

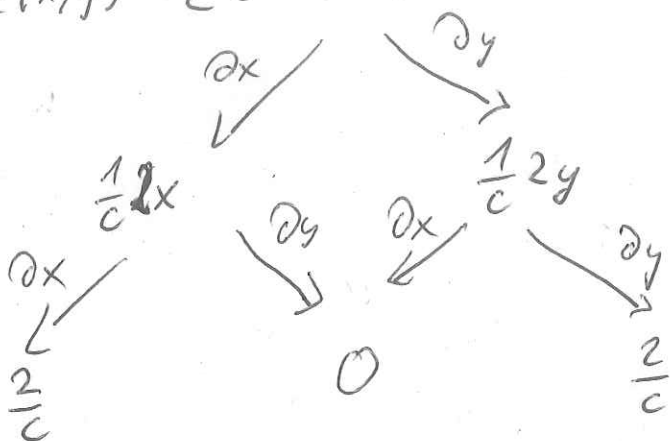
# Aufgabe 1:

$$f(x,y) = \frac{1}{c}(x^2 + y^2)$$

$$df = \frac{2}{c}x dx + \frac{2}{c}y dy$$

$$d^2f = \frac{2}{c}dx^2 + \frac{2}{c}dy^2$$

a)



b)  $f(x,y) = x^2 + 2xy + \cos(xy)$



$$2x + 2y + y \sin(xy)$$

$$2x + x \sin(xy)$$

~~5/1~~

$$\frac{\partial}{\partial x} \rightarrow 2 + y^2 \cos(xy)$$

$$\frac{\partial}{\partial y} \rightarrow 2 + \sin(xy) + y \cdot x \cdot \cos(xy)$$

$$\frac{\partial}{\partial y} \rightarrow x^2 \cos(xy)$$

$$df = (2x + 2y + y \sin(xy)) dx + (2x + x \sin(xy)) dy$$

$$d^2f = (2 + y^2 \cos(xy)) dx^2 + 2 \cdot (2 + \sin(xy) + xy \cos(xy)) \cdot dx dy + (x^2 \cos(xy)) \cdot dy^2$$

~~cn f(x,y) = x^2 + y^2~~

$$c) f(x) = \ln(x+y) \cdot \cos(xy)$$

$$\frac{\partial}{\partial x} \left( \frac{1}{x+y} \cdot \cos(xy) \right)$$

 $\frac{\partial}{\partial y}$ 

$$\frac{1}{x+y} \cdot \cos(xy)$$

$$y \ln(x+y) \sin(xy)$$

$$-x \cdot \ln(x+y) \sin(xy)$$

 $\frac{\partial}{\partial x}$ 
 $\frac{\partial}{\partial y}$ 
 $\frac{\partial}{\partial x}$ 
 $\frac{\partial}{\partial y}$ 

$$\frac{-\cos(xy)}{(x+y)^2} - \frac{2y \sin(xy)}{x+y}$$

$$-y^2 \cos(xy) \ln(x+y)$$

$$\frac{-\cos(xy)}{(x+y)^2} - \ln(x+y) \sin(xy)$$

$$- \frac{x \sin(xy)}{x+y} - \frac{y \sin(xy)}{x+y}$$

$$-xy \cos(xy) \ln(x+y)$$

$$\frac{-\cos(xy)}{(x+y)^2} - \frac{(2x \sin(xy))}{x+y}$$

$$-x^2 \cos(xy) \ln(x+y)$$

Aufgabe 2:

$$z = x^3 + y^3 + 2$$

$$\frac{\partial z}{\partial x} = 3x^2$$

$$\frac{\partial z}{\partial y} = 3y^2$$

a)

$$\frac{dy}{dx} = - \frac{(\partial z / \partial x)}{(\partial z / \partial y)} = - \left( \frac{x}{y} \right)^2$$

b)  $z = xy + y^2 + \ln(y)$

$$\frac{\partial z}{\partial x} = y$$

$$\frac{\partial z}{\partial y} = x + 2y + \frac{1}{y}$$

$$\begin{aligned} \frac{dy}{dx} &= - \frac{(\partial z / \partial x)}{(\partial z / \partial y)} = \frac{-y}{x + 2y + \frac{1}{y}} \\ &= \frac{-y^2}{xy + 2y^2 + 1} \end{aligned}$$

Aufgabe 3 wie beim impliziten Ableiten:

$$z = \left(p + \frac{a}{V^2}\right)(V-b) - RT$$

$$\left(\frac{\partial z}{\partial T}\right)_p = -R \quad \left(\frac{\partial z}{\partial V}\right)_p = p + \frac{a}{V^2} - (2a(V-b)) : V^3$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p + \frac{a}{V^2} - (2a(V-b)) : V^3}$$

Aufgabe 4:

Formel 1: Totales Differential

Formel 2: einfache Rechnung

Formel 3: implizite Ableitung + Formel 2

Formel 4: Satz von Schwarz

$$\left(\frac{\partial x}{\partial y}\right)_z = - \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$

$$\Leftrightarrow - \left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$

$$\Leftrightarrow -1 = \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x \frac{1}{\left(\frac{\partial x}{\partial y}\right)_z}$$

$$= \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_z$$

Rollen von y und z vertauschen