$$\mathcal{F}[f'](\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f'(x) e^{-i\omega x}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\left(\left[f(x) e^{-i\omega x}\right]_{-\infty}^{\infty} + i\omega \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\,dx \right)$$

$$= i\omega \cdot \mathcal{F}[f](\omega)$$

$$\mathcal{F}[f](\omega) \cdot \mathcal{F}[g](\omega) \cdot \sqrt{2\pi}$$

$$= \frac{1}{\sqrt{2\pi}}\left(\int_{-\infty}^{\infty} f(z) e^{-i\omega z}\,dz\right)\cdot\left(\int_{-\infty}^{\infty} g(y) e^{-i\omega y}\,dy\right)$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(z)\cdot g(y)\, e^{-i\omega z}\cdot e^{-i\omega y}\,dy\,dz$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(z) g(y)\, e^{-i\omega(z+y)}\,dy\,dz \qquad \begin{array}{l|l} y = t & t = y \\ z = x - t & x = z + y \end{array}$$

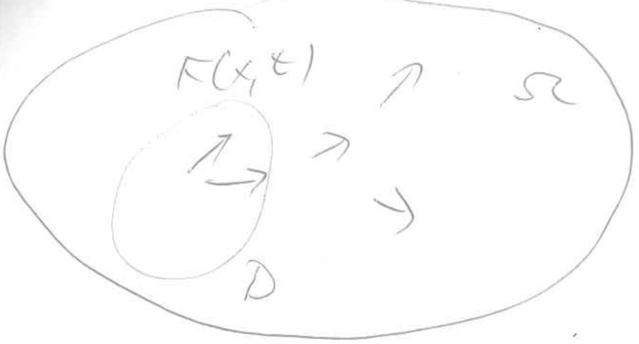
$$\frac{d(t,x)}{d(y,z)} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\underbrace{\int_{-\infty}^{\infty} f(x-t) g(t)\, e^{-i\omega x}\,dt}_{f*g}\,dx$$

$$= \mathcal{F}[f*g](\omega)$$

erklären: Moving window Faltungssatz

Herleitung der Diffusions-/Wärmeleitungsgleichung



$F(x,t) \in \mathbb{R}^4$
Richtung/Stärke der Wärme-/Stoffbewegung
$f(x,t,u(x,t)) \in \mathbb{R}$
Reaktionsrate
$u(x,t) \in \mathbb{R}$ Stoff/Wärmemenge

$$\int_D u(x,t)\,dx$$

Stoffmenge in D

$$\frac{d}{dt}\int_D u(x,t)\,dx = -\oint_{\partial D} \vec{F}(x,t)\,d\vec{\nu}(x) + \int_D f(x,t,u(x,t))\,dx$$

‖ Satz von Gauß

$$\int_D \operatorname{div}_x F(x,t)$$

div = Summe der Ableitungen $\sum \frac{\partial}{\partial x_i} F_i$

$$\left.\begin{aligned} \frac{\partial u}{\partial t} &= -\operatorname{div}_x F + f \\ &\Downarrow \\ F &= -\lambda \nabla u(x,t) \end{aligned}\right\} \quad u_t = \lambda \Delta u + f$$

$\boxed{\text{Summe der 2. Ableitungen}}$

Mehrdim. Faltung
Mehrdim. F-Trafo

Lösung der Diffusionsgleichung

$$\boxed{u_t = \Delta u}$$

$\mathcal{F}$ nur in x

$$\frac{\partial}{\partial t} \mathcal{F}[u] = \left(-\sum_{i=1}^{n} \omega_i^2\right) \cdot \mathcal{F}[u]$$

gewöhnliche DGL in $\mathcal{F}[u]$

$f' = af \Rightarrow f = c \cdot e^{at}$

$$\mathcal{F}[u] = \hat{c}(\omega) \cdot e^{-\left(\sum_{i=1}^{n} \omega_i^2\right) \cdot t}$$

$$\Rightarrow u(x,t) = \mathcal{F}^{-1}\Big[\underbrace{\hat{c}(\omega)}_{\mathcal{F}[f]} \cdot \underbrace{e^{-\left(\sum_{i=1}^{n} \omega_i^2\right) t}}_{\mathcal{F}[g]}\Big]$$

$\mathcal{F}[f * g] = \left(\sqrt{2\pi}\right)^n \mathcal{F}[f] \cdot \mathcal{F}[g]$

$\Rightarrow \mathcal{F}^{-1}[\mathcal{F}[f] \cdot \mathcal{F}[g]] = \frac{1}{\sqrt{2\pi}^n} f * g$

$$= \frac{1}{(\sqrt{2\pi})^n} \mathcal{F}^{-1}[\hat{c}(\omega)] * \mathcal{F}^{-1}\left[e^{-\left(\sum_{i=1}^{n} \omega_i^2\right) \cdot t}\right]$$

$\Rightarrow \frac{1}{(2t)^{n/2}} \cdot e^{-\frac{\|x\|^2}{4t}}$

$$= \frac{1}{(4\pi t)^{n/2}} \, c(x) * e^{-\frac{\|x\|^2}{4t}}$$

$$= \frac{1}{(4\pi t)^{n/2}} \int_{\mathbb{R}^n} c(y) \cdot e^{-\frac{\|x-y\|^2}{4t}} \, dy$$