Optimal Design of Experiments

LV 19086

Guillaume Sagnol

Organization

- Timing: Wednesday, 12:15–13:45 ?
- Website: Follow link on `http://www.zib.de/sagnol`
- Language: English
- Examination: Oral ?
- Prerequisites in stats: None.

Optimal Design of Experiments

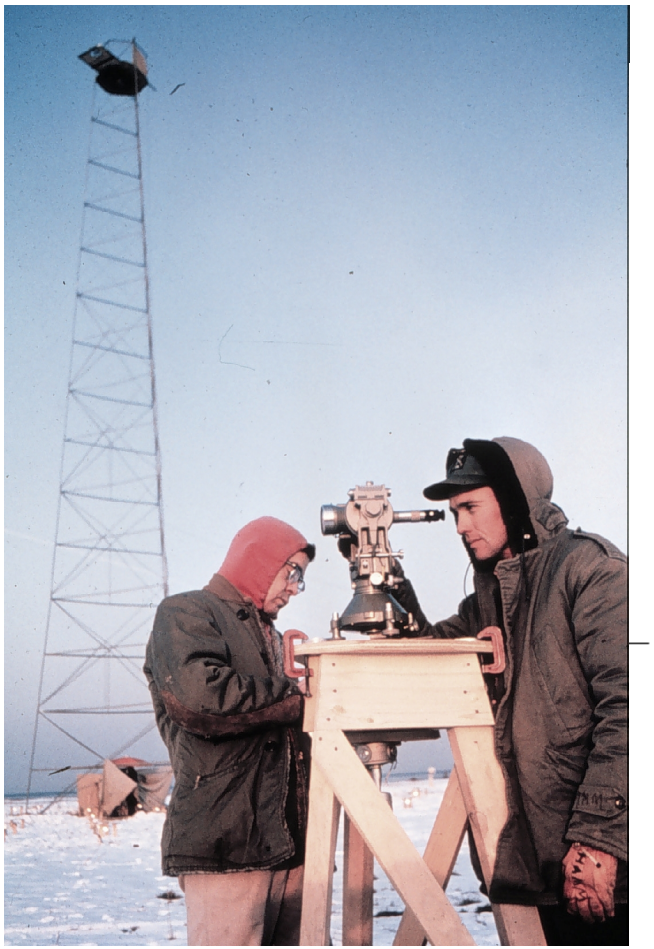
What it is:

- Optimization theory applied to the design of a statistical experiment

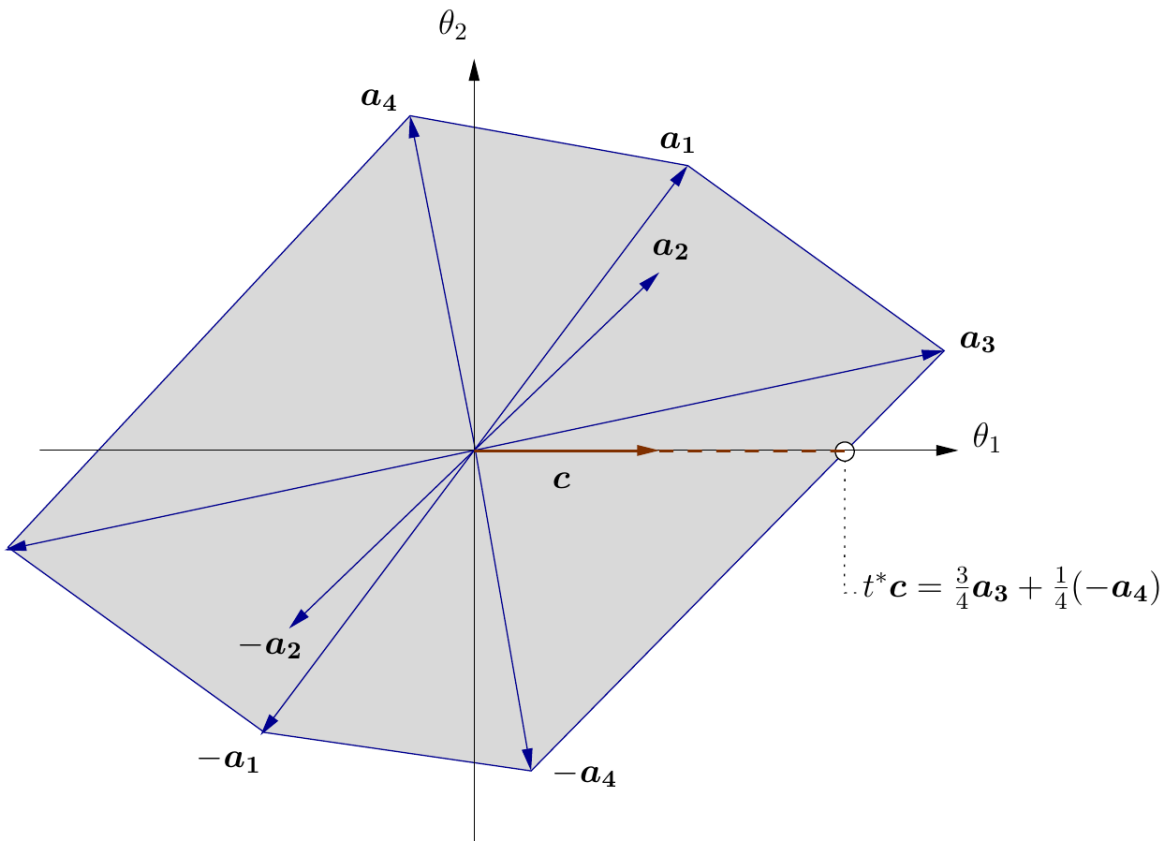
What it is not:

- Optimization theory applied to analyze the data of a statistical experiment

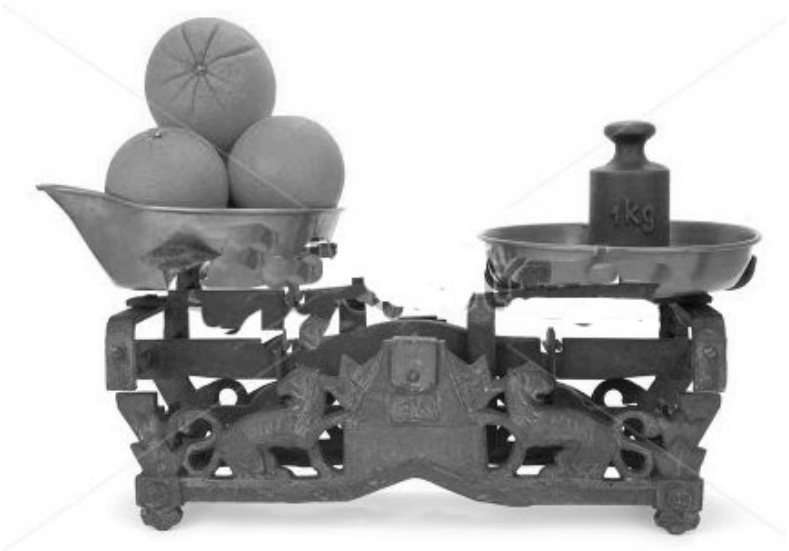
History



Gustav Elfving



Applications...



How many weighings to identify a heavier orange ?

Weighings

Harold Hotelling

Estimate the weights of N objects in $p \geq N$ weighings.

Is the strategy of weighing each object separately optimal ?

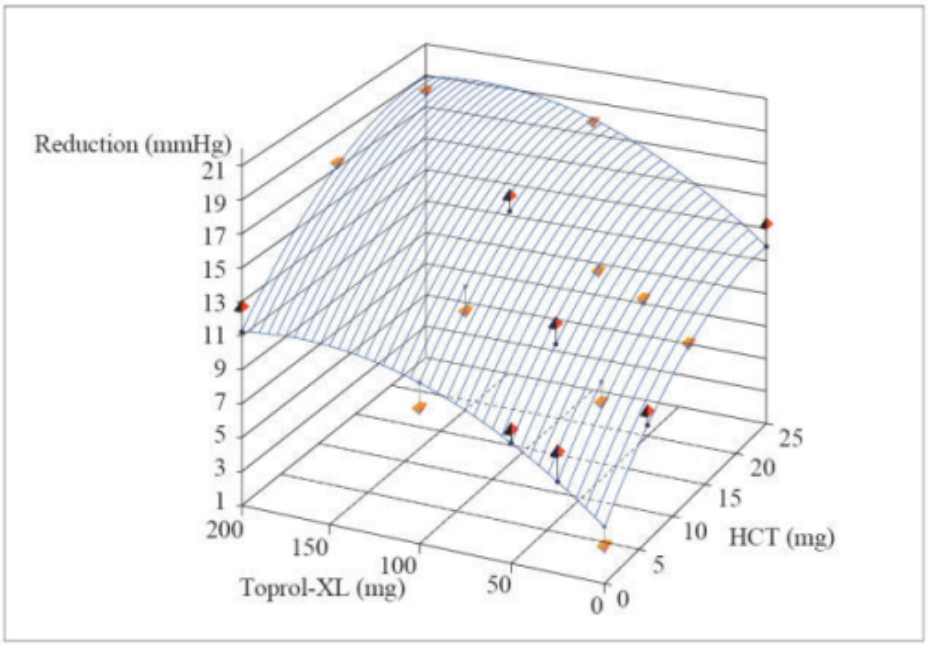
Weighings

Which strategy is better ?

	Design 1	Design 2
Weighing 1	1	1,2,3,4,5,6,7,8,
Weighing 2	2	1,2,3,8 vs. 4,5,6,7
Weighing 3	3	1,4,5,8 vs. 2,3,6,7
Weighing 4	4	1,6,7,8 vs. 2,3,4,5
Weighing 5	5	2,4,6,8 vs. 1,3,5,7
Weighing 6	6	2,5,7,8 vs. 1,3,4,6
Weighing 7	7	3,4,7,8 vs. 1,2,5,6
Weighing 8	8	3,5,6,8 vs. 1,2,4,7

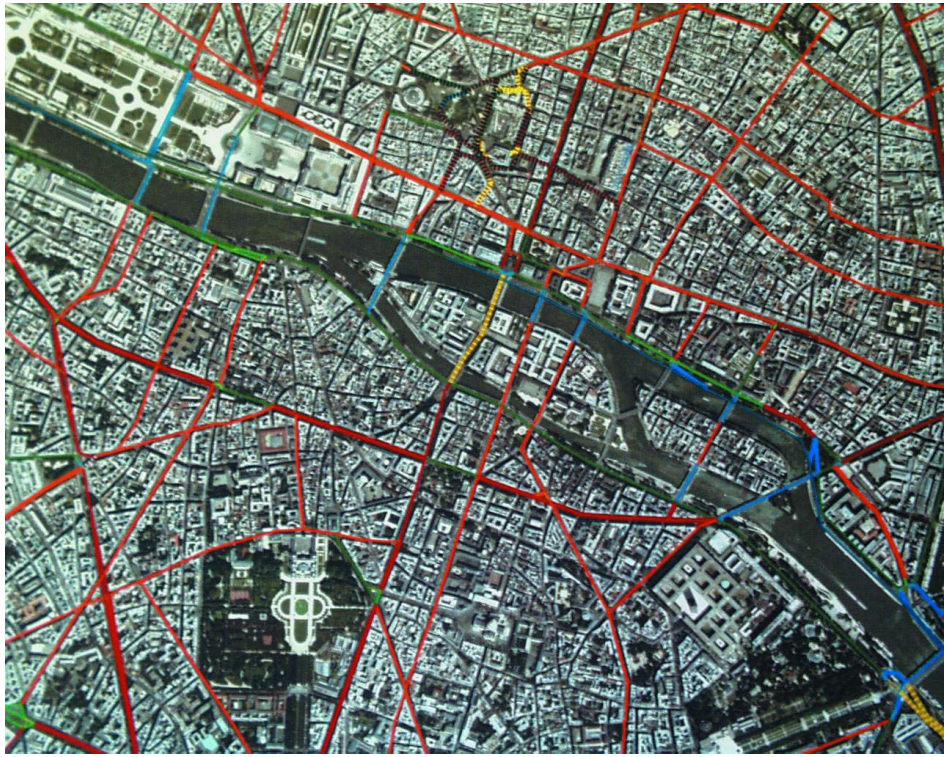
Drug design

Dose response surface:



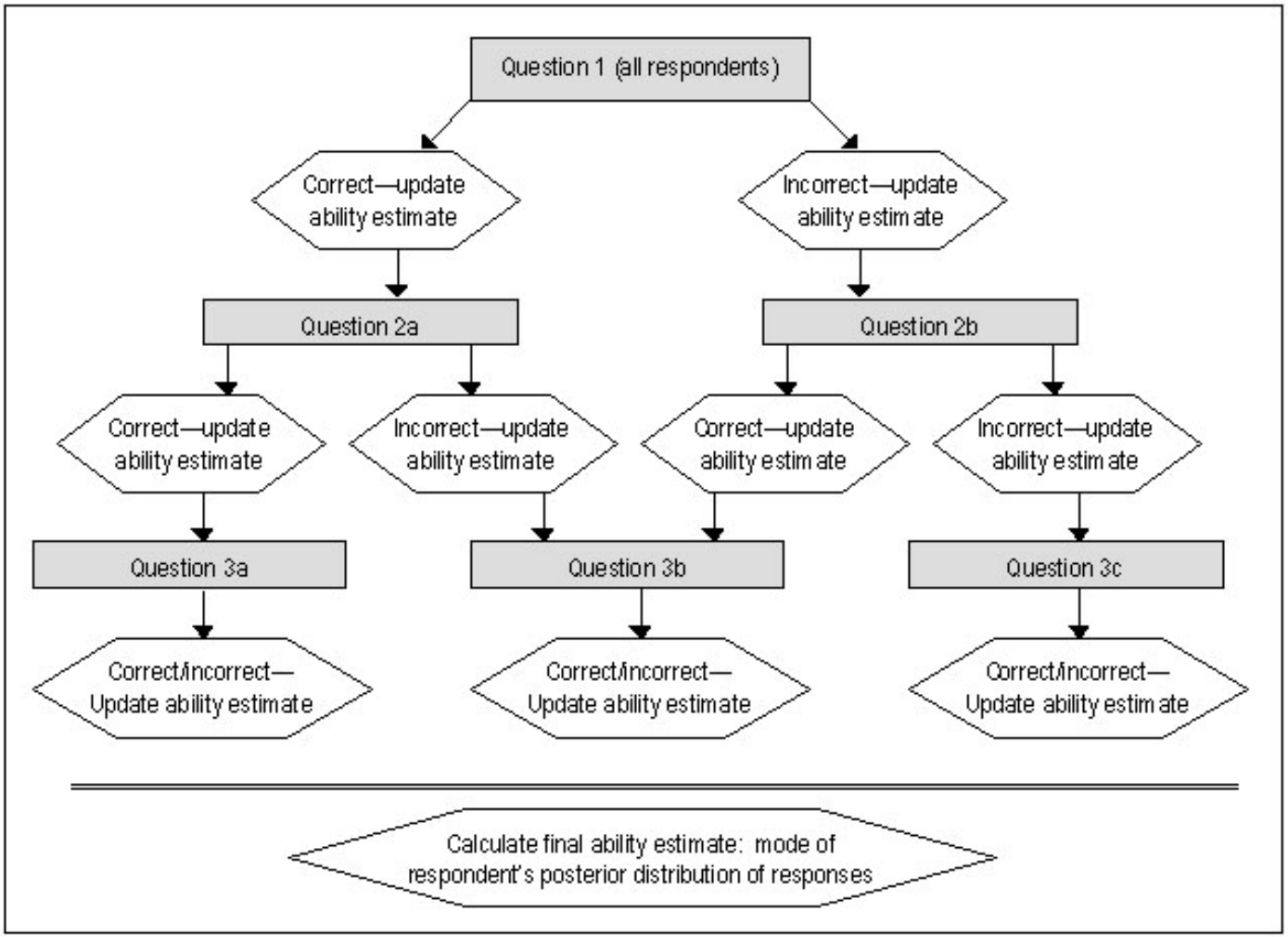
How many patients with what dose ?

Estimate Source to Destination traffic



Design a poll to estimate the Origin-Destination traffic.

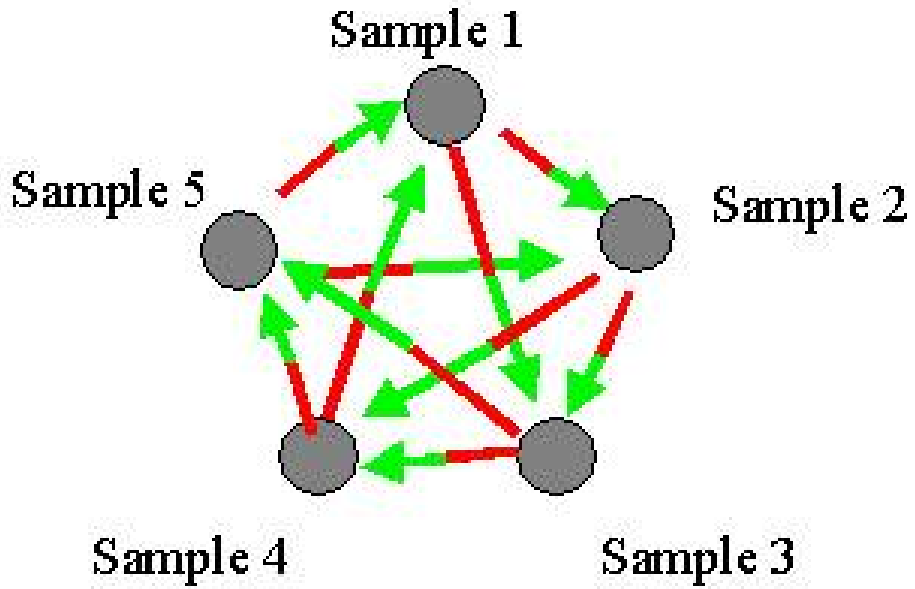
Computerize adaptive Testing



Choose the next question in order to estimate the capability of the examinee acurately.

Design of DNA microarrays

Goal: Compare differences of gene expressions



Select a subset of pairwise differences
to measure with a red/green dye.

Agricultural and plant-breeding experiment

Choose the trials to do in every plot.
(Knowing that the plots might affect the efficiency of the treatments...)

Mathematics of Optimal designs

General idea:

- Minimize the "size" of some confidence ellipsoids
  $\rightsquigarrow$ Geometric problems

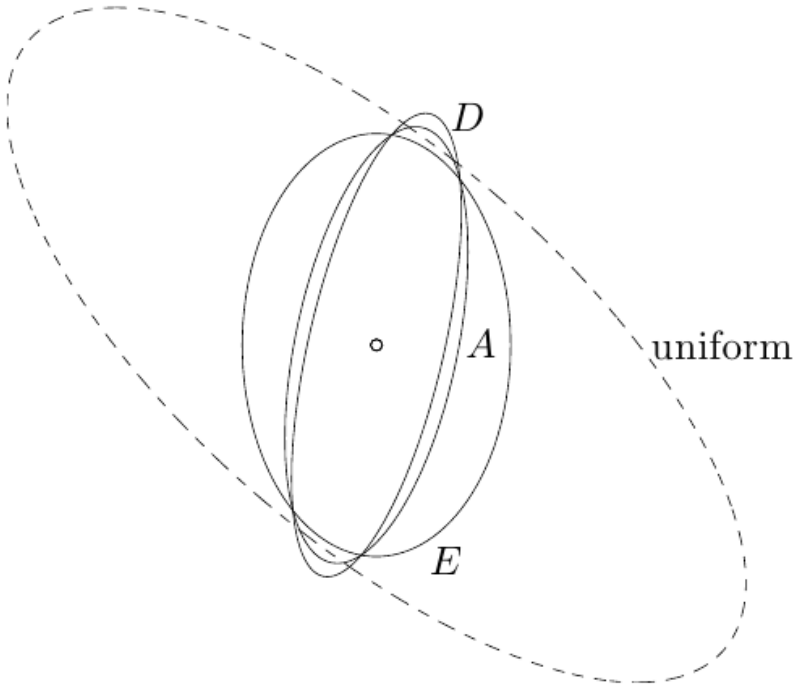
- "Maximize" Fisher Information Matrices
  $\rightsquigarrow$ Linear Algebra

Tools:

- Optimize some convex function of the eigenvalues
  $\rightsquigarrow$ Conic Programming

- For block designs, study the connectivity between treaments
  $\rightsquigarrow$ Graph theory

Geometric problems



Which confidence ellipsoid is the best ?

$\rightsquigarrow$ Most natural criteria can be expressed as a *convex function* of the eigenvalues of the *information matrix*

Eigenvalue optimization

Eigenvalue optimization problems:
For some concave function Φ, find a design $\mathsf{w} \in \mathcal{W}$ that maximizes

$$\Phi\Big(\lambda(\underbrace{M(\mathsf{w})}_{\text{Information matrix of the design w}})\Big),$$

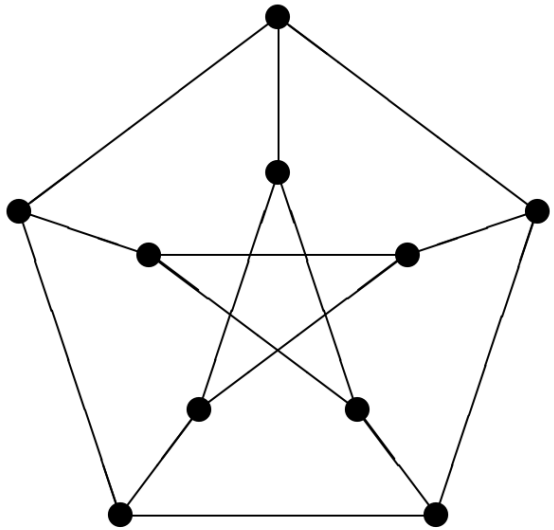
Often, this can be solved by *semidefinite programming*

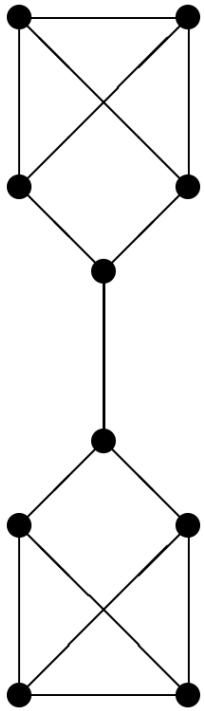
$$\begin{aligned} \min_{X} \quad & \langle C, X \rangle \\ \text{s. t.} \quad & \langle A_i, X \rangle = b_i, \quad i = 1, \dots, m \\ & X \succeq_{\mathbb{S}_n^+} 0 \end{aligned}$$

$$\begin{aligned} \max_{\mathsf{y}} \quad & \langle \mathsf{b}, \mathsf{y} \rangle \\ \text{s. t.} \quad & \sum_i y_i A_i \preceq_{\mathbb{S}_n^+} C \end{aligned}$$

Graph Theory

You must construct a network to connect 10 nodes with 15 edges.
Which graph is better ?





We will see that there is a nice relation between a "good network" and a "good experimental design".

Conclusion

- An exciting topic

  Optimal Design of Experiments offers a rare blend of linear algebra, convex analysis, and statistics.

  Friedrich Pukelsheim

- And in addition, a lot of combinatorics
- Many Applications in various areas

... Now, let's solve the Hotelling's weighing problem !