Aufgabe 1:

a)

$$\begin{tikzcd}[column sep=small]
 & & f(x,y) = \frac{1}{c}(x^2+y^2) \arrow[dl, "\partial x"'] \arrow[dr, "\partial y"] & & \\
 & \frac{1}{c}2x \arrow[dl, "\partial x"'] \arrow[dr, "\partial y"] & & \frac{1}{c}2y \arrow[dl, "\partial x"'] \arrow[dr, "\partial y"] & \\
\frac{2}{c} & & 0 & & \frac{2}{c}
\end{tikzcd}$$

$$df = \frac{2}{c}x\,dx + \frac{2}{c}y\,dy$$

$$d^2f = \frac{2}{c}dx^2 + \frac{2}{c}dy^2$$

b)

$$\begin{tikzcd}[column sep=small]
 & & f(x,y) = x^2 + 2xy - \cos(xy) \arrow[dl, "\partial x"'] \arrow[dr, "\partial y"] & & \\
 & 2x+2y+y\sin(xy) \arrow[dl, "\partial x"'] \arrow[dr, "\partial y"] & & 2x + x\sin(xy) \arrow[dl, "\partial x"'] \arrow[dr, "\partial y"] & \\
2+y^2\cos(xy) & & 2+\sin(xy) + y\cdot x\cdot\cos(xy) & & x^2\cos(xy)
\end{tikzcd}$$

$$df = (2x+2y+y\sin(xy))\,dx + (2x + x\sin(xy))\,dy$$

$$d^2f = (2+y^2\cos(xy))\,dx^2 + 2\cdot(2+\sin(xy)+xy\cos(xy))\cdot dx\,dy + (x^2\cos(xy))\cdot dy^2$$

~~$d^2f(x,y) = xy + y^2$~~

c) $f(x) = \ln(x+y) \cdot \cos(xy)$

∂x:

$$\frac{1}{x+y} \cdot \cos(xy) - y \ln(x+y) \sin(xy)$$

∂y:

$$\frac{1}{x+y} \cdot \cos(xy) - x \cdot \ln(x+y) \sin(xy)$$

∂x:

$$-\frac{\cos(xy)}{(x+y)^2} - \frac{2y\sin(xy)}{x+y} - y^2 \cos(xy) \ln(x+y)$$

∂y:

$$\frac{-\cos(xy)}{(x+y)^2} - \ln(x+y)\sin(xy) - \frac{x\sin(xy)}{x+y} - \frac{y\sin(xy)}{x+y} - xy\cos(xy)\ln(x+y)$$

∂x:

$$\frac{-\cos(xy)}{(x+y)^2} - \frac{(2x\sin(xy))}{x+y} - x^2\cos(xy)\ln(x+y)$$

∂y

Aufgabe 2:

$z = x^3 + y^3 + 2$ $\qquad \frac{\partial z}{\partial x} = 3x^2 \qquad \frac{\partial z}{\partial y} = 3y^2$

a)

$$\frac{dy}{dx} = -\frac{(\partial z/\partial x)}{(\partial z/\partial y)} = -\left(\frac{x}{y}\right)^2$$

b) $z = xy + y^2 + \ln(y)$ $\qquad \frac{\partial z}{\partial x} = y \qquad \frac{\partial z}{\partial y} = x + 2y + \frac{1}{y}$

$$\frac{dy}{dx} = -\frac{(\partial z/\partial x)}{(\partial z/\partial y)} = \frac{-y}{x+2y+1/y}$$

$$= \frac{-y^2}{xy+2y^2+1}$$

Aufgabe 3 wie beim impliziten Ableiten:

$$z = \left(p + \frac{a}{V^2}\right)(V-b) - RT$$

$$\left(\frac{\partial z}{\partial T}\right)_p = -R \qquad \left(\frac{\partial z}{\partial V}\right)_p = p + \frac{a}{V^2} - (2a(V-b)) : V^3$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_p = \frac{R}{p + \frac{a}{V^2} - (2a(V-b)) : V^3}$$

Aufgabe 4. Formel 1: Totales Differential
Formel 2: einfache Rechnung
Formel 3: implizite Ableitung + Formel 2

Formel 4: Satz von Schwarz

$$\left(\frac{\partial x}{\partial y}\right)_z = -\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$

$$\Leftrightarrow -\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x$$

$$\Leftrightarrow -1 = \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x \frac{1}{\left(\frac{\partial x}{\partial y}\right)_z}$$

$$= \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x \left(\frac{\partial y}{\partial x}\right)_z$$

Rollen von y und z vertauschen