A MIXED INTEGER PROGRAMMING MODEL FOR A LASER WELDING PROBLEM IN CAR BODY MANUFACTURING

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Abstract. We address the problem of scheduling laser welding robots in car manufacturing. We are given a set of weld seams that have to be produced on a car body within a manufacturing cell. The weld seams are applied by several robots working in parallel. The task is to find, for each robot, a sequence and the timing of its moves such that all seams are produced within the cycle time of the cell and as few external laser sources as possible are employed. In the variant of the problem we consider here, to each welding robot, one fixed laser source feeding it must be assigned. A laser source can feed several robots, but, at any point in time, only one of its associated robots is allowed to weld. In our special case, each robot must follow, in addition, a predefined sequence of weld seams. We propose a mixed integer programming formulation for this special welding problem that can be solved to optimality very quickly for the practical cases we encountered.

1. Introduction

Welding is an important part of car body manufacturing. The set of weld seams that are necessary to produce the body are subdivided in the design phase of the production line in two steps. First, a group of weld seams is assigned to each welding cell; second, this group is subdivided into subgroups of weld seams, one for each welding robot of the cell (typically 2–6 robots per cell). The subgroups must be chosen such that every robot can complete the associated welding jobs within the given cycle time. Various side constraints have to be satisfied in addition, for instance, robot moves must be collision free.

We consider here laser welding robots. Our goal is to schedule and time the robot moves such that the number of external laser sources is as small as possible.

A welding robot is always situated in one of the following two possible working states: either the robot is welding, or it is not. In the second case it may be idle or moving. Whenever a robot is welding, it must be fed by an external laser source. Due to technical restrictions, each robot is fed by one fixed laser source. While robots are inflexible in this respect, laser sources can feed several (in our case up to six) robots, but only one at a time. This option is rarely utilized in practice. In most cases we know of, one laser source supplies just one robot.

In this paper we investigate whether it is possible to reduce the number of laser sources by coordinating the moves of several robots in such a way that one laser source can feed more than one robot.

The reason for looking at this special objective is that laser sources are by far the most expensive devices of a manufacturing cell and their capacity should be utilized as much as possible. Scheduling algorithms for welding robots usually aim at maximizing the work done in a production cycle or minimizing the length of the traveled distance. Hence, these algorithms are inadequate for the problem considered here.

2. Problem Description

In this paper, we do not the consider the problem of configuring the manufacturing cells of the overall production line and assign the weld seams to cells and welding robots. Instead, we address the problem of processing the weld seams assigned to a single cell. A production line has a fixed cycle time by which all weld seams in each cell have to be produced. At the beginning of each cycle, each welding robot of the manufacturing cell is in a predefined home position. From this position the robot starts its welding sequence and the robot must reach its home position again before the cycle time is over.

We assume that each welding robot has already given a fixed sequence of weld seams specifying the order by which the seams are processed as well as the corresponding starting times. This sequence is called *robot welding path* and specifies the following data:

- a *welding time* for each weld seam and
- a *minimal moving time* for moving between each two adjacent weld seams, or one weld seam and the robot's home position.

Given a fixed sequence of weld seams for each robot, there is little left to decide. A robot welding paths can only be modified by making the corresponding robot *wait*: While the welding robot is moving, it is possible to slow down the robot such that processing the next weld seam is arbitrarily postponed. Therefore, the moving times given by a robot welding path are minimal.

We investigate whether or not it is possible to save laser sources by sharing them among the robots. However, our model does not directly provide an answer to this question: Instead of minimizing the number of required laser sources, we aim to minimize for a fixed number of laser sources the required processing time in the laser cell, i. e., the time for processing all weld seams and moving the robots back to their home positions. With this objective we are able to tell to what extent the cycle time is violated when a certain number of laser sources is used.

The decisions that must be made in the problem are the following:

- Assign each robot to one laser source.
- Set the starting time for each weld seam.

Next we summarize the restrictions that must be satisfied:

(i) Each robot is assigned exactly one laser source.
(ii) Each moving time of a robot is at least the minimal moving time.
(iii) Each two robots that have been assigned to the same laser source do not weld simultaneously. Moreover, when one robot welds directly after the other one there is a time gap of at least the switching time.

3. Related Work

Note that a positive switching time means no further restriction in the problem setting if it is does not exceed the minimal moving time. In this case the problem is easily transformed into an equivalent problem without required switching times by increasing the welding time of each weld seam by the switching time and decreasing the consecutive moving time by the same value.

If there is only laser source, this transformed problem is a so-called *single-machine scheduling problem with minimum time lags.* Table 1 gives an overview on correlations with our problem. In the literature, several versions of this scheduling problem have been studied (see e. g. [BK99, MS03]). These problem vary in their objective, their type of precedence constraints, and their structure of processing times and time lags. The paper [BK99] also gives an overview on the complexity of many variants. About half of them are NP-hard.

laser welding	scheduling
laser source	machine
weld seams	jobs
welding time	processing time
moving time	time lag
robot welding path	precedence constraints

Table 1. Connection between the terms from laser welding and scheduling.

In our case the objective is the makespan, the precedence constraints form chains, and the processing times and time lags may be arbitrary. For only two chains of jobs, the complexity of the scheduling problem is still open. However, in [Wik92] Wikum proposed a pseudopolynomial algorithm. But if the number of chains is part of the input, the problem is already NP-hard if one of the chains consists of more than one job, see [WLN94]. To the best of our knowledge, no mixed integer programming models have yet been proposed in this context. [MSSU01] gives integer programming models incorporating time lags, but the considered scheduling problem is quite different.

4. Mixed Integer Programming Model

In this section we model the considered laser welding problem using as a mixed integer program (MIP). Given is a set of welding robots R and a set of laser sources L. Furthermore, an amount of δ time units is required to switch the output of one laser source between two robots. Each robot $r \in R$ specifies:

- a set of associated weld seams J_r,
- a weld seam $\mathit{first}_r \in J_r$ processed first in the welding path of robot r,
- a weld seam $\mathit{last}_r \in J_r$ processed last in the welding path of robot r,
- a minimal moving time t_r for reaching weld seam first_r from the home position of robot r.

For a given robot $r \in R$, each weld seam $j \in J_r$ has the following data:

- a welding time p_j,
- a weld seam $\mathit{next}_j \in J_r$ succeeding j in the robot welding path if $j \neq \mathit{last}_r$,
- a minimal moving time d_j which is required by robot r to reach weld seam next_j if $j \neq \mathit{last}_r$, or the robot's home position if $j = \mathit{last}_r$.

Before we present our MIP formulation for the general problem, let us consider the restricted case, where only one laser source is available. Our model is formulated in terms of the following variables. The continuous variable x_j denotes the starting time of weld seam j, and for each two distinct robots $r, s \in R$ and weld seams $j \in J_r$ and $k \in J_s$, the binary variable

$$y_{jk} := \begin{cases} 0, & \text{if weld seam } j \text{ is processed not later than weld seam } k, \\ 1, & \text{otherwise.} \end{cases}$$

Using these variables the restrictions of the problem can be formulated as described in the following. Restriction (ii) concerning the moving times is formulated using two types of constraints: One for moving a robot from its home position to the first weld seam and a second for moving between adjacent weld seams:

$$\begin{aligned} x_{\mathit{first}_r} &\geq t_r \quad \forall r \in R \\ x_{\mathit{next}_j} &\geq x_j + p_j + d_j \quad \forall r \in R, j \in J_r \setminus \{\mathit{last}_r\}. \end{aligned}$$

The condition for switching the laser source (restriction (iii)) is modeled using a formulation with big M:

$$x_k + My_{jk} \geq x_j + p_j + \delta \quad \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \tag{1}$$

$$x_j + M(1 - y_{jk}) \geq x_k + p_k + \delta \quad \forall r, s \in R : r \neq s, j \in J_r, k \in J_s. \tag{2}$$

Note that for M large enough exactly one of the above constraints is trivially satisfied for each feasible assignment of variables depending on the order the weld seam j and k are produced. For instance, if y_{jk} equals zero, i. e., weld seam j is produced before seam k, then inequality (2) is always satisfied. In this case inequality (1) requires that weld seam k (processed by another robot) cannot be processed earlier than the completion time of weld seam j plus the switching time δ.

Adding constraints that determine together with the objective the maximal processing time of a welding robot, the following MIP models the problem for only one laser source:

$$\text{(SL)} \qquad \begin{aligned} \min\ z \quad & \text{s. t.} \\ z \geq x_{last_r} + p_{last_r} + d_{last_r} \quad & \forall r \in R \\ x_{first_r} \geq t_r \quad & \forall r \in R \\ x_{next_j} \geq x_j + p_j + d_j \quad & \forall r \in R, j \in J_r \setminus \{last_r\} \\ x_k + My_{jk} \geq x_j + p_j + \delta \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \\ x_j + M(1 - y_{jk}) \geq x_k + p_k + \delta \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \\ y_{jk} \in \{0, 1\} \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \end{aligned}$$

In the following, we describe how to extend the program (SL) (single laser) for multiple laser sources. We need additional variables for modeling the case that two robots r and s share one laser source. In this case the continuous variable v_{rs} should equal at least 1. This is realized by using, for each robot r and each laser source l, the following binary variables:

$$u_{rl} := \begin{cases} 1, & \text{if robot } r \text{ is fed by laser source } l, \\ 0, & \text{otherwise.} \end{cases}$$

The following constraints reflect that each robot is assigned exactly one laser source (restriction (i)) and that v_{rs} equals at least 1 if the robots r and s are fed by the same laser source:

$$\begin{aligned} \sum_{l \in L} u_{rl} = 1 \quad & \forall r \in R \\ v_{rs} \geq u_{rl} + u_{sl} - 1 \quad & \forall r, s \in R : r \neq s, l \in L. \end{aligned}$$

Note that v_{rs} may be set to zero if the robots r and s are fed by different laser sources.

What remains to be done is to modify inequality (1) and inequality (2) such that these constraints need not be satisfied for robots r and s that are fed by different laser sources, i. e., v_{rs} can be set to zero. This is realized by adding the term $M(1 - v_{rs})$ on the left hand side of both constraints:

$$\begin{aligned} x_k + M(1 + y_{jk} - v_{rs}) \geq x_j + p_j + \delta \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \\ x_j + M(2 - y_{jk} - v_{rs}) \geq x_k + p_k + \delta \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s. \end{aligned}$$

Finally, the mixed integer program (ML) (multiple laser) modeling the general problem reads as follows:

$$
\text{(ML)} \qquad \begin{aligned}
\min \ z \quad & \text{s.t.} \\
z \geq x_{last_r} + p_{last_r} + d_{last_r} \quad & \forall r \in R \\
\sum_{l \in L} u_{rl} = 1 \quad & \forall r \in R \\
v_{rs} \geq u_{rl} + u_{sl} - 1 \quad & \forall r, s \in R : r \neq s, l \in L \\
x_{first_r} \geq t_r \quad & \forall r \in R \\
x_{next_j} \geq x_j + p_j + d_j \quad & \forall r \in R, j \in J_r \setminus \{last_r\} \\
x_k + M(1 + y_{jk} - v_{rs}) \geq x_j + p_j + \delta \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \\
x_j + M(2 - y_{jk} - v_{rs}) \geq x_k + p_k + \delta \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \\
y_{jk} \in \{0, 1\} \quad & \forall r, s \in R : r \neq s, j \in J_r, k \in J_s \\
u_{rl} \in \{0, 1\} \quad & \forall r \in R, l \in L
\end{aligned}
$$

5. Results

In this section, we apply the proposed model (ML) to a real-life instance. The manufacturing cell we consider consists of three welding robots. Currently, three laser sources are employed each of which supplies just one robot. In total 34 weld seams have to be produced. The cycle time of the considered cell equals 36 seconds and the time for switching a laser source between two robots equals 0.1 seconds. Figure 1 depicts the currently applied robot welding paths: The blocks represent the weld seams and the lines in between show the robot moves. In the following we will refer to the robots as the blue, red, and green robot according to the colors used in the pictures.

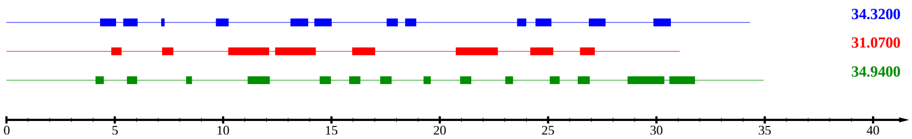

Figure 1. Original robot welding paths of the considered instance.

For two laser sources, the commercial software CPLEX solves the corresponding mixed integer program to optimality within only one second. The optimal robot welding paths for this case are show in Figure 2. In the solution the red and the

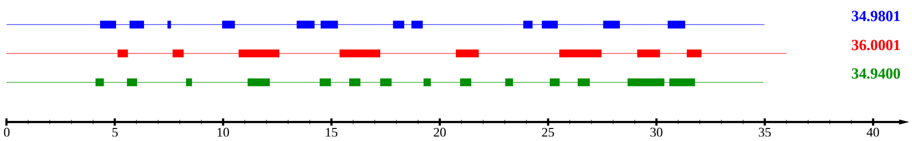

Figure 2. Optimized robot welding paths for two laser sources.

blue robot share one laser source, while the green robot is fed by the second laser source alone, and thus its robot welding path has not been modified. Note that the processing time for the red welding robot equals quite exactly the cycle time of 36 seconds. Therefore, it is not directly possible to reduce the number of employed laser sources to two. However, if one weld seam can be slightly shortened or one weld seam can be transfered into another manufacturing cell, two laser sources suffice.

Finally, we investigate the possibility to feed all robots by only one laser source. For the corresponding mixed integer program CPLEX needs with standard settings less than three seconds to compute an optimal solution. The resulting robot welding paths are depicted in Figure 3. Note that the maximal processing time is about

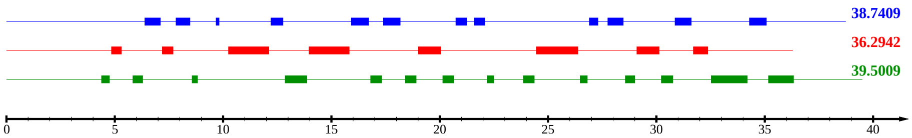

FIGURE 3. Optimized robot welding paths for one laser source.

39.5 seconds. Hence, with our approach a reduction to only one laser source is impossible. However, it is open for the considered manufacturing cell whether one laser source can suffice when all possible freedoms of choice are exploited. The total welding time (sum of all welding times) in the considered cell is about 25 seconds. Since the time between processing two weld seams is at least the switching time, additionally 3.3 seconds are required. In total, at least 28.3 seconds are needed from start of the first weld seam until the last is produced. Because of the time for the robot moves from and to the home positions, there is hardly time left. Therefore, it is rather unlikely that one laser source can suffice.

6. CONCLUSIONS

We have proposed a mixed integer programming formulation for the problem of scheduling welding robots with external laser sources that can be solved to optimality very quickly. The main drawback of our model is that it is based on a heuristic approach leaving much potential unused: We only modify existing robot welding paths by adding little waiting times in the robot moves. For the instance we considered, however, this method already allows to reduce the number of required laser sources from three to two, when, e. g., one weld seam is shortened a little bit! We were not able to apply more exact approaches because of missing data.

More sophisticated planning for minimizing the number of necessary laser sources would be as follows. **First**, the problem yields more potential when the sequence of weld seams for each robot need not be fixed. That is, sequencing the weld seams for each robot is part of the optimization. In a **second** step, one would also allow arbitrary assignments of weld seams to robots, i. e., the subgroups of weld seams associated with the robots are not fixed, but can be optimized as well. With these decisions all freedom of choice in the manufacturing cell can be exploited. We are sure that a customized *column generation algorithm* can solve the corresponding problem at least near-optimal.

In a **third** step, one should extend the focus to the whole production line, instead of considering only a single manufacturing cell. Optimizing the assignment of weld seams to cells too, will yield a lot more possibilities. By taking into account all these decisions we expect significant potential for reducing the number of required laser sources. Even if this global approach is not used, the problem features much more potential than we have exploited in this paper.

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