevin equation

- Describe the Kac-Zwanzig model and the Generalized Langevin equation.
- Describe the memory kernel, the noise term and the Fluctuation-Dissipation theorem.
- Under what conditions can the generalised Langevin equation be reduced to the Markovian Langevin equation?

References

[1] Lecture notes 4a, 5, 6.
[2] Tuckerman M.E., Statistical Mechanincs: Theory and Molecular Simulation 2010, Chapter 15.
[3] Kupferman R. 2002.
[4] Kubo R. 1966.

5 Escape rate problem: high friction regime

- Describe the escape rate problem.
- Describe the Pontryagin theory for the high friction regime.

References

[1] Lecture notes 7.
[2] Baron P., Reaction Rate Theory and Rare Events Simulations, chapter 18.

6 Escape rate problem: moderate friction regime

- Describe the escape rate problem.
- Describe the Kramers theory for the moderate friction regime.

References

[1] Lecture notes 8.
[2] Baron P., Reaction Rate Theory and Rare Events Simulations, chapter 16.

7 Escape rate problem: low friction regime

- Describe the escape rate problem.
- Describe the Kramers theory for the low friction regime.

References

[1] Lecture notes 9.
[2] Baron P., Reaction Rate Theory and Rare Events Simulations, chapter 16.

- Describe the Fokker-Planck operator, the infinitesimal generator, the propagator, and the transfer operator.
- Describe the SqRA method to discretize the inifitisimal generator.

References

[1] Lecture notes 10, 11.
[2] Donati L. 2018.
[3] Donati L. 2021.

Other topics not requested for the exam

- PCCA+
- ISOKANN