1

$$-x_1 + 2x_2 + 7x_3 - 4x_4 = 1$$
$$x_2 - x_3 - 2x_4 = 1$$
$$-2x_1 + 7x_2 + 11x_3 - 14x_4 = 0$$

$$A = \begin{pmatrix} -1 & 2 & 7 & -4 \\ 0 & 1 & -1 & -2 \\ -2 & 7 & 11 & -14 \end{pmatrix}$$ Koeffizienten-Matrix

$$b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$ „rechte Seite"

Bild-Kern-Algorithmus

$$\begin{array}{cccc|cccc} -1 & 2 & 7 & -4 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 & 0 & 0 \\ -2 & 7 & 11 & -14 & 0 & 0 & 1 & 0 \\ & & & & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & 1 & 2 & 7 & -4 \\ 0 & 1 & -1 & -2 & 0 & 1 & 0 & 0 \\ -2 & 3 & -3 & -6 & 0 & 0 & 1 & 0 \\ & & & & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{cccc|cccc} -1 & 0 & 0 & 0 & 1 & 2 & 9 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 2 \\ -2 & 3 & 0 & 0 & 0 & 0 & 1 & 0 \\ & & & & 0 & 0 & 0 & 1 \end{array}$$

BILD

KERN

AR | R
RANG=2 | DEFEKT=2

$ARy = b \qquad Ry = x$ ②

$$\begin{aligned} -y_1 &= 1 \\ y_2 &= 1 \\ -2y_1 + 3y_2 &= 0 \end{aligned}$$

nicht lösbar (von "oben nach unten" lösen)

$$b = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$$

$$\begin{aligned} -y_1 &= 1 \\ y_2 &= 1 \\ -2y_1 + 3y_2 &= 5 \end{aligned} \quad \Rightarrow \quad \begin{aligned} y_1 &= -1 \\ y_2 &= 1 \end{aligned}$$

$$x = \underbrace{\begin{pmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} -1 \\ 1 \\ y_3 \\ y_4 \end{pmatrix}}_{y} = \underbrace{\underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{\text{spezielle Lösung}} \underbrace{+ y_3 \begin{pmatrix} 4 \\ 1 \\ 1 \\ 0 \end{pmatrix} + y_4 \begin{pmatrix} 0 \\ 2 \\ 0 \\ 1 \end{pmatrix}}_{\text{KERN}}}_{\text{allgemeine Lösung}}$$

$$\min \|Ax-b\|^2$$

$$= \min \langle Ax-b \mid Ax-b \rangle$$

$$= \min \left(x^T A^T A x - 2(Ax)^T b + b^T b \right)$$

Differenzieren:

$$0 = 2A^TAx - 2A^Tb$$

$$\Leftrightarrow \boxed{A^TAx = A^Tb}$$ **Normalgleichung**

$$x_1 + x_2 t_i + x_3 t_i^2 = b_i$$

$$\begin{pmatrix} 1 & t_1 & t_1^2 \\ \vdots & \vdots & \vdots \\ 1 & t_n & t_n^2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$$A \qquad x \quad = \quad b$$

$$A^TA = \begin{pmatrix} 5 & -16 & -24 & 32 \\ -16 & 54 & 90 & -108 \\ -24 & 90 & 171 & -180 \\ 32 & -108 & -180 & 216 \end{pmatrix}$$

$$A^Tb = \begin{pmatrix} -1 \\ 3 \\ 6 \\ -6 \end{pmatrix}$$

<u>Lösung:</u> $\begin{pmatrix} \\ \\ \end{pmatrix} + \lambda \begin{pmatrix} \\ \\ \end{pmatrix} + \mu \begin{pmatrix} \\ \\ \end{pmatrix}$

$$\operatorname{rang}(A^TA) = \operatorname{rang}(A) = 2$$

$$4 - 2 = \operatorname{defekt}(A^TA) = 2$$