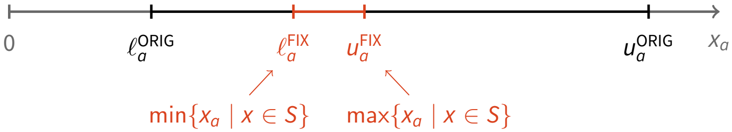
Timetable Merging for the Periodic Event Scheduling Problem



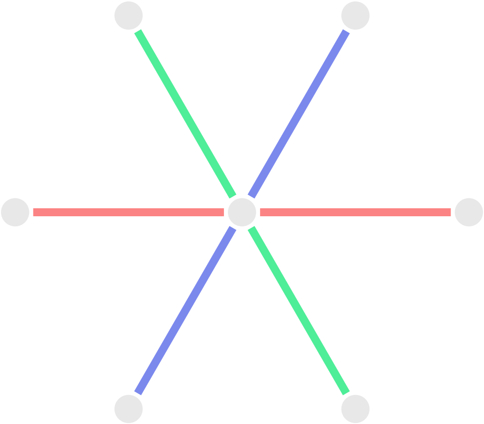
Niels Lindner
Zuse Institute Berlin

Christian Liebchen
Technical University of Applied Sciences Wildau

RailBeijing 2021
November 5, 2021

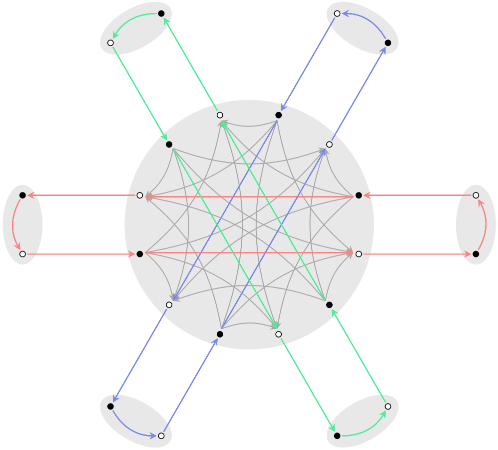
Periodic Timetabling in Public Transport

Periodic Timetabling in Public Transport



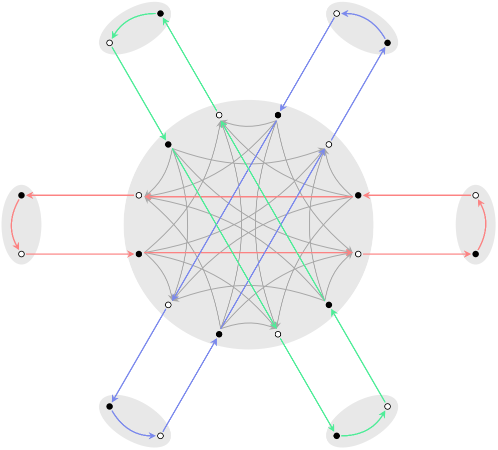
Line Network, 3 bidirectional lines

Periodic Timetabling in Public Transport



Event-Activity Network

Periodic Timetabling in Public Transport

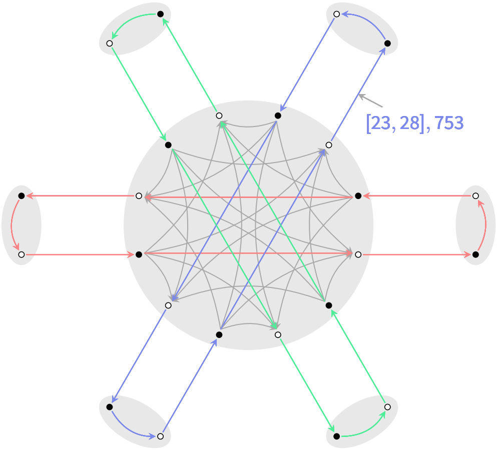


Event-Activity Network

Events:
- ● arrival
- ○ departure

Activities:
- → drive, dwell, turn
- → transfer
- . . .

Periodic Timetabling in Public Transport



Periodic Event Scheduling Instance

Bounds:

- driving times
- minimum transfer times
- maximum dwell times
- minimum headway times
- ...

Weights:

- passenger load
- turnaround penalties
- ...

Period time:

- e.g., $T = 60$ for 1 hour, resolution of 1 minute

Periodic Event Scheduling Problem (PESP)

Given

$G = (V, A)$	event-activity network,
$T \in \mathbb{N}$	period time,
$\ell \in \mathbb{R}^A$	lower bounds,
$u \in \mathbb{R}^A$	upper bounds,
$w \in \mathbb{R}^A_{\geq 0}$	weights,

find

$\pi \in [0, T)^V$	periodic timetable,
$x \in \mathbb{R}^A$	periodic tension

such that

(1) $\pi_j - \pi_i \equiv x_{ij} \bmod T$ for all $ij \in A$,

(2) $\ell \leq x \leq u$,

(3) $w^\top x$ is minimum,

or decide that no such (π, x) exists.

(Serafini and Ukovich, 1989)

Periodic Event Scheduling Problem (PESP)

Given

$G = (V, A)$	event-activity network,
$T \in \mathbb{N}$	period time,
$\ell \in \mathbb{R}^A$	lower bounds,
$u \in \mathbb{R}^A$	upper bounds,
$w \in \mathbb{R}^A_{\geq 0}$	weights,

find

$\pi \in [0, T)^V$	periodic timetable,
$x \in \mathbb{R}^A$	periodic tension

such that

(1) $\pi_j - \pi_i \equiv x_{ij} \mod T$ for all $ij \in A$,

(2) $\ell \leq x \leq u$,

(3) $w^\top x$ is minimum,

or decide that no such (π, x) exists.

(Serafini and Ukovich, 1989)

Incidence-based MIP formulation:

$$
\begin{array}{lrl}
\text{Minimize} & w^\top x & \\
\text{s.t.} & \pi_j - \pi_i = x_{ij} - Tp_{ij}, & ij \in A, \\
& \ell_{ij} \leq x_{ij} \leq u_{ij}, & ij \in A, \\
& 0 \leq \pi_i < T, & i \in V, \\
& p_{ij} \in \{0, 1, 2\}, & ij \in A.
\end{array}
$$

Cycle-based MIP formulation:

(Nachtigall, 1994, Liebchen and Peeters, 2009)

$$
\begin{array}{lr}
\text{Minimize} & w^\top x \\
\text{s.t.} & \Gamma x = Tz, \\
& \ell \leq x \leq u, \\
& z \in \mathbb{Z}^B.
\end{array}
$$

$B \subseteq \mathbb{Z}^A$	integral cycle basis of G
$\Gamma \in \mathbb{Z}^{B \times A}$	cycle matrix of B
$z \in \mathbb{Z}^B$	cycle offsets/modulo parameters

Periodic Event Scheduling Problem (PESP)

Given

$G = (V, A)$	event-activity network,
$T \in \mathbb{N}$	period time,
$\ell \in \mathbb{R}^A$	lower bounds,
$u \in \mathbb{R}^A$	upper bounds,
$w \in \mathbb{R}^A_{\geq 0}$	weights,

find

$\pi \in [0, T)^V$	periodic timetable,
$x \in \mathbb{R}^A$	periodic tension

such that

(1) $\pi_j - \pi_i \equiv x_{ij} \bmod T$ for all $ij \in A$,

(2) $\ell \leq x \leq u$,

(3) $w^\top x$ is minimum,

or decide that no such (π, x) exists.

(Serafini and Ukovich, 1989)

Incidence-based MIP formulation:

$$
\begin{array}{lll}
\text{Minimize} & w^\top x & \\
\text{s.t.} & \pi_j - \pi_i = x_{ij} - Tp_{ij}, & ij \in A, \\
& \ell_{ij} \leq x_{ij} \leq u_{ij}, & ij \in A, \\
& 0 \leq \pi_i < T, & i \in V, \\
& p_{ij} \in \{0, 1, 2\}, & ij \in A.
\end{array}
$$

Cycle-based MIP formulation:

(Nachtigall, 1994, Liebchen and Peeters, 2009)

$$
\begin{array}{ll}
\text{Minimize} & w^\top x \\
\text{s.t.} & \Gamma x = Tz, \\
& \ell \leq x \leq u, \\
& z \in \mathbb{Z}^B.
\end{array}
$$

$B \subseteq \mathbb{Z}^A$	integral cycle basis of G
$\Gamma \in \mathbb{Z}^{B \times A}$	cycle matrix of B
$z \in \mathbb{Z}^B$	cycle offsets/modulo parameters

Hardness of PESP

Theory:

- NP-hard for fixed $T \geq 3$
  (Odijk, 1994, Nachtigall, 1996)
- NP-hard if G has treewidth ≥ 2
  (L. and Reisch, 2020)
- NP-hard cutting plane separation (cycle, change-cycle, flip)
  (Borndörfer et al., 2020, L. and Liebchen, 2020)

Hardness of PESP

Theory:

- NP-hard for fixed $T \geq 3$
  (Odijk, 1994, Nachtigall, 1996)
- NP-hard if G has treewidth ≥ 2
  (L. and Reisch, 2020)
- NP-hard cutting plane separation (cycle, change-cycle, flip)
  (Borndörfer et al., 2020, L. and Liebchen, 2020)

Practice:

- rich literature on algorithms:
  - MIP
  - CP
  - SAT (also MaxSAT and SAT+ML)
  - Modulo Network Simplex
  - Matching, Merging, Maximum Cuts, Graph Partitioning, ...
- several success stories (Berlin, Copenhagen, Netherlands, Switzerland, ...)
- none of the 22 instances of `PESPlib` (est. 2012 by Goerigk) solved to proven optimality yet

Hardness of PESP

Theory:

- NP-hard for fixed $T \geq 3$
  (Odijk, 1994, Nachtigall, 1996)
- NP-hard if G has treewidth ≥ 2
  (L. and Reisch, 2020)
- NP-hard cutting plane separation (cycle, change-cycle, flip)
  (Borndörfer et al., 2020, L. and Liebchen, 2020)

Practice:

- rich literature on algorithms:
  - MIP
  - CP
  - SAT (also MaxSAT and SAT+ML)
  - Modulo Network Simplex
  - Matching, Merging, Maximum Cuts, Graph Partitioning, . . .
- several success stories (Berlin, Copenhagen, Netherlands, Switzerland, . . .)
- none of the 22 instances of `PESPlib` (est. 2012 by Goerigk) solved to proven optimality yet

Summary:

There are many ways to heuristically optimize periodic timetables, but it is hard to assess the actual quality.

Solving PESP in Practice

ConcurrentPESP

- parallel solver for PESP instances developed at Zuse Institute Berlin
  (Borndörfer et al., 2020, presented at RailNorrköping 2019)
- combines a plenty of PESP algorithms
- computed best primal and dual bounds for all `PESPlib` instances
- current `PESPlib` incumbents are locally optimal for the implemented heuristics

Solving PESP in Practice

ConcurrentPESP

- parallel solver for PESP instances developed at Zuse Institute Berlin
  (Borndörfer et al., 2020, presented at RailNorrköping 2019)
- combines a plenty of PESP algorithms
- computed best primal and dual bounds for all `PESPlib` instances
- current `PESPlib` incumbents are locally optimal for the implemented heuristics

Our approach: Merging

Can we escape local optima by combining several good timetables to even better timetables?

Solving PESP in Practice

ConcurrentPESP

- parallel solver for PESP instances developed at Zuse Institute Berlin
  (Borndörfer et al., 2020, presented at RailNorrköping 2019)
- combines a plenty of PESP algorithms
- computed best primal and dual bounds for all `PESPlib` instances
- current `PESPlib` incumbents are locally optimal for the implemented heuristics

Our approach: Merging

Can we escape local optima by combining several good timetables to even better timetables?

Inspiration

- *Tour Merging* for the Traveling Salesman Problem (Cook, Seymour, 2003)
- *Crossover* in Mixed-Integer Programming (Rothberg, 2007)
- *Arc Selection* heuristic for PESP via MaxSAT (Roth, 2019)

Merging Periodic Timetables

PESP instance (G, T, ℓ, u, w)

↓

set S of good solutions x_1 x_2 $\cdots$ x_n

↓

new MIP

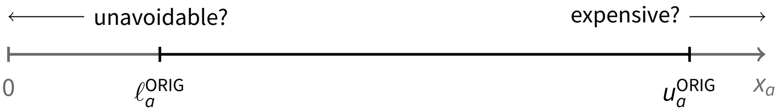
should ...

- ... be easier to solve
- ... contain all $x \in S$ as feasible solutions
- ... have a feasible set contained in the feasible set of the original PESP instance

Restricting the MIP

Idea

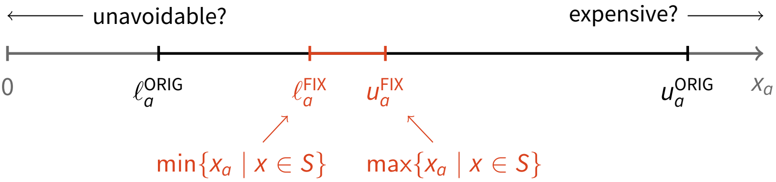
Restrict the x (periodic tension/activity durations) variables according to the values in S.



Restricting the MIP

Idea

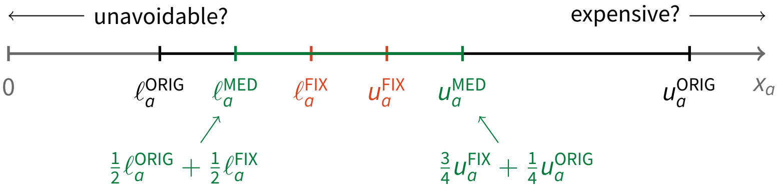
Restrict the x (periodic tension/activity durations) variables according to the values in S.



Restricting the MIP

Idea

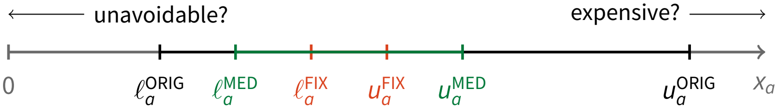
Restrict the x (periodic tension/activity durations) variables according to the values in S.



Restricting the MIP

Idea

Restrict the x (periodic tension/activity durations) variables according to the values in S.



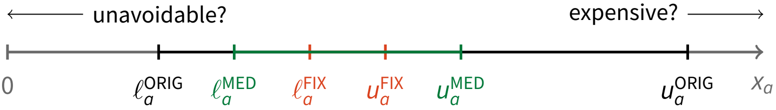
Tension-restricted scenarios

Consider arbitrary combinations of $\{\mathrm{ORIG}, \mathrm{MED}, \mathrm{FIX}\}$ for lower and upper bounds.

Restricting the MIP

Idea

Restrict the x (periodic tension/activity durations) variables according to the values in S.



Tension-restricted scenarios

Consider arbitrary combinations of $\{$ORIG, MED, FIX$\}$ for lower and upper bounds.

Cycle-offset restricted scenarios

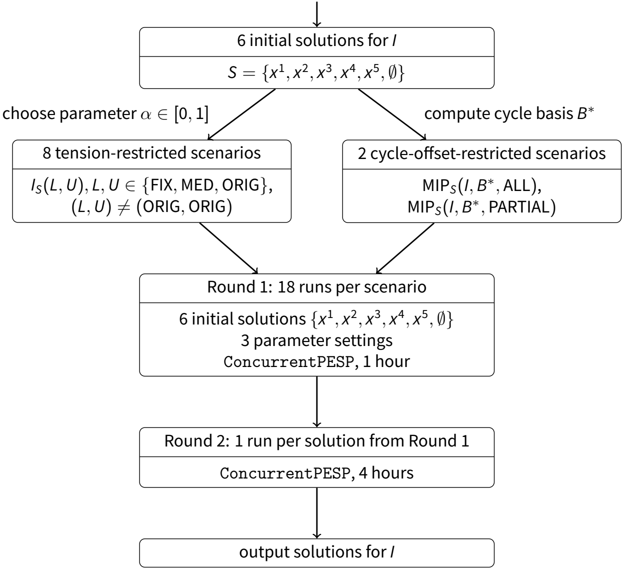
Two restriction schemes $\{$ALL, PARTIAL$\}$ for the cycle offsets z, tightening the bounds from Odijk's cycle inequalities (Odijk, 1994), and making use of minimum cycle bases.

Span Analysis

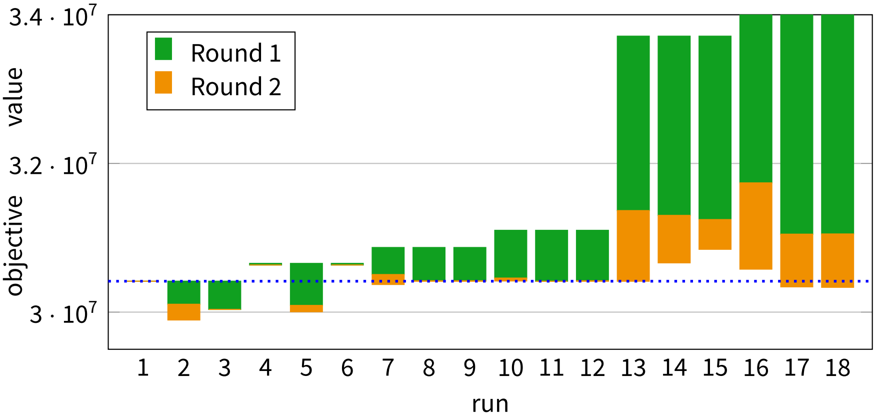
I	L	U	fixed activities	free activities	avg. span	avg. wt. span
R1L1	FIX	FIX	55.18 %	28.03 %	18.98	6 511
	MED	FIX	53.78 %	28.05 %	19.76	7 608
	ORIG	FIX	53.78 %	28.07 %	20.49	8 646
	FIX	MED	10.34 %	29.13 %	21.03	15 285
	MED	MED	10.12 %	29.24 %	21.81	16 382
	ORIG	MED	10.12 %	29.79 %	22.55	17 420
	FIX	ORIG	10.34 %	35.07 %	26.19	35 391
	MED	ORIG	10.12 %	35.61 %	26.96	36 488
	ORIG	ORIG	10.12 %	44.28 %	27.70	37 526
R4L4	FIX	FIX	42.78 %	32.38 %	24.99	3 802
	MED	FIX	42.66 %	32.40 %	25.76	4 073
	ORIG	FIX	42.66 %	32.54 %	26.46	4 320
	FIX	MED	8.93 %	34.36 %	27.02	7 643
	MED	MED	8.86 %	34.70 %	27.79	7 915
	ORIG	MED	8.86 %	36.53 %	28.50	8 162
	FIX	ORIG	8.93 %	40.91 %	31.84	16 221
	MED	ORIG	8.86 %	42.01 %	32.61	16 493
	ORIG	ORIG	8.86 %	54.27 %	33.32	16 740

Analysis of spans $u_a - \ell_a$ for the tension-restricted R1L1 and R4L4 `PESPlib` scenarios

Experimental Setup



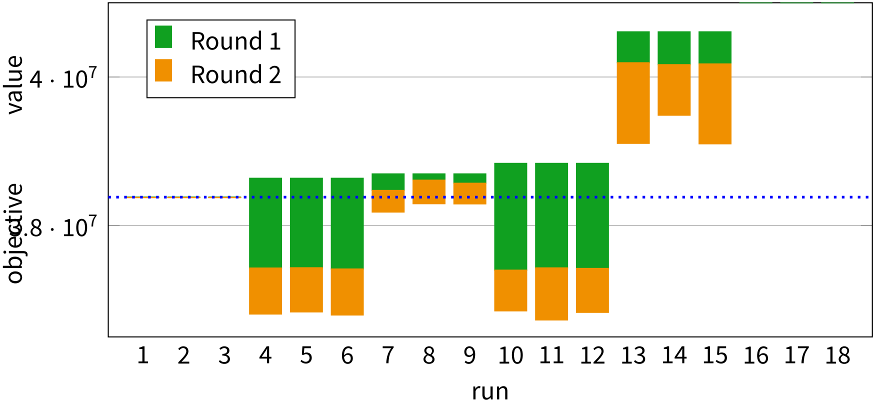
Results for R1L1



Objective values of the solutions obtained from R1L1(ORIG, MED)

- after Round 1: #2, #3, #5 produced solutions better than the PESPlib incumbent (value 30 415 672, dotted blue line)
- after Round 2: in total **7 better timetables**

Results for R4L4



Objective values of the solutions obtained from R4L4(MED, ORIG)

- after Round 1: 6 solutions better than the PESPlib incumbent (value 38 381 922, dotted blue line
- after Round 2: in total **9 better timetables**

The Last Slide

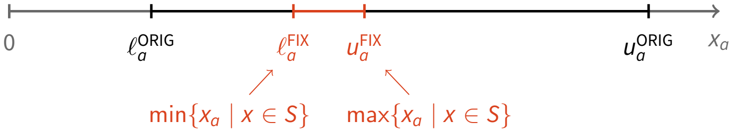
Some more details

- All scenarios with u^{FIX} could be solved to optimality: The best solution in S was already optimal.
- The best solutions found are not necessarily the ones with the best solution in S as initial solution for the solver.
- The cycle-offset restriction schemes produced comparable results.

Conclusion

- Timetable Merging can produce better periodic timetables.
- Timetable Merging escapes local minima for the modulo network simplex method and several other heuristics.
- While Timetable Merging is in concept similar to MIP crossover, it requires more computational power due to the difficulty of PESP.

Timetable Merging for the Periodic Event Scheduling Problem



Niels Lindner
Zuse Institute Berlin

Christian Liebchen
Technical University of Applied Sciences Wildau

RailBeijing 2021
November 5, 2021