I. SUMMARY FIRST LECTURE

- Hamiltonian dynamics of one particle of mass m (units: kg) on x-axis.
- Definition of position (units: m)

$$x(t) \tag{1}$$

- Definition of velocity (units: $\mathrm{m\,s^{-1}}$)

$$v(t) = \frac{d}{dt}x(t) = \dot{x}(t)\,. \tag{2}$$

- Definition of acceleration (units: $\mathrm{m\,s^{-2}}$)

$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t) = \ddot{x}(t)\,. \tag{3}$$

- Definition of momentum (units: $\mathrm{kg\,m\,s^{-1}}$)

$$p(t) = mv(t) = m\frac{d}{dt}x(t) = m\dot{x}(t)\,. \tag{4}$$

- Definition of force (units: $\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$)

$$F(t) = ma(t) = m\frac{d^2}{dt^2}x(t) = m\ddot{r} = \frac{d}{dt}p(t) = \dot{p}(t)\,. \tag{5}$$

- Force and potential energy function (units: J)

$$F = -\frac{d}{dx}V(x)\,. \tag{6}$$

- Kinetic energy (units: J)

$$E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}\,. \tag{7}$$

- Hamiltonian function or total energy (units: J)

$$\begin{aligned}\mathcal{H}(x,p) &= E_k + V(x)\\ &= \frac{p^2}{2m} + V(x)\,,\end{aligned} \tag{8}$$

- Equations of motion

$$\begin{cases}\dot{x} &= \dfrac{\partial \mathcal{H}}{\partial p} = \dfrac{p(t)}{m}\\ \dot{p} &= -\dfrac{\partial \mathcal{H}}{\partial x} = F(t) = -\dfrac{d}{dx}V(x(t))\end{cases}\,. \tag{9}$$

II. HARMONIC OSCILLATOR

Potential energy function of an harmonic oscillator:

$$V(x) = \frac{1}{2}kx^2\,, \tag{10}$$

where k is the spring constant (units: $\mathrm{N\,m^{-1}} = \mathrm{kg\,s^{-2}}$).

The second equation of the motion of the harmonic oscillator can be written as

$$m\ddot{x} + kx = 0. \tag{11}$$

This is a second-order linear homogeneous ordinary differential equation with constant coefficients and the analytical (exact) solution is

$$x(t) = x(0)\cos(\omega t) + \frac{p(0)}{m\omega}\sin(\omega t)\,, \tag{12}$$

where we introduced the angular frequency of the oscillation (units: s^{-1})

$$\omega = \sqrt{\frac{k}{m}}\,. \tag{13}$$

Finally, from the first equation of motion

$$p = m\dot{x} \tag{14}$$

we obtain the solution for the momentum

$$p(t) = -m\omega x(0)\sin(\omega t) + p(0)\cos(\omega t)\,. \tag{15}$$