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Accumulation of Solid Particles in Convective Flows

Accumulation of solid particles suspended in unsteady convective flows is under theoretical investigation. The principal goal is to understand and interpret recent experiments by D. Schwabe [1,2]. Providing that volume particle concentration, nonisothermality, and relative size of particles is small, an effective single-fluid theoretical model is developed. The peculiarity of the obtained model is taking into account the distinction between fluid and particle inertia. This model is further applied to study particle accumulation in different flow setups: in a model oscillatory flow in a canal heated from below and subjected to the modulated gravity and in the Marangoni flow in a half-zone under microgravity conditions. These problems are investigated numerically by means of finite difference technique. We demonstrate, that the developed theoretical model properly describes generic features of particle accumulation in unsteady flows. Particularly, heavy particles tend to leave the centers of vortices, where the flow vorticity flow is maximal, and accumulate at their periphery. From numerical simulations in a floating zone, we try to clarify particle dynamics in Schwabe's setup.

INTRODUCTION

The behavior of small solid particles in isothermal fluid flows has been the subject of a great deal of study. The dynamics of a particle in a nonuniform unsteady flow is generally very complex [3]. In a trivial case of overdamped dynamics, the particle behaves as a fluid element. Under gravity a heavy particle settles down, so the velocity of fluid and the sedimentation velocity dominates its dynamics [4,5]. However, when sedimentation of particles does not play a principal role, the particle accumulation effect that is essential due to a difference in the inertial properties of the phases, becomes significant. Particularly this situation puts into effect in microgravity conditions.

Particle accumulation is such a behavior of the disperse particles in the flow when they completely leave some flow domains and concentrate in the other domains, forming agglomerates. Quite unusual accumulation of suspended particles in a flow was for the first time observed in the experiments by D. Schwabe [1,2], who studied thermocapillary flows in a liquid zone. The particles accumulate in disperse clouds which topology and temporal dynamics at some conditions become substantially nontrivial. Under subcritical conditions the clouds of particles form an asymmetric toroidal cloud, whereas at supercritical behavior the clouds rotate and resemble elongated bunches of particles.

Generally, particle accumulation have been considered theoretically in a number of studies (see, for example, [4,5] and references therein). For some simple two-dimensional flows, the analytical results describing major peculiarities of the phenomenon were obtained. However, the theoretical model of the accumulation effect observed by D. Schwabe is still absent. The goal of our research is to develop theory of the accumulation effect observed experimentally. Below we develop an appropriate theoretical model and apply it to study particle accumulation in convective flows of different nature.

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THEORETICAL MODEL

Initial Model and Basic Assumptions

Let us consider the behavior of small solid particles suspended in fluid (liquid or gas). We assume that particles are solid spheres of radius r_s, which is much smaller than the characteristic flow length-scale L. On a scale much greater than r_s the particles are regarded as a continuous medium with volume fraction $\varphi = 4/3\pi r_s^3 n$, where n is the number of particles per unit volume of medium. Denoting the volume fraction of the fluid by μ, we come to the relation $\mu + \varphi = 1$.We suppose that volume concentration of particles n is small so that one can neglect particle-particle interactions. It is also assumed that carrier phase is incompressible and the solid particles cannot deform as well as combine into agglomerates. After averaging over space the equations for mass, momentum and energy balance of both phases in the frames of the above assumptions can be written in dimensionless form as follows [6]

$$\partial(\mu\rho)/\partial t + \mathrm{div}(\mu\rho v) = 0, \quad \partial\varphi/\partial t + \mathrm{div}(\varphi v_s) = 0, \qquad (1)$$

$$\rho\,(dv/dt) = -\nabla p + \mathrm{Div}(\mu e) - \rho Ga\gamma + \varphi F,$$
$$\Delta(d_s v_s/dt) - \rho(dv/dt) = -(D-\rho)Ga\gamma - F, \qquad (2)$$

$$\mu\rho(dT/dt) = (1/Pr)\mathrm{div}(\mu\nabla T) +$$
$$(3/Pr)(L/r_s)^2\,\varphi(T_s - T),$$

$$DB(d_sT_s/dt) = -(3/Pr)(L/r_s)^2(T_s - T),$$
$$F = (9/2)(L/r_s)^2(v_s - v) + O(r_s/L), \qquad (3)$$

where the subscript "s" stands for the solid phase, dimensionless density ρ, time t, velocities v and v_s pressure p and temperatures T and T_s have been introduced using the following units respectively: the reference density of fluid ρ_0, (ρ_0L^2/η), $(\eta/(\rho_0L))$, $(\eta^2/(\rho_0L^2))$, the reference temperature difference θ (η is the dynamic viscosity of fluid). The vector γ is a unit vector, directed against gravity. The shear rate tensor e is given by the components $e_{ik} = (\partial v_k/\partial x_i + \partial v_i/\partial x_k)$, i, k = 1, 2, 3. Eqs.(1)-(4) contain the following dimensionless parameters: the Galilei number $Ga = gL^3\rho_0^2/\eta^2$, the Prandtl number $Pr = \eta c/\kappa$, the ratio of densities of phases $D = \rho_s/\rho_0$, the ratio of the specific heats of phases $B = c_s/c$ and the ratio of characteristic partice size to the flow length-scale (r_s /L). Since unsteady forces such as the Basset history term and the added mass force are negligibly small for small particles [3], the phase interaction term in Eq.(4) includes only the Stokes drag. We also do not explicitly take into account the Einstein correction term to the viscosity of fluid due to impurity. Such an approximation is valid in the accepted case of small volume concentration of solid phase. The time derivatives $d/dt = \partial/\partial t + (v\cdot\nabla)$, $d_s/dt = \partial/\partial t + (v_s\cdot\nabla)$ are different. These derivatives are used to denote the time derivatives following an element of the fluid and an element of the solid phase respectively. The equation of state we use here is typical for natural convection. We consider weak convection, which means that the variations of density due to non-isothermality are small. Assuming that the fluid density is a function of only temperature, we restrict ourselves to a linear law:

$$\rho = 1 - (\beta\theta)T, \quad \beta\theta \ll 1. \qquad (5)$$

Of special not is the form of equation for φ (see Eq.(1)). The density of solid phase is assumed to be constant. This assumption means that we neglect thermal expansion of the solid phase, which can be done due to the smallness of volume particle concentration.

Single-Fluid Approximation

The two-phase theoretical model (1)-(5) is quite complex and can be simplified. Taking into account the smallness of particle concentration, non-isothermality and relative size of particles, we retain only the leading terms in the momentum Eqs. (2). As a result we obtain:

$$\nabla p + \rho Ga\gamma = 0, \quad (D-1)Ga\gamma + (9/2)(L/r_s)^2(v_s - v) = 0,$$

The first relationship means that to the leading order the pressure disribution is pure hydrostatic, and the second one gives the relation for the velocities of phases

$$v_s = v - S\gamma, \quad S = (2/9)(r_s/L)^2(D-1)Ga, \qquad (6)$$

where S is the dimensionless sedimentation velocity. In the accepted approximation the particle velocity is a sum of the constant sedimentation velocity and the fluid velocity. Taking into account Eq.(6), we obtain in the next order the momentum equation for the fluid:

$$dv/dt = -\nabla p' + \Delta v + (\beta\theta T - (D-1)\,\varphi)Ga\gamma, \qquad (7)$$

where p' is the convective addition to the hydrostatic pressure. According to the idea of the Boussinesq approximation we can state that at large values of the Galilei number and small density inhomogeneities we can consider the density of medium to be constant everywhere except for the buoyancy force, where these small inhomogeneities are multiplied by Ga. By introducing the thermal Grashof number $Gr = \beta\theta Ga$ and its concentration analog $Gc = \varphi_0(D-1)Ga$ (where φ_0 is the reference value of the particle volume fraction) we rewrite Eq.(7) in the form:

$$\partial v/\partial t + (v\cdot\nabla)v = -\nabla p + \Delta v + GrT\gamma - Gc\,\varphi\gamma,$$

where the prime for p is omitted and the volume fraction is normalized by φ_0. From Eq.(3) we obtain to the leading order $T_s =$

T. We restrict our theory to this approximation. The latter means that for small particles ($(r_s/L) << 1$) the time to equlibiriate the fluid and particle temperatures is much less than viscous hydrodynamic time scale. Thus, the energy equation reads:

$$\partial T/\partial t + (v\cdot\nabla)T = (1/Pr)\,\Delta T.$$

It follows from Eq.(1) for fluid phase, that to the leading order the fluid flow is incompresible: div v = 0. This fact together with (6) manifests that the accepted approximation is too rough to describe particle accumulation, since the particle velocity field is solenoidal div $v_s = 0$, which means that there are no any sources or sinks in the field vs. Let us take into account the difference in the inertial properties of phases. We assume the fluid velocity to be given and rewrite Eq.(6) in the form:

$$v_s = v - S\gamma + v_s',$$

where v_s' is a small correction term to be found. Substituting this expression in Eq.(2) for vs we obtain:

$$v_s' = -(2/9)(r_s/L)^2((D-1)dv/dt - DS(\gamma\cdot\nabla)v + GrT\gamma).$$

The physical processes of sedimentation and backward influence of disperse phase onto convective flow, which are governed by the parameters S and Gc respectively, do not play a significant role for accumulation. We can neglect these phenomena if thermal convection, defined by Gr, is strong enough:

$$(r_s/L)^2(D-1) << \beta\theta,\quad \varphi_0(D-1) << \beta\theta.$$

Finally, we arrive at a set of equations describing accumulation of small solid particles in an unsteady convective flow:

$$\partial v/\partial t + (v\cdot\nabla)v = -\nabla p + \Delta v + GrT\gamma,$$
$$\text{div } v = 0, \tag{8}$$

$$\partial T/\partial t + (v\cdot\nabla)T = (1/Pr)\Delta T, \tag{9}$$

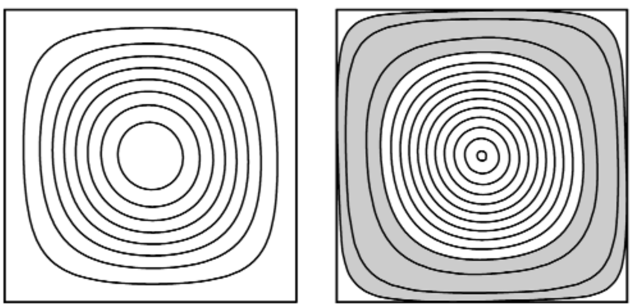

Fig. 1. Isolines of stream function (left) and particle concentration (right) in a steady state in the absence of gravity modulation.

$$\partial\varphi/\partial t + \text{div}(\varphi v_s) = 0,$$
$$v_s = v - \alpha\,(dv/dt + Gr/(D-1)T\gamma), \tag{10}$$

where the small parameter $\alpha = (2/9)(r_s/L)^2(D-1)$ stands for the difference in the inertial properties of phases. In the limiting case of heavy particles ($D >> 1$) this parameter reduces to the Stokes number $St = (2/9)(r_s/L)^2D$, the ratio of particle response time to a characteristic timescale of the flow. It is seen, that the accumulation effect vanishes in the case when both phases are of the same density ($D = 1$).

APPLICATION OF THEORETICAL MODEL

Accumulation under gravity modulation

Consider a model problem of particle accumulation in a convective flow in a square canal subjected to modulation of gravity. We suppose that a suspension completely fills the cavity and restrict our consideration to the two-dimensional case. We introduce Cartesian reference frame, which moves together with the cavity and has the origin in the left lower corner of the square. The axes x and y are aligned along the sides of the square. The behavior of two-phase system is described by the single-fluid model (8)-(10), where the replacement $Gr\,\gamma \rightarrow Gr\,(1 + a \sin(\omega t))\,\gamma$ is done. Here a and ω are the dimensionless amplitude and the frequency of modulation, the unit vector $\gamma = (0,1,0)$.We investigate the role of only one term in Eq.(10), accounting for the difference in the inertia properties of phases, i.e. we suppose that

$$v_s = v - \alpha\,(dv/dt).$$

On the border $\partial\Gamma$ of the canal Γ we impose the no-slip conditions for the velocity of fluid $v\,|_{\partial\Gamma} = 0$. We assume, that upper and lower boundaries are kept at constant, but different temperatures: $T = 0$ at $y = 0$ and $T = -1$ at $y = 1$, whereas the sidewalls are supposed to be adiabatic: $(\partial T/\partial x) = 0$ at $x = 0$ and $x = 1$. The initial conditions correspond to the state with uniform distribution of particles and a flow in the form of a vortex in the center of the cavity. The set of nonlinear partial differential equations with the boundary conditions is solved numerically by means of finite difference technique. The equations of fluid motion are treated in terms of stream function and vorticity [7]; the transfer equation for particles is integrated according to the scheme, proposed by LeVeque [8]. Numerical simulation was performed for the following values of the governing parameters: $Gr = 3000$, $Pr = 1$, $\alpha = 0.0005$. The calculations show that in the absence of gravity modulation ($a = 0$) one-vortex convective flow leads to migration of heavy particles to the periphery due to the inertia. With time, the particle concentration in the center of the convective vortex decreases and tends to zero. As a result, the particles are concentrated at the outer border of the vortex, forming a cloud of a ring-like shape (Figure 1). Dark zone corresponds to the regions where particle concentration exceeds its initial value. The ring thickness and concentration of the particles located in this ring, which is higher than the initial concentration, depend on intensity of the convective flow: larger the flow intensity, thinner the ring and higher the concentration of particles in it. In the case of gravity modulation, the convective flow is oscillatory. The calculations for this case were made for values of modulation parameters corresponding to the instability zone of subharmonic resonance ($\alpha = 35$, $\omega = 50$), when oscillations occur with the period twice larger than the modulation period. As the calculations show, in this case most part of time the convective flow is one-vortex and the particles form a cloud at the vortex periphery, however the inner border of the cloud is no more symmetric; it takes the shape of a deformed ring. When the vorticity changes the sign, the transformation of convective pattern occurs at which the flow changes its direction to the opposite one. Such a flow transformation is accompanied by formation of the three-vortex patterns (Figure 2). At such moments the shape of the cloud is subjected to the largest deformation. When the flow again becomes one-vortex, the cloud takes an almost ring-like shape again. Principal features of the particle behavior in the unsteady flow are similar to those observed in the case of no modulation: particles leave the centers of the vortices, where the flow vorticity is maximal, and accumulate at the periphery of vortices. Unsteady patterns, which are formed by the particles during evolution, reflect the instantaneous structure of the flow carrying the particles.

Accumulation in a fluid zone

In the last part, the developed theoretical model is applied to the Schwabe setup. We study the process of particle accumulation in a 3D liquid half zone, heated from above under microgravity conditions. Free surface is assumed to be nondeformable and adiabatic. In the accepted approximations, the system is described by Eqs. (8)-(10) with Gr = 0; the boundary conditions are formulated as follow:

$$z = 0:\ v_r = 0,\ v_\theta = 0,\ v_z = 0,\ T = 0;$$
$$z = A:\ v_r = 0,\ v_\theta = 0,\ v_z = 0,\ T = 1; \qquad (11)$$

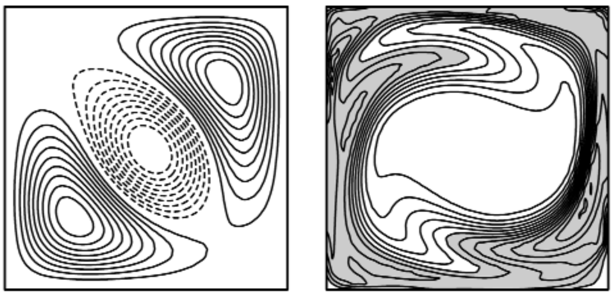

Fig. 2. Isolines of stream function (left) and particle concentration (right), corresponding to the threevortex regime under gravity modulation.

$$r = 1:\ \partial v_\theta/\partial r - v_\theta = -\,\mathrm{Ma/Pr}\ \partial T/\partial\theta,$$
$$\partial v_z/\partial r = -\,\mathrm{Ma/Pr}\ \partial T/\partial z,\ \partial T/\partial r = 0, \qquad (12)$$

where v_r, v_θ, v_z are the components of the fluid velocity in cylindrical reference frame. Thermocapillary flow is generated through the boundary conditions on a free sufrace, governed by the Marangoni number; A is the aspect ratio. The initial state corresponds to the quiescent state with uniform distribution of particles. The set of nonlinear equations and boundary conditions (8)-(12) was integrated numerically in terms of primitive variables [7] for a fixed Pr = 1 and A = 1. Under subcritical conditions the flow is axisymmetric, and the particles accumulate in the periphery of a toroidal vortex. In the vertical cross-section the particle structure looks like a ring (Figure 3). Topologically, the same result was found in a model problem discussed above. However, there is a difference because of the free surface. As well as in the experiments, the particle ring is not symmetric. It is significantly thick near the axis of the fluid zone, and extremely thin near the free surface. Such an asymmetry is caused by large gradients of velocity near the free surface.

At supercritical conditions, the symmetry of the flow is broken. For our particular values of the parameters A and Pr there appears time-dependent azimuthal velocity component [9]. The instability takes the form of a pair of a hydrothermal waves travelling azimuthally. As a result of instability, the flow becomes unsteady and is no longer axisymmetric. Due to numerical difficulties our numerical scheme does not allow us to integrate the equation for particle concentration for a long enough time. The reason is that our technique ensures the divergence for the fluid velocity to be quite small but not perfectly zero. Such accuracy is enough for studying the pure thermocapillary convection without particles. For particles this condition becomes important: although small, nonzero divergence of fluid velocity, corresponding to a false sink or source of particle concentration, distorts particle dynamics with time.

However, the obtained data allow us to guess what quatitatively happens: due to three-dimensional character of the flow particles behave in much the same way as in Schwabe's setup: disperse structures in a vertical cross-section are no longer in

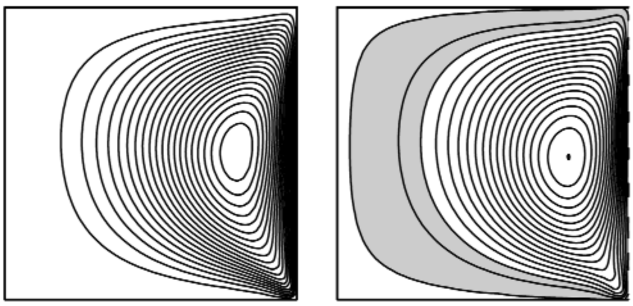

Fig. 3. Isolines of stream function (left) and particle concentration (right) in a steady state at Ma = 1000.

the form of a closed unsteady ring. Contrastingly, the azimuthal velocity component tears the particle ring, and it transforms to an elongated bunch of particles. Such a bunch of particles rotate with the frequency corresponding to oscillations of the Marangoni flow.

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